

Resolución Deber

① Resuelva

$$y''' + 12y'' + 36y' = 0$$

$$m^3 + 12m^2 + 36m = 0$$

$$m(m^2 + 12m + 36) = 0$$

$$m = 0$$

$$m^2 + 12m + 36 = 0$$

$$m = \frac{-12 \pm \sqrt{144 - 4(36)}}{2(i)}$$

$$m = \frac{-12 \pm \sqrt{144 - 144}}{2}$$

$$m = -\frac{12}{2} + 0 \Rightarrow m = -6$$

$$m = -\frac{12}{2} - 0 \Rightarrow m = -6$$

$$y = C_1 e^{0x} + C_2 e^{-6x} + C_3 e^{-6x} x$$

$$y = C_1 + C_2 e^{-6x} + C_3 x e^{-6x}$$

$$y' = -6Ce^{-6x} - 6(3x)e^{-6x} + 3e^{-6x}$$

$$y'' = 366e^{-6x} + 363xe^{-6x} - 63e^{-6x} - 63e^{-6x}$$

$$y'' = 36(2)e^{-6x} + 36(3)xe^{-6x} - 12(3)e^{-6x}$$

$$0 = C_1 + C_2 + 0$$

$$1 = -6C_2 + C_3$$

$$-7 = 36C_2 - 12C_3$$

$$0 = C_1 + C_2 \quad (1)$$

$$\begin{array}{l} 6 \quad 4 = -6C_2 + C_3 \Rightarrow 6 = -36C_2 + 6C_3 \\ -7 = 36C_2 - 12C_3 \Rightarrow -7 = 36C_2 - 12C_3 \\ -1 = -6C_3 \end{array}$$

$$C_3 = \frac{1}{6}$$

$$1 = -6C_2 + \left(\frac{1}{6}\right)$$

$$GC_2 = -1 + \frac{1}{6}$$

$$\begin{array}{r} m^2 + 12m + 36 \\ \downarrow \qquad \qquad \downarrow \\ m \qquad \qquad 6 \end{array}$$

$$(m+6)(m+6)=0$$
$$m=-6 \quad m=-6$$

$$6C_2 = -\frac{5}{8}$$

$$C_2 = -\frac{5}{36}$$

$$C_1 + C_2 = 0$$

$$C_1 = \frac{5}{36}$$

$$y = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}xe^{-6x}$$

$$\textcircled{2} \quad y''' + 2y'' - 5y' - 6y = 0$$

$$m^3 + 2m^2 - 5m - 6 = 0$$

$$\begin{array}{r|l} 1 & 1 \quad 2 \quad -5 \quad -6 \\ -1 & -1 \quad -1 \quad +6 \\ \hline 1 & 1 \quad -6 \end{array}$$

$$(m+1)(m^2+m-6)$$

$$(m+1)(m+3)(m-2) = 0$$

$$m = -1 \quad m = -3 \quad m = 2$$

$$y = C_1e^{2x} + C_2e^{-x} + C_3e^{-3x}$$

$$y' = 2C_1e^{2x} - C_2e^{-x} - 3C_3e^{-3x}$$

$$y'' = 4C_1e^{2x} + C_2e^{-x} + 9C_3e^{-3x}$$

$$\begin{cases} 0 = C_1 + C_2 + C_3 & \textcircled{1} \\ 0 = 2C_1 - C_2 - 3C_3 & \textcircled{2} \\ 1 = 4C_1 + C_2 + 9C_3 & \textcircled{3} \end{cases}$$

