Harvard Business Review

Decision Making And Problem Solving

Decision Trees for Decision-Making

by John F. Magee

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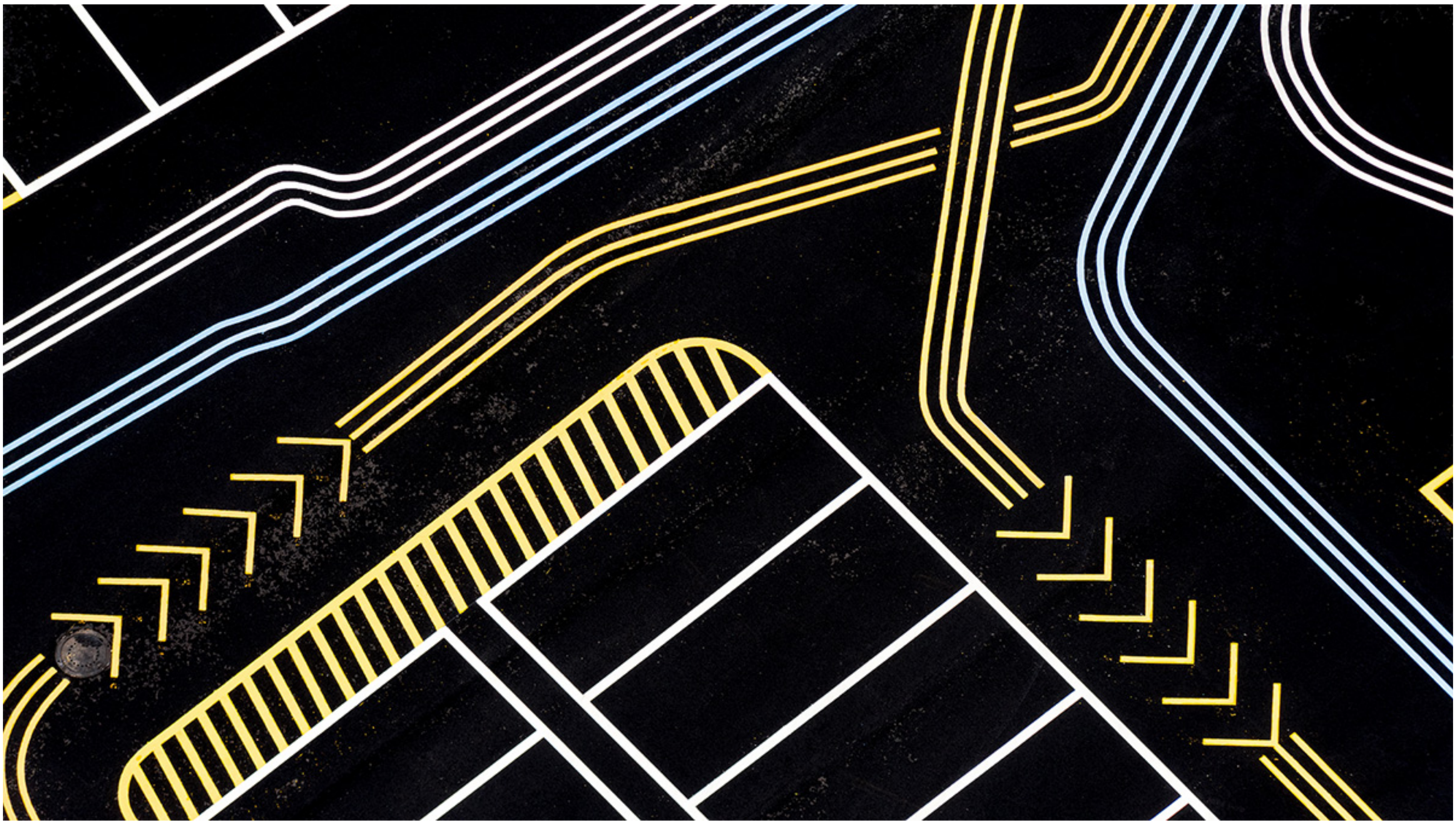

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The management of a company that I shall call Stygian Chemical Industries, Ltd., must decide whether to build a small plant or a large one to manufacture a new product with an expected market life of 10 years. The decision hinges on what size the market for the product will be.

Possibly demand will be high during the initial two years but, if many initial users find the product unsatisfactory, will fall to a low level thereafter. Or high initial demand might indicate the possibility of a sustained high-volume market. If demand is high and the company does not expand within the first two years, competitive products will surely be introduced.

If the company builds a big plant, it must live with it whatever the size of market demand. If it builds a small plant, management has the option of expanding the plant in two years in the event that demand is high during the introductory period; in the event that demand is low during the introductory period, the company will maintain operations in the small plant and make a tidy profit on the low volume.

Management is uncertain what to do. The company grew rapidly during the 1950s; it kept pace with the chemical industry generally. The new product, if the market turns out to be large, offers the present management a chance to push the company into a new period of profitable growth. The development department, particularly the development project engineer, is pushing to build the large-scale plant to exploit the first major product development the department has produced in some years.

