

Study on reliability of bracing structure for foundation pit by Excel optimization algorithm

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Abstract. On the basis of the theory of structure reliability, the optimization algorithm according to the geometric significance of the reliability index and the checking point method are studied to calculate the reliability of the bracing structure for foundation pit. Combined with a soil nailed wall engineering example, the primary data calculation table is established to solve the non-linear programming problem adopting the embedded planning solving method of Excel software and the results indicate that the reliability index and the coordinates of the checking point are same nearly using the checking point method and Excel planning solving method, but the latter is more efficient and convenient than the former, it means that it is feasible to calculate the reliability index of the bracing structure for foundation pit by Excel optimization algorithm.

Introduction

The traditional design method of bracing structure for foundation pit is the safety coefficient method^[1], it is mainly according to the relevant experience of engineering practice, by means of calculating the fixed value of safety coefficient, namely the ratio of structure resistance and load effect to judge the safety degree of the design project. For the complexity of geotechnical engineering itself, the advantages of the safety coefficient method is simple and quick, but because of the heterogeneity of geotechnical material, the complicity and the nature can not be duplicated of geological conditions makes the study of this field has strong random uncertainty, using the safety coefficient method to evaluate the stability of the foundation pit has inevitable defects, and its becoming a necessary trend to introduce random analysis for the research of foundation pit engineering. According to the structural reliability theory and probability theory^[2], the stability problems of the bracing structure for foundation pit are random events, and the reliability of the structure, i.e. the probability of the structure resistance is greater than the load effect should be calculated to evaluate the safety degree of the structure, it is more scientific and objective compared with the safety coefficient method.

The correlative theory of structure reliability

Structural reliability means the ability of structure could undertake load, resist environment and effectively work under the specified conditions, environment and the design service life. The quantitative measurement of structure reliability is the probability of reliability that could be expressed as P_r , i.e. the probability of structure could complete the prospective function within the prescribed time and conditions. In contrast, the probability of failure expressed as P_f means the probability of structure could not complete the prospective function. P_r and P_f are complementary, namely $P_r + P_f = 1$. If the structural performance function is $Z = R - S$, in which R and S are structure resistance and load effect respectively. It is supposed that $Z < 0$ and $Z > 0$ are severally corresponding to the state of structure failure and reliability, $Z = 0$ corresponds to the limit state of structure. If R and S are two independent random variables, the joint probability density function could be expressed as $f_R(r)f_S(s)$, and thus the failure probability of structure could be calculated by the integral formulas (1a) and (1b).

$$P_f = P(Z < 0) = \iint_{Z < 0} f_R(r)f_S(s)drds = \int_0^{+\infty} f_S(s) \left(\int_0^s f_R(r)dr \right) ds = \int_0^{+\infty} f_S(s)F_R(s)ds \quad (1a)$$

$$P_f = P(Z < 0) = \int_0^{+\infty} f_R(r) \left(\int_r^{+\infty} f_S(s) ds \right) dr = \int_0^{+\infty} f_R(r) (1 - F_S(r)) dr \quad (1b)$$

Where, $F_R(s)$ and $F_S(r)$ denote respectively the probability distribution function values of R and S .

Since there are a number of difficulties to use the direct numerical integration function to calculate the failure probability, for example, the joint probability density function of a certain number of basic random variables could not be ascertained easily, a great deal of performance function are nonlinear and the nonlinear level may be higher, some approximate method are adopted to solve these conundrums such as the first-order second-moment method (FOSM), the optimization algorithm and so on. All of these approximate method calculate the failure probability of structure via calculating the reliability index, i.e. β , because of when Z obey normal distribution, the reliability index and the failure probability of structure have the one-to-one relationship as show in formula (2).

$$P_f = \int_{-\infty}^{\frac{\mu_z}{\sigma_z}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \Phi\left(-\frac{\mu_z}{\sigma_z}\right) = \Phi(-\beta) \quad (2)$$

Among it, μ_z and σ_z are the means and standard variances of Z respectively, $\Phi(\cdot)$ is the value of standard normal distribution function. When random variables are not normal distribution, the normal tail approximation method could be used to transform these variables into equivalent normal distribution random variables so as to calculate the failure probability.

Checking point method. Checking point method is a calculation method with higher accuracy belonging to FOSM, and its central idea is based on the random variables' actual distribution type to develop the nonlinear performance function Z into Taylor series of which should be reserved the linear item at the checking point on the situs of failure surface, and then the reliability index of structure could be calculate by the random variables' means, i.e. the first-order moments and standard variances, i.e. the second-order central moments. Because the reliability index and also the design checking point need to be calculated, the iterative calculation should be involved in the checking point method to complete the solution procedure.