$$0 = C_1 + C_2 + C_3$$

$$0 = 2C_1 - C_2 - 3C_3$$

$$0 = 3C_1 - 2C_3 \quad \textcircled{4}$$

$$0 = 2C_1 - C_2 - 3C_3$$

$$1 = 4C_1 + C_2 + 9C_3$$

$$1 = 6C_1 + 6C_3 \quad \textcircled{5}$$

$$\textcircled{4} \wedge \textcircled{5}$$

$$3/ \quad 0 = 3C_1 - 2C_3 \Rightarrow 0 = 9C_1 - 6C_3$$

$$/ \quad 1 = 6C_1 + 6C_3 \Rightarrow \underline{1 = 6C_1 + 6C_3}$$

$$1 = 15C_1$$

$$C_1 = \frac{1}{15}$$

$$0 = 3C_1 - 2C_3$$

$$3C_1 = 2C_3$$

$$3\left(\frac{1}{15}\right) = 2C_3$$

$$2C_3 = \frac{1}{5}$$

$$C_3 = \frac{1}{10}$$

$$0 = C_1 + C_2 + C_3$$

$$0 = \frac{1}{15} + C_2 + \frac{1}{10}$$

$$C_2 = -\frac{1}{15} - \frac{1}{10}$$

$$C_2 = -\frac{1}{6}$$

$$y = \frac{1}{15} e^{2x} - \frac{1}{6} e^{-x} + \frac{1}{10} e^{-3x}$$

$$y^{(4)} + 2y'' - y' + 2y = 0$$

$$m^4 + 2m^2 - m + 2 = 0$$

$$(m^2 + am + b)(m^2 + cm + d) = 0$$

$$m^4 + cm^3 + dm^2 + am^3 + acm^2 + adm + bm^2 + bcm + bd = 0$$

$$m^4 + (a+c)m^3 + (ac+b+d)m^2 + (ad+bc)m + bd = 0$$

$$bd = 2$$

$$b = 1 \quad d = 2$$

$$a + c = 0$$

$$a = -c$$

$$ac + b + d = 2$$

$$ad + bc = -1$$

$$(-c)c + 1 + 2 = 2$$

$$-c^2 + 1 + 2 = 2$$

$$-c^2 = 2 - 2 - 1$$

$$-c^2 = -1$$

$$c^2 = 1$$

$$c = \pm 1$$

$$a = -1 \quad c = 1$$

$$(m^2 - m + 1)(m^2 + m + 2) = 0$$

$$m = \frac{-1 \pm \sqrt{1 - 4(1)}}{2}$$

$$m = \frac{-1 \pm \sqrt{1 - 8}}{2}$$

$$m = \frac{1 \pm \sqrt{3}}{2} i$$

$$m = \frac{1 \pm \sqrt{7}}{2} i$$

$$y = C_1 e^{\frac{1}{2}x} \cos \frac{\sqrt{3}}{2}x + C_2 e^{\frac{1}{2}x} \sin \frac{\sqrt{3}}{2}x + C_3 e^{\frac{1}{2}x} \cos \frac{\sqrt{7}}{2}x + C_4 e^{\frac{1}{2}x} \sin \frac{\sqrt{7}}{2}x$$

$$④ \quad y'' - 5y' = 2x^3 - 4x^2 - x + 6$$

$$m^2 - 5m = 0$$

$$m(m-5) = 0$$

$$m=0 \quad m-5=0$$

$$m=5$$

$$y_c = C_1 e^{5x} + C_2 e^{0x}$$

$$y_c = C_1 e^{5x} + C_2$$

$$y_p = Ax^3 + Bx^2 + Cx + D$$

$$y_p = Ax^3 + Bx^2 + Cx + D$$

$$y' = 4Ax^2 + 3Bx + C$$

$$y'' = 8Ax + 3B$$

$$8Ax + 3B - 5(4Ax^2 + 3Bx + C) = 2x^3 - 4x^2 - x + 6$$

$$-20A = 2$$

$$A = -\frac{1}{10}$$

$$12A - 15B = -4$$

$$-15B = -4 - 12\left(-\frac{1}{10}\right)$$

$$-15B = -4 + \frac{6}{5}$$

$$-15B = -\frac{14}{5}$$

$$B = \frac{14}{75}$$

$$6B - 10C = -1$$

$$-10C = -1 - 6B$$

$$-10C = -1 - 6\left(\frac{14}{75}\right)$$

$$-10C = -1 - \frac{98}{25}$$

$$-10C = -\frac{53}{25}$$

$$C = \frac{53}{250}$$

$$2C + D = 6$$

$$-5D = 6 - 2C$$

$$-5D = 6 - 2\left(\frac{53}{250}\right)$$

$$-5D = 6 - \frac{53}{125}$$

$$D = \frac{697}{625}$$

$$y_p = -\frac{1}{10}x^3 + \frac{14}{75}x^2 - 5x + 16$$

$$y = C_1 e^{5x} + C_2 - \frac{1}{10}x^3 + \frac{14}{75}x^2 + \frac{53}{250}x - \frac{697}{625}$$

