

Workshop in class

Numerical Methods

Date: 2024/06/02

Topic:

Gauss Elimination Method.

LU Factorization Methods

Iterative Solution of Linear Systems

Workshop Objective:

This workshop aims to have students apply the concepts of numerical solving of linear systems to practical problems. Students will work on solving specific problems and then present their solutions in a collaborative environment.

Total duration: 90 minutes

PROBLEM 1:

Consider the linear 2×2 system


$$\mathbf{Ax} = \mathbf{b}, \quad \mathbf{A} = \begin{bmatrix} \epsilon & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}, \quad \mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

where $\epsilon > 0$ is a very small constant.

- Solve by Gauss elimination without partial pivoting.
- Solve by Gauss elimination with partial pivoting. Compare the results and discuss their validity.

Credit: (a) 1 point, (b) 1point: 2 points

PROBLEM 2:

 Using Doolittle's method solve each linear system $\mathbf{Ax} = \mathbf{b}$.

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 3 & -1 \\ -4 & 5 & 6 & 0 \\ 4 & -2 & -1 & -1 \\ -2 & 13 & -12 & 18 \end{bmatrix}, \quad \mathbf{b} = \begin{Bmatrix} 0 \\ 7 \\ -11 \\ 65 \end{Bmatrix}$$

Credit: 2 points

PROBLEM 3:

 For each linear system find the components of the first vector generated by the Jacobi method.

$$\begin{bmatrix} 3 & 0 & 1 & -1 \\ 0 & -4 & 2 & 1 \\ 1 & -2 & 5 & 0 \\ -1 & 3 & 2 & 6 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ -4 \\ 16 \end{bmatrix}, \quad \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Credit: 3 points

PROBLEM 4:

 For each linear system find the components of the first vector generated by the Gauss-Seidel method.

$$\begin{bmatrix} 6 & 3 & -2 & 0 \\ 3 & 7 & 1 & -2 \\ -2 & 1 & 8 & 3 \\ 0 & -2 & 3 & 9 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 \\ -5 \\ 1 \\ 20 \end{bmatrix}, \quad \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Credit: 3 points

Good Teamwork!!!