The chairman, a principal stockholder, is wary of the possibility of large unneeded plant capacity. He favors a smaller plant commitment, but recognizes that later expansion to meet high-volume demand would require more investment and be less efficient to operate. The chairman also recognizes that unless the company moves promptly to fill the demand which develops, competitors will be tempted to move in with equivalent products.

The Stygian Chemical problem, oversimplified as it is, illustrates the uncertainties and issues that business management must resolve in making investment decisions. (I use the term "investment" in a broad sense, referring to outlays not only for new plants and equipment but also for large, risky orders, special marketing facilities, research programs, and other purposes.) These decisions are growing more important at the same time that they are increasing in complexity. Countless executives want to make them better—but how?

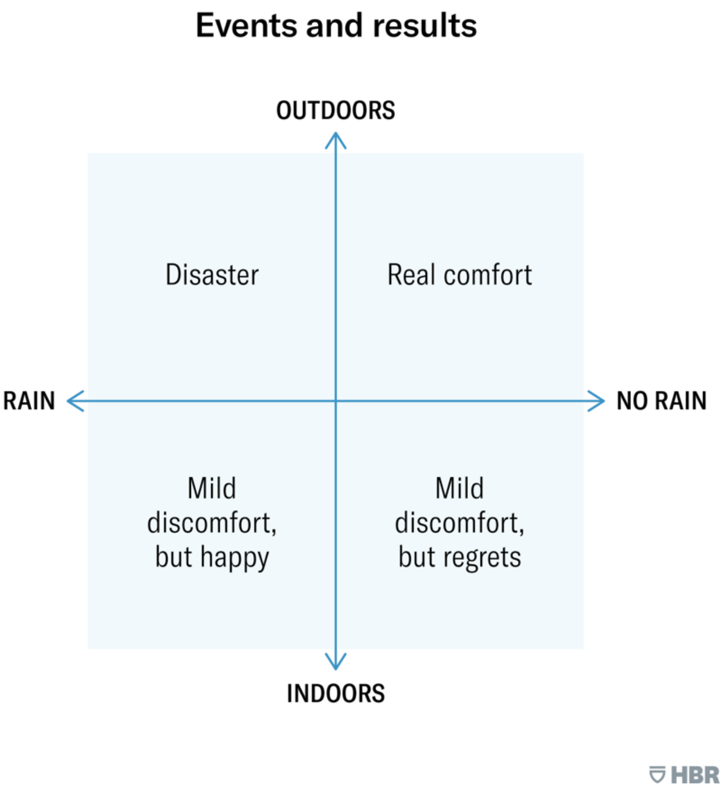
In this article I shall present one recently developed concept called the "decision tree," which has tremendous potential as a decision-making tool. The decision tree can clarify for management, as can no other analytical tool that I know of, the choices, risks, objectives, monetary gains, and information needs involved in an investment problem. We shall be hearing a great deal about decision trees in the years ahead. Although a novelty to most businesspeople today, they will surely be in common management parlance before many more years have passed.

Later in this article we shall return to the problem facing Stygian Chemical and see how management can proceed to solve it by using decision trees. First, however, a simpler example will illustrate some characteristics of the decision-tree approach.

Displaying Alternatives

Let us suppose it is a rather overcast Saturday morning, and you have 75 people coming for cocktails in the afternoon. You have a pleasant garden and your house is not too large, so if the weather permits, you would like to set up the refreshments in the garden and have the party there. It would be more pleasant, and your guests would be more comfortable. On the other hand, if you set up the party in the garden and after all the guests are assembled it begins to rain, the refreshments will be ruined, your guests will get damp, and you will heartily wish you had decided to have the party in the house. (We could complicate this problem by considering the possibility of a partial commitment to one course or the other and opportunities to adjust estimates of the weather as the day goes on, but the simple problem is all we need.)

This particular decision can be represented in the form of a "payoff" table:



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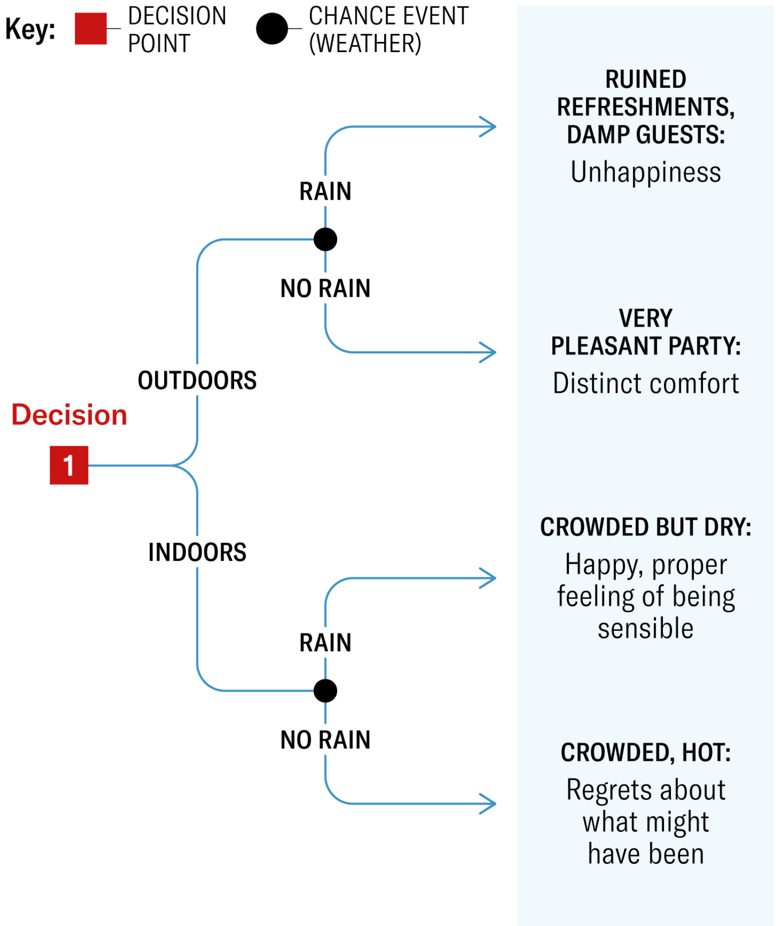
Much more complex decision questions can be portrayed in payoff table form. However, particularly for complex investment decisions, a different representation of the information pertinent to the problem—the decision tree—is useful to show the routes by which the various possible outcomes are achieved. Pierre Massé, Commissioner General of the National Agency for Productivity and Equipment Planning in France, notes:

> The decision problem is not posed in terms of an isolated decision (because today's decision depends on the one we shall make tomorrow) nor yet in terms of a sequence of decisions (because under uncertainty, decisions taken in the future will be influenced by what we have learned in the meanwhile). The problem is posed in terms of a tree of decisions.

Exhibit I illustrates a decision tree for the cocktail party problem. This tree is a different way of displaying the same information shown in the payoff table. However, as later examples will show, in complex decisions the decision tree is frequently a much more lucid means of presenting the relevant information than is a payoff table.

Exhibit I: Decision Tree for Cocktail Party



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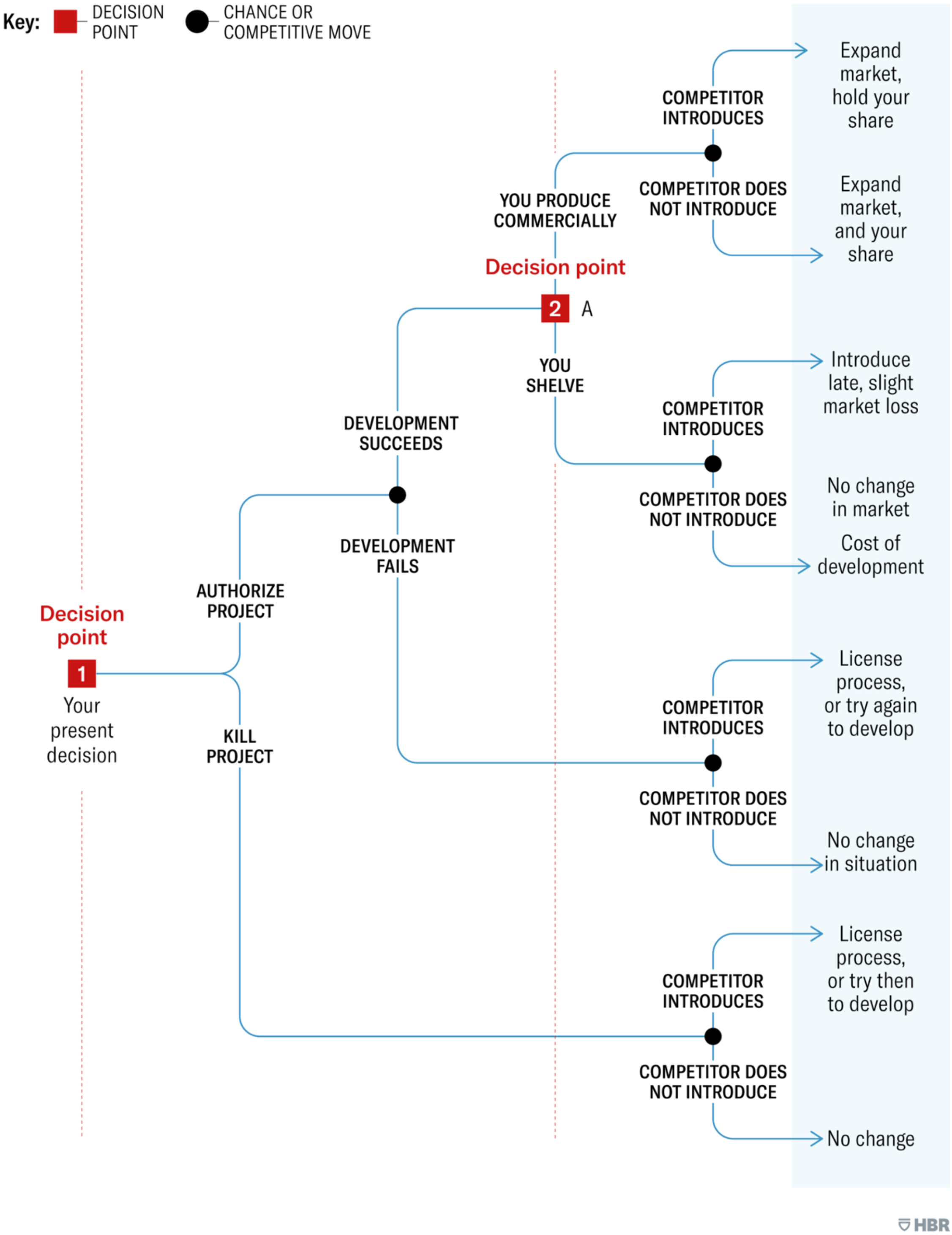
The tree is made up of a series of nodes and branches. At the first node on the left, the host has the choice of having the party inside or outside. Each branch represents an alternative course of action or decision. At the end of each branch or alternative course is another node representing a chance event—whether or not it will rain. Each subsequent alternative course to the right represents an alternative outcome of this chance event. Associated with each complete alternative course through the tree is a payoff, shown at the end of the rightmost or terminal branch of the course.