$$5) \quad y'' + 2y' + y = \text{Sen } x + 3 \text{Cos } 2x$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1 \quad m = -1$$

$$y_c = C_1 e^{-x} + C_2 x e^{-x}$$

$$y_p = A \text{Cos } x + B \text{Sen } x + C \text{Cos } 2x + D \text{Sen } 2x$$

$$y' = -A \text{Sen } x + B \text{Cos } x - 2C \text{Sen } 2x + 2D \text{Cos } 2x$$

$$y'' = -A \text{Cos } x - B \text{Sen } x - 4C \text{Cos } 2x - 4D \text{Sen } 2x$$

$$\begin{aligned} & -A \text{Cos } x - B \text{Sen } x - 4C \text{Cos } 2x - 4D \text{Sen } 2x - 2A \text{Sen } x + 2B \text{Cos } x - 4C \text{Sen } 2x \\ & + 4D \text{Cos } 2x + A \text{Cos } x + B \text{Sen } x + C \text{Cos } 2x + D \text{Sen } 2x = \text{Sen } x + 3 \text{Cos } 2x \end{aligned}$$

$$-2A = 1$$

$$A = -\frac{1}{2}$$

$$2B = 0$$

$$B = 0$$

$$-3C + 4D = 3$$

$$-4C - 3D = 0$$

$$3/-3C + 4D = 3 \Rightarrow$$

$$-9C + 12D = 9$$

$$4/-4C - 3D = 0 \Rightarrow$$

$$-16C - 12D = 0$$

$$-25C = 9$$

$$C = -\frac{9}{25}$$

$$-3\left(-\frac{9}{25}\right) + 4D = 3$$

$$4D = 3 - \frac{27}{25}$$

$$4D = \frac{48}{25}$$

$$D = \frac{48}{100} = \frac{12}{25}$$

$$y_p = -\frac{1}{2} \text{Cos } x - \frac{9}{25} \text{Cos } 2x + \frac{12}{25} \text{Sen } 2x$$

$$y = C_1 e^{-x} + C_2 x e^{-x} - \frac{1}{2} \text{Cos } x$$

$$- \frac{9}{25} \text{Cos } 2x + \frac{12}{25} \text{Sen } 2x$$

$$(6) \quad y'' + 2y' - 24y = 16 - (x+2)e^{4x}$$

$$m^2 + 2m - 24 = 0$$

$$(m+6)(m-4) = 0$$

$$m = -6 \quad m = 4$$

$$y_c = C_1 e^{4x} + C_2 e^{-6x}$$

$$y_p = A - (Bx + C)e^{4x}$$

$$y_p = A - Bx^2 e^{4x} - Cx e^{4x}$$

$$y' = -4Bx^2 e^{4x} - 2Bx e^{4x} - 4Cx e^{4x} - C e^{4x}$$

$$y'' = -16Bx^2 e^{4x} - 8Bx e^{4x} - 8Bx e^{4x} - 2B e^{4x} - 16Cx e^{4x} - 4C e^{4x} - 4C e^{4x}$$

$$y'' = -16Bx^2 e^{4x} - 16Bx e^{4x} - 2B e^{4x} - 16Cx e^{4x} - 8C e^{4x}$$

$$-16Bx^2 e^{4x} - 16Bx e^{4x} - 2B e^{4x} - 16Cx e^{4x} - 8C e^{4x} - 8Bx^2 e^{4x} - 4Bx e^{4x} - 8Cx e^{4x}$$