Optimization algorithm. The reliability index could be approximately acquired by checking point method on a high level precision. But it is cockamamie due to the iterative calculation repeatedly and in addition, the iterative calculation may be misconvergence on many circumstances, so optimization algorithm is becoming a better solution way to calculate the reliability index according to the geometric significance of the reliability index itself^[3,4,5].

According to structure reliability theory, the reliability index's geometric significance is the shortest distance from the origin of coordinate to limit-state curved surface in the standard normal coordinate system, thus the calculation problem of the reliability index could be changed into nonlinear planning problem with constraints, namely the checking point, i.e. P^* should be acquired by searching the point on the limit-state curved surface $Z=0$ which have the shortest distance from the origin of coordinate in the standard normal coordinate system, while the distance between the checking point P^* and the origin of coordinates is the structure reliability index. Thereout, the optimization calculation model of the reliability index could be expressed as formula (3).

$$\begin{cases} \text{Find} & X = (x_1, x_2, \dots, x_n) \\ \text{Minimize} & \beta = \sqrt{(X - \mu_X)^T C^{-1} (X - \mu_X)} \\ \text{Subject to} & Z = g(x_1, x_2, \dots, x_n) = 0 \end{cases} \quad (3)$$

Among them, C is the multiple random variables' covariance matrix, i.e. $C = [C_{X_i X_j}]_{n \times n}$, the diagonal elements of it are the random variables' variances, i.e. $\sigma_{X_i}^2$, and another elements of it are covariance, i.e. $\rho_{X_i X_j} \sigma_{X_i} \sigma_{X_j}$, in which, $\rho_{X_i X_j}$ is correlation coefficient between the random variables.

The structural reliability index by optimization algorithm could be realized conveniently in Excel and Matlab software. Using the optimization algorithm tools embedded in Excel software such as the planning solution function^[6] to calculate the reliability index, the software needs to be full installed to

solve the linear and nonlinear planning problem efficiently and conveniently. And therefore, the tedious manual computation, programming and debugging process are avoided, the computation efficiency is greatly improved .

Analysis and calculation of an engineering example

An excavation engineering in Zhengdong New Area using soil nailed structure^[7], the excavation depth of it is approximately 9m and the height of the soil nailed wall is designed as $H = 9.0\text{m}$. Six rows of soil nails are setted totally, the horizontal and vertical distances between these soil nails are 1.5m and 1.4m respectively, the embedding depth of the first row is 1.4m, the drilling diameter is 0.15m, the length of these soil nails is 8m and the dip angle of them is 10° . The several soil layers' physical and mechanical parameters acquired by the situs testing and laboratory data are calculated by weighted average method according to the soil thickness, and thus the data are ascertained, including the soil severe is $\gamma=18.25\text{kN}\cdot\text{m}^{-3}$, the ground overloading is $q=20\text{kPa}$, the means of soil cohesion c and internal friction angle φ are each $\mu_c=1\text{kPa}$ and $\mu_\varphi=26^\circ$, the standard variances of c and φ are $\sigma_c=5.13\text{kPa}$ and $\sigma_\varphi=3.7^\circ$ respectively when assuming the soil below ground is homogeneous.

Since variation coefficient of soil thickness, soil severe, the length of soil nail and overloading are small, only soil cohesion and internal friction angle are supposed as random variables so as to simplify the calculation, and it is also assumed they're independent and obeying normal distribution.

According to the calculation results of the literature [7], the most likely failure mode of this soil nailed structure may be the overall slippage instability among all kinds of the possible external failure modes, therefore, the overall slippage instability issue, i.e. the controlling failure mode is analyzed to ascertain its structural reliability by FOSM and Excel software method in the light of optimization algorithm.

Analysis on overall anti-slippage stability. The working principle of soil nailed structure is similar to the gravity retaining wall. The force analysis of overall anti-slippage mode includes that the active earth pressure on the back of the soil nail wall brings the sliding force, the friction caused by the weight of the wall and the ground surface overloading produces the stabilizing force. Based on the limit equilibrium theory, the soil nailed wall would reach the failure limit state when sliding force equals to stabilizing force, and thus the longitudinal unit length along the soil nail wall is taken to establish the performance function of overall anti-slippage stability as shown in formula (4).

$$Z = R - S = g(c, \varphi) = B(\gamma H + q) \tan \varphi + cB - \left[\left(\frac{1}{2} \gamma H^2 + qH \right) K_a - 2cH \sqrt{K_a} \right] \quad (4)$$

Among it, the active earth pressure coefficient is $K_a = \tan^2(45^\circ - \frac{\varphi}{2})$, B is 11/12 times of the soil nail length(m).