When I am drawing decision trees, I like to indicate the action or decision forks with square nodes and the chance-event forks with round ones. Other symbols may be used instead, such as single-line and double-line branches, special letters, or colors. It does not matter so much which method of distinguishing you use so long as you do employ one or another. A decision tree of any size will always combine (a) *action* choices with (b) different possible *events* or *results* of action which are partially affected by chance or other uncontrollable circumstances.

Decision-event chains. The previous example, though involving only a single stage of decision, illustrates the elementary principles on which larger, more complex decision trees are built. Let us take a slightly more complicated situation.

You are trying to decide whether to approve a development budget for an improved product. You are urged to do so on the grounds that the development, if successful, will give you a competitive edge, but if you do not develop the product, your competitor may—and may seriously damage your market share. You sketch out a decision tree that looks something like the one in Exhibit II.

Exhibit II:
Decision Tree with Chains of Actions and Events



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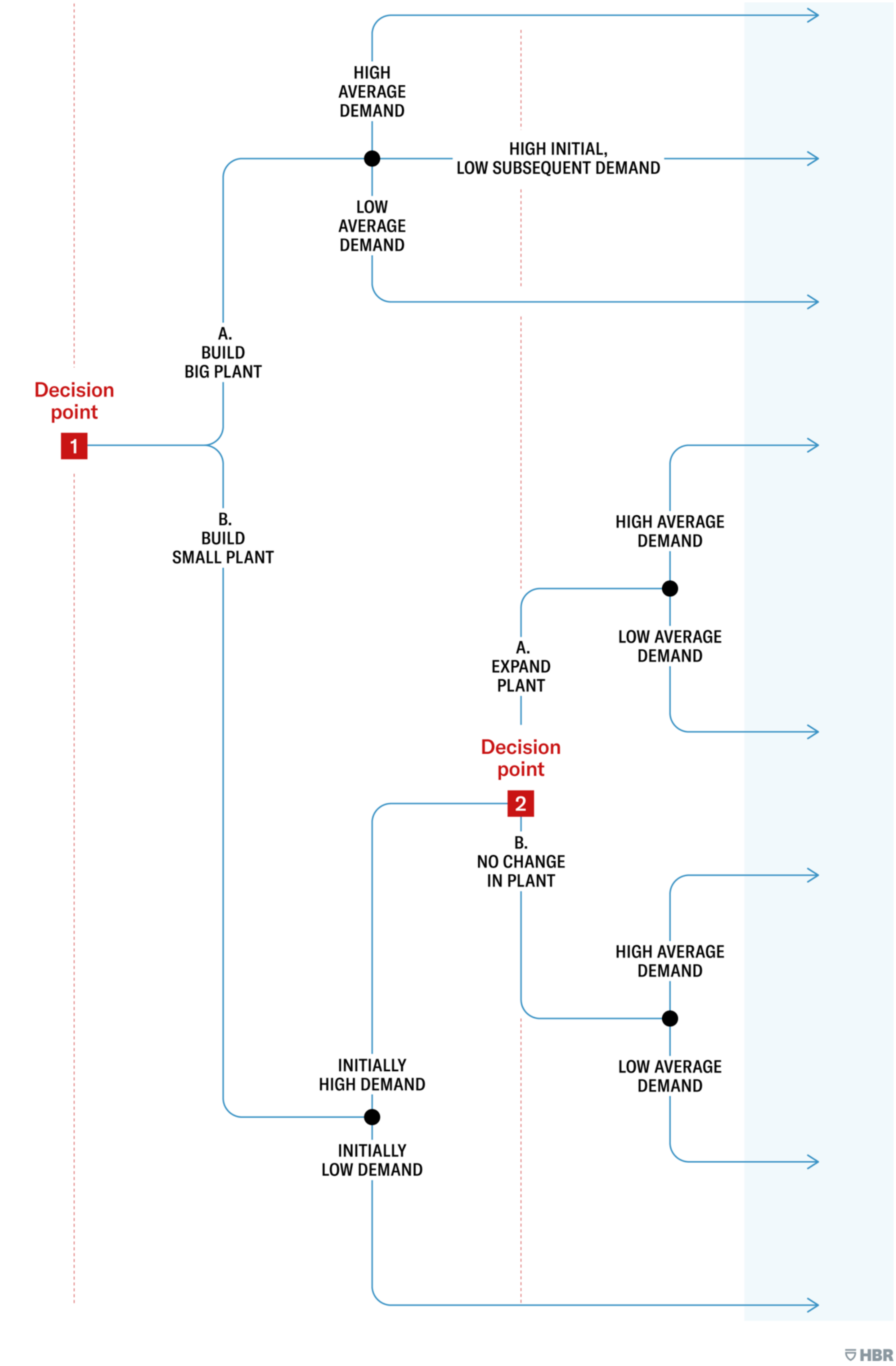
Your initial decision is shown at the left. Following a decision to proceed with the project, if development is successful, is a second stage of decision at Point A. Assuming no important change in the situation between now and the time of Point A, you decide now what alternatives will be important to you at that time. At the right of the tree are the outcomes of different sequences of decisions and events. These outcomes, too, are based on your present information. In effect you say, "If what I know now is true then, this is what will happen."