$$-2C e^{4x} - 24A + 24Bx^2 e^{4x} + 24Cx e^{4x} = 16 - (x+2)e^{4x}$$

$$-20B = -1$$

$$B = \frac{1}{20}$$

$$-2B - 10C = -2$$

$$-2\left(\frac{1}{20}\right) - 10C = -2$$

$$-\frac{1}{10} - 10C = -2$$

$$-10C = -2 + \frac{1}{10}$$

$$-10C = -\frac{19}{10}$$

$$C = \frac{19}{100}$$

$$-24A = 16$$

$$A = -\frac{16}{24}$$

$$A = -\frac{2}{3}$$

$$y_p = -\frac{2}{3} + \frac{1}{20}x^2 e^{4x} + \frac{19}{100}x e^{4x}$$

$$y = C_1 e^{4x} + C_2 e^{-6x} - \frac{2}{3} + \frac{1}{20}x^2 e^{4x} + \frac{19}{100}x e^{4x}$$

$$⑦ \quad y''' - 2y'' - 4y' + 8y = 6xe^{2x}$$

$$m^3 - 2m^2 - 4m + 8 = 0$$

$$\begin{array}{r|l} 1 & -2 & -4 & +8 & \\ +2 & +0 & -8 & & \\ \hline 1 & +0 & -4 & & \end{array} \quad +2$$

$$(m-2)(m^2-4)=0$$

$$(m-2)(m-2)(m+2)=0$$

$$m=2 \quad m=2 \quad m=-2$$

$$y_c = C_1 e^{2x} + C_2 x e^{2x} + C_3 e^{-2x}$$

$$y_p = (Ax+B)e^{2x}$$

$$y_p = (Ax+B)e^{2x} x^2$$

$$y_p = Ax^3 e^{2x} + Bx^2 e^{2x}$$

$$y' = 2Ax^3 e^{2x} + 3Ax^2 e^{2x} + 2Bx^2 e^{2x} + 2Bx e^{2x}$$

$$y'' = 4Ax^3 e^{2x} + 6Ax^2 e^{2x} + 6Ax^2 e^{2x} + 6Ax e^{2x} + 4Bx^2 e^{2x} + 4Bx e^{2x} + 4Be^{2x}$$

$$y'' = 4Ax^3 e^{2x} + 12Ax^2 e^{2x} + 6Ax e^{2x} + 4Bx^2 e^{2x} + 8Bx e^{2x} + 4Be^{2x}$$

$$y''' = 8Ax^3 e^{2x} + 12Ax^2 e^{2x} + 24Ax^2 e^{2x} + 24Ax e^{2x} + 12Ax e^{2x} + 6Ae^{2x} + 8Bx^2 e^{2x} + 8Bx e^{2x} + 16Bx e^{2x} + 8Be^{2x} + 8Be^{2x}$$

$$y''' = 8Ax^3 e^{2x} + 36Ax^2 e^{2x} + 36Ax e^{2x} + 6Ae^{2x} + 8Bx^2 e^{2x} + 24Bx e^{2x} + 16Be^{2x}$$

$$\begin{aligned} & 8Ax^3 e^{2x} + 36Ax^2 e^{2x} + 36Ax e^{2x} + 6Ae^{2x} + 8Bx^2 e^{2x} + 24Bx e^{2x} + 16Be^{2x} \\ & - 8Ax^3 e^{2x} - 24Ax^2 e^{2x} - 12Ax e^{2x} - 8Bx^2 e^{2x} - 16Bx e^{2x} - 8Be^{2x} \\ & - 8Ax^3 e^{2x} - 12Ax^2 e^{2x} - 8Bx^2 e^{2x} - 8Bx e^{2x} + 8Ax^3 e^{2x} + 8Bx^2 e^{2x} = 6xe^{2x} \end{aligned}$$

$$24Ax e^{2x} + 8Be^{2x} + 6Ae^{2x} = 6xe^{2x}$$

$$24A = 6$$

$$A = \frac{1}{4}$$

$$6A + 8B = 0$$

$$8B = -6A$$

$$8B = -6\left(\frac{1}{4}\right)$$

$$B = -\frac{3}{16}$$

$$y_p = \frac{1}{4}x^3e^{2x} - \frac{3}{16}x^2e^{2x}$$

$$y = C_1e^{2x} + C_2xe^{2x} + C_3e^{-2x} + \frac{1}{4}x^3e^{2x} - \frac{3}{16}x^2e^{2x}$$