The reliability calculation of overall anti-slippage. At first, reliability calculation by Excel software based on optimization algorithm needs to establish the table as shown in Fig.1, i.e. the primary data calculation table in Excel window in accordance with the nonlinear planning problem as previously stated in formula (3). And then the button of "planning solution" in "tools" of Excel menu bar needs to be selected, in which, the target cell, changeable cell and constraint cell should be assumed in the dialog box that is popped, these three items are the elements in the "C17", "B5, B6" and "A17" respectively according to Fig.1. The iterative result of the reliability index is reserved in the target cell, the design checking point's coordinates are reserved in the changeable cell, while the restraint requirements of planning calculation is reserved in the constraint cell. At last, when the button of "minimum value" is selected, the optimization results of planning calculation would be completed as shown in Fig.2 and Table 1.

The reliability calculation of overall anti-slippage stability by checking point method should adopt Matlab software programming and the results are also shown in Table 1.

From Table 1, the reliability index β and the coordinates of checking point are same nearly using checking point method and optimization algorithm by Excel software. When assuming $10^{-3} \sim 10^{-2}$ is the standard of failure probability for foundation pit^[8], the probability of this engineering example

does not exceed the upper limit, i.e. 10^{-2} , since it is a small probability event, the bracing structure would not happen to overall anti-slippage, this conclusion is consistent with the actual situation of the engineering practice.

	A	B	C	D	E	F	G	H	I
1	y	H	B	q	Ka	S	R		
2	18.25	9	7.333333	20	0.390462	235.159	739.6747		
3									
4	Random va.X's values		Means	Standard variances	Covariance matrix C				
5	c (kPa)	11	11	5.13	26.3169	0	0		
6	φ (rad)	0.453786	0.453786	0.064577	0	0.00417			
7									
8	$(X-\mu_X)^T$				Inverse matrix of, i.e. C^{-1}		$(X-\mu_X)$		
9	0	0			0.037998	0	0		0
10					0	239.7959			0
11									
12					$C^{-1}(X-\mu_X)$				
13					0				
14					0				
15									
16	Z		β		$(X-\mu_X)^T C^{-1}(X-\mu_X)$				
17	504.5157		0		0				

Fig.1 The primary data table in Excel software

	A	B	C	D	E	F	G	H	I
1	y	H	B	q	Ka	S	R		
2	18.25	9	7.333333	20	0.537116	450.6261	450.6261		
3									
4	Random va.X's values		Means	Standard variances	Covariance matrix C				
5	c (kPa)	3.263426	11	5.13	26.3169	0			
6	φ (rad)	0.305886	0.453786	0.064577	0	0.00417			
7									
8	$(X-\mu_X)^T$				Inverse matrix of, i.e. C^{-1}		$(X-\mu_X)$		
9	-7.73657	-0.1479			0.037998	0			-7.73657
10					0	239.7959			-0.1479
11									
12					$C^{-1}(X-\mu_X)$				
13					0				
14					0				
15									
16	Z		β		$(X-\mu_X)^T C^{-1}(X-\mu_X)$				
17	-1.3E-08		2.742219		7.519766				

Fig.2 The planning results by Excel software

Table 1 The results of checking point method and Excel optimization algorithm of overall anti-slippage stability of soil nailed wall

Computational method	Coordinates of checking point x^*		β	$P_f(\%)$
	c*(kPa)	φ*(rad)		
Checking point method	3.3482	0.3053	2.7426	0.3048
Excel optimization algorithm	3.2634	0.3059	2.7422	0.3051

Conclusions

The calculation problem of reliability index is transformed into a nonlinear programming problem with constraints in the light with it's geometric significance. Based on the planning function embedded in Excel software, the primary data calculation table in Excel software of soil nailed wall's overall anti-slippage is established, the optimization analysis and calculation on reliability index and coordinates of checking point are completed. The results of this example shows that comparing with the checking point method needs programming and debugging program, the optimization algorithm by Excel software is simple and convenient undoubtedly, it is practicable when using Excel optimization algorithm to calculate the reliability of bracing structure for foundation pit .

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