Of course, you do not try to identify all the events that can happen or all the decisions you will have to make on a subject under analysis. In the decision tree you lay out only those decisions and events or results that are important to you and have consequences you wish to compare. (For more illustrations, see the Appendix.)

Adding Financial Data

Now we can return to the problems faced by the Stygian Chemical management. A decision tree characterizing the investment problem as outlined in the introduction is shown in Exhibit III. At Decision #1 the company must decide between a large and a small plant. This is all that must be decided *now*. But if the company chooses to build a small plant and then finds demand high during the initial period, it can in two years—at Decision #2—choose to expand its plant.

Exhibit III:
Decisions and Events for Stygian Chemical Industries, Ltd.



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But let us go beyond a bare outline of alternatives. In making decisions, executives must take account of the probabilities, costs, and returns which appear likely. On the basis of the data now available to them, and assuming no important change in the company's situation, they reason as follows:

- Marketing estimates indicate a 60% chance of a large market in the long run and a 40% chance of a low demand, developing initially as follows:

  Initially high demand, sustained high: **60%**
  Initially high demand, long-term low: **10%**
  Initially low and continuing low: **30%**
  Initially low and subsequently high: **0%**

- Therefore, the chance that demand initially will be high is 70% (60 + 10). *If* demand is high initially, the company estimates that the chance it will continue at a high level is 86% (60 ÷ 70). Comparing 86% to 60%, it is apparent that a high initial level of sales changes the estimated chance of high sales in the subsequent periods. Similarly, if sales in the initial period are low, the chances are 100% (30 ÷ 30) that sales in the subsequent periods will be low. Thus the level of sales in the initial period is expected to be a rather accurate indicator of the level of sales in the subsequent periods.

- Estimates of annual income are made under the assumption of each alternative outcome:

  1. A large plant with high volume would yield $1,000,000 annually in cash flow.
  2. A large plant with low volume would yield only $100,000

because of high fixed costs and inefficiencies.

3. A small plant with low demand would be economical and would yield annual cash income of $400,000.
4. A small plant, during an initial period of high demand, would yield $450,000 per year, but this would drop to $300,000 yearly in the long run because of competition. (The market would be larger than under Alternative 3, but would be divided up among more competitors.)
5. If the small plant were expanded to meet sustained high demand, it would yield $700,000 cash flow annually, and so would be less efficient than a large plant built initially.
6. If the small plant were expanded but high demand were not sustained, estimated annual cash flow would be $50,000.