$$\textcircled{8} \quad x^3y''' + 4x^2y'' - 7xy' - 9y = 0$$

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$y''' = m(m-1)(m-2)x^{m-3}$$

$$x^3[m(m-1)(m-2)x^m] + 4x^2[m(m-1)x^m] - 7x[mx^m] - 9x^m = 0$$

$$x^m[(m^3 - m^2 - m^2 + 2m) + 4(m^2 - m) - 7m - 9] = 0$$

$$m^3 - 2m^2 - m^2 + 2m + 4m^2 - 4m - 7m - 9 = 0$$

$$m^3 + m^2 - 9m - 9 = 0$$

$$\begin{array}{r|l} 1 & 1 & 1 & -9 & -9 \\ & -3 & +6 & +9 & \\ \hline & 1 & -2 & -3 & \end{array}$$

$$(m+3)(m^2 - 2m - 3) = 0$$

$$(m+3)(m-3)(m+1) = 0$$

$$m = -3 \quad m = 3 \quad m = -1$$

$$y = C_1x^3 + C_2x^{-1} + C_3x^{-3}$$

$$y = C_1x^3 + \frac{C_2}{x} + \frac{C_3}{x^3}$$

$$9) x^4 y^{(4)} + 6x^3 y''' + 3x^2 y'' - 3xy' + 4y = 0$$

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$y''' = m(m-1)(m-2)x^{m-3}$$

$$y^{(4)} = m(m-1)(m-2)(m-3)x^{m-4}$$

$$x^4 [m(m-1)(m-2)(m-3)x^m \cdot x^{-4}] + 6x^3 [m(m-1)(m-2)x^m \cdot x^{-3}] + 3x^2 [m(m-1)x^m \cdot x^{-2}] - 3x(m x^{m-1}) + 4x^m = 0$$

$$(m^2 - m)(m-2)(m-3)x^m + 6(m^2 - m)(m-2)x^m + 3(m^2 - m)x^m - 3mx^m + 4x^m = 0$$

$$(m^3 - 2m^2 - m^2 + 2m)(m-3) + 6(m^3 - 2m^2 - m^2 + 2m) + 3m^2 - 3m - 3m + 4 = 0$$

$$(m^3 - 3m^2 + 2m)(m-3) + 6m^3 - 18m^2 + 12m + 3m^2 - 6m + 4 = 0$$

$$m^4 - 3m^3 - 3m^3 + 9m^2 + 2m^2 - 6m + 6m^3 - 18m^2 + 12m + 3m^2 - 6m + 4 = 0$$

$$m^4 - 4m^2 + 4 = 0$$

$$(m^2 - 2)^2 = 0$$

$$m^2 - 2 = 0$$

$$m^2 - 2 = 0$$

$$m = \pm \sqrt{2}$$

$$m = \pm \sqrt{2}$$

$$y = C_1 x^{\sqrt{2}} + C_2 x^{\sqrt{2}} \ln x + C_3 x^{-\sqrt{2}} + C_4 x^{-\sqrt{2}} \ln x$$

$$(10) \quad 5x^2 y'' - 3xy' + y = 0$$

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$5x^2 [m(m-1)x^m \cdot x^{-2}] - 3x [mx^m \cdot x^{-1}] + x^m = 0$$

$$5(m^2 - m)x^m - 3mx^m + x^m = 0$$

$$5m^2 - 5m - 3m + 1 = 0$$

$$5m^2 - 8m + 1 = 0$$

$$m = \frac{+8 \pm \sqrt{64 - 20}}{10}$$

$$m = \frac{8 \pm \sqrt{44}}{10}$$

$$m = \frac{4 \pm 2\sqrt{11}}{5}$$

$$m = \frac{4 \pm \sqrt{11}}{5}$$

$$y = C_1 x^{\frac{4+\sqrt{11}}{5}} + C_2 x^{\frac{4-\sqrt{11}}{5}}$$