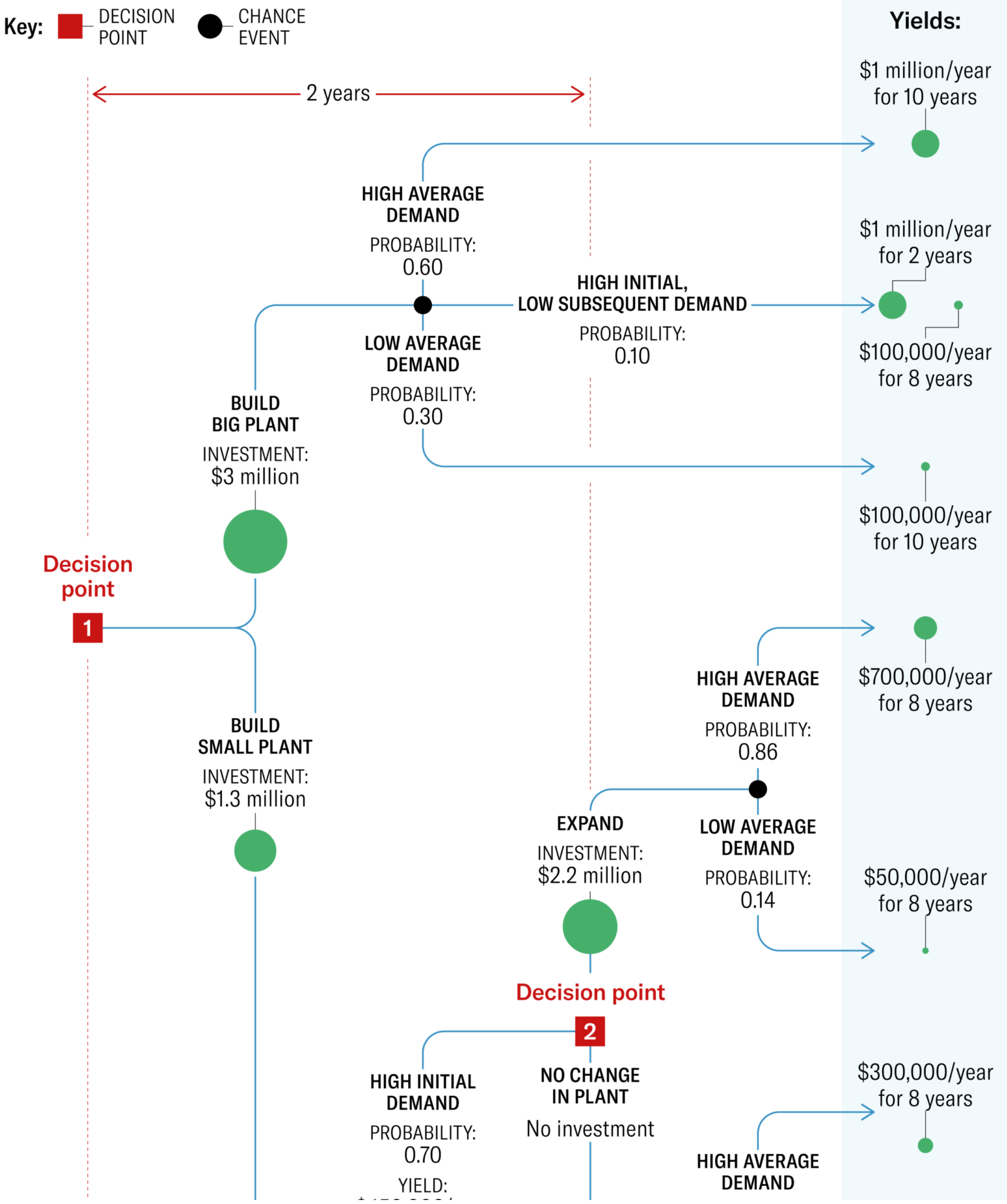
- It is estimated further that a large plant would cost $3 million to put into operation, a small plant would cost $1.3 million, and the expansion of the small plant would cost an additional $2.2 million.

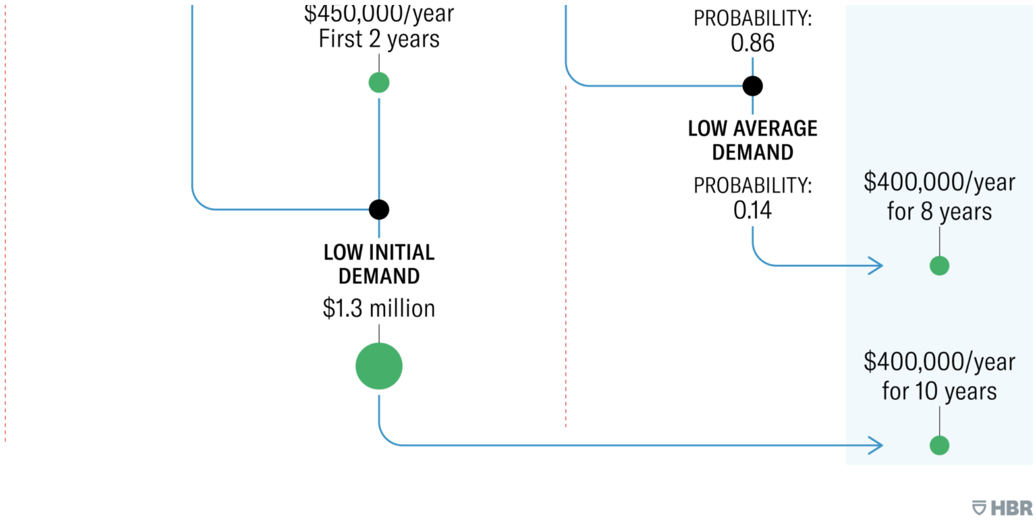
When the foregoing data is incorporated, we have the decision tree shown in Exhibit IV. Bear in mind that nothing is shown here which Stygian Chemical's executives did not know before; no numbers have been pulled out of hats. However, we are beginning to see dramatic evidence of the value of decision trees in *laying out* what management knows in a way that enables more-systematic analysis and leads to better decisions. To sum up the requirements of making a decision tree, management must:

1. Identify the points of decision and alternatives available at each point.

2. Identify the points of uncertainty and the type or range of alternative outcomes at each point.
3. Estimate the values needed to make the analysis, especially the probabilities of different events or results of action and the costs and gains of various events and actions.
4. Analyze the alternative values to choose a course.

Exhibit IV:
Decision Tree with Financial Data





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Choosing Course of Action

We are now ready for the next step in the analysis—to compare the consequences of different courses of action. A decision tree does not give management the answer to an investment problem; rather, it helps management determine which alternative at any particular choice point will yield the greatest expected monetary gain, given the information and alternatives pertinent to the decision.

Of course, the gains must be viewed with the risks. At Stygian Chemical, as at many corporations, managers have different points of view toward risk; hence they will draw different conclusions in the circumstances described by the decision tree shown in Exhibit IV. The many people participating in a decision —those supplying capital, ideas, data, or decisions, and having different values at risk—will see the uncertainty surrounding the decision in different ways. Unless these differences are recognized and dealt with, those who must make the decision, pay for it, supply data and analyses to it, and live with it will judge the issue, relevance of data, need for analysis, and criterion of success in different and conflicting ways.

For example, company stockholders may treat a particular investment as one of a series of possibilities, some of which will work out, others of which will fail. A major investment may pose risks to a middle manager—to his job and career—no matter what decision is made. Another participant may have a lot to gain from success, but little to lose from failure of the project. The nature of the risk—as each individual sees it—will affect not only the assumptions he is willing to make but also the strategy he will follow in dealing with the risk.

The existence of multiple, unstated, and conflicting objectives will certainly contribute to the "politics" of Stygian Chemical's decision, and one can be certain that the political element exists whenever the lives and ambitions of people are affected. Here, as in similar cases, it is not a bad exercise to think through who the parties to an investment decision are and to try to make these assessments:

- *What is at risk?* Is it profit or equity value, survival of the business, maintenance of a job, opportunity for a major career?

- *Who is bearing the risk?* The stockholder is usually bearing risk in one form. Management, employees, the community—all may be bearing different risks.

- *What is the character of the risk that each person bears?* Is it, *in his terms,* unique, once-in-a-lifetime, sequential, insurable? Does it affect the economy, the industry, the company, or a portion of the company?

Considerations such as the foregoing will surely enter into top management's thinking, and the decision tree in Exhibit IV will not eliminate them. But the tree will show management what decision today will contribute most to its long-term goals. The tool for this next step in the analysis is the concept of "rollback."

"Rollback" concept. Here is how rollback works in the situation described. At the time of making Decision #1 (see Exhibit IV), management does not have to make Decision #2 and does not even know if it will have the occasion to do so. But if it *were* to have the option at Decision #2, the company would expand the plant, in view of its current knowledge. The analysis is shown in Exhibit V. (I shall ignore for the moment the question of discounting future profits; that is introduced later.) We see that the total expected value of the expansion alternative is $160,000 greater than the no-expansion alternative, over the eight-year life remaining. Hence that is the alternative management would choose if faced with Decision #2 with its existing information (and thinking only of monetary gain as a standard of choice).

Exhibit V:
Analysis of Possible Decision #2

Using maximum expected total cash flow as criterion

Choice: Expansion

CHANCE EVENT	PROBABILITY (1)	TOTAL YIELD, 8 YEARS (2)	EXPECTED VALUE (1) x (2)
HIGH AVERAGE DEMAND	0.86	$5,600,000	$4,816,000
LOW AVERAGE DEMAND	0.14	400,000	56,000
		TOTAL:	4,872,000
		LESS INVESTMENT:	2,200,000
		NET:	**$2,672,000**

Choice: No expansion

CHANCE EVENT	PROBABILITY (1)	TOTAL YIELD, 8 YEARS (2)	EXPECTED VALUE (1) x (2)
HIGH AVERAGE DEMAND	0.86	$2,400,000	$2,064,000
LOW AVERAGE DEMAND	0.14	3,200,000	448,000
		TOTAL:	2,512,000
		LESS INVESTMENT:	0
		NET:	**$2,512,000**

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Readers may wonder why we started with Decision #2 when today's problem is Decision #1. The reason is the following: We need to be able to put a monetary value on Decision #2 in order to "roll back" to Decision #1 and compare the gain from taking the lower branch ("Build Small Plant") with the gain from taking the upper branch ("Build Big Plant"). Let us call that monetary value for Decision #2 its *position value*. The position value of a decision is the expected value of the preferred branch (in this case, the plant-expansion fork). The expected value is simply a kind of average of the results you would expect if you were to repeat the situation over and over—getting a $5.6 million yield 86% of the time and a $400,000 yield 14% of the time.

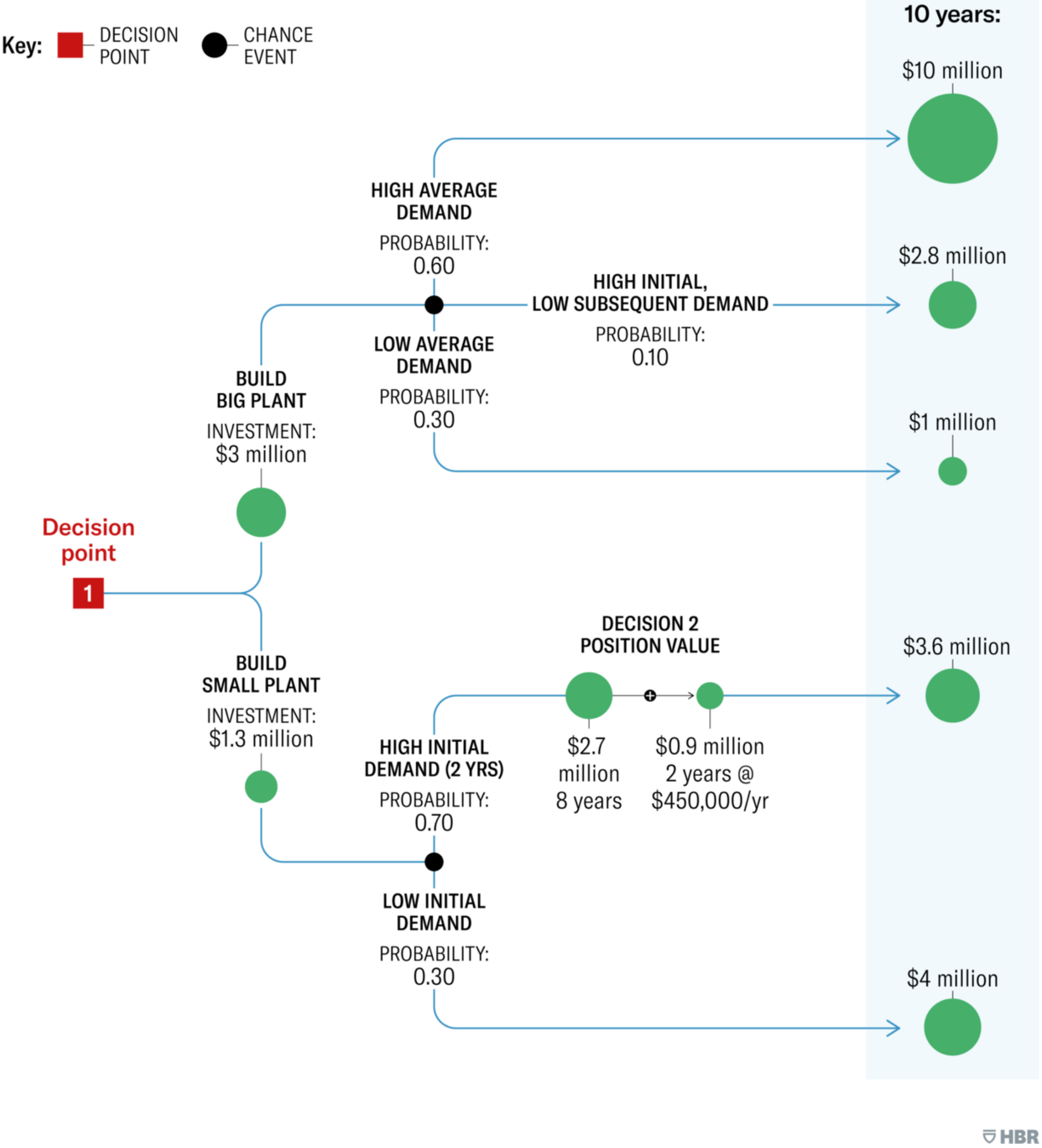
Stated in another way, it is worth $2,672,000 to Stygian Chemical to get to the position where it can make Decision #2. The question is: Given this value and the other data shown in Exhibit IV, what now appears to be the best action at Decision #1?

Turn now to Exhibit VI. At the right of the branches in the top half we see the yields for various events if a big plant is built (these are simply the figures in Exhibit IV multiplied out). In the bottom half we see the small plant figures, including Decision #2 position value plus the yield for the two years prior to Decision #2. If we reduce all these yields by their probabilities, we get the following comparison:

Build big plant:
($10 million × .60) + ($2.8 million × .10) + ($1 million × .30) – $3 million = $3.6 million

Build small plant:
($3.6 million × .70) + ($4 million × .30) – $1.3 million = $2.4 million

Exhibit VI: Cash Flow Analysis for Decision #1



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The choice which maximizes expected total cash yield at Decision #1, therefore, is to build the big plant initially.

Accounting for Time

What about taking differences in the *time* of future earnings into account? The time between successive decision stages on a decision tree may be substantial. At any stage, we may have to weigh differences in immediate cost or revenue against differences in value at the next stage. Whatever standard of choice is applied, we can put the two alternatives on a comparable basis if we discount the value assigned to the next stage by an appropriate percentage. The discount percentage is, in effect, an allowance for the cost of capital and is similar to the use of a discount rate in the present value or discounted cash flow techniques already well-known to businesspeople.

When decision trees are used, the discounting procedure can be applied one stage at a time. Both cash flows and position values are discounted.

For simplicity, let us assume that a discount rate of 10% per year for all stages is decided on by Stygian Chemical's management. Applying the rollback principle, we again begin with Decision #2. Taking the same figures used in previous exhibits and discounting the cash flows at 10%, we get the data shown in Part A of Exhibit VII. Note particularly that these are the present values *as of the time Decision #2 is made.*

Exhibit VII: Analysis of Decision #2 with Discounting

A: Present values of cash flows

CHOICE	OUTCOME	YIELD	PRESENT VALUE
EXPAND	HIGH DEMAND	$700,000/yr, 8 yrs	$4,100,000
EXPAND	LOW DEMAND	50,000/yr, 8 yrs	300,000
NO CHANGE	HIGH DEMAND	300,000/yr, 8 yrs	1,800,000
NO CHANGE	LOW DEMAND	400,000/yr, 8 yrs	2,300,000

B: Obtaining discounted expected values

Choice: Expansion

CHANCE EVENT	PROBABILITY (1)	PRESENT VALUE YIELD (2)	DISCOUNTED EXPECTED VALUE (1) x (2)
HIGH AVERAGE DEMAND	0.86	$4,100,000	$3,526,000
LOW AVERAGE DEMAND	0.14	300,000	42,000
		TOTAL:	3,568,000
		LESS INVESTMENT:	2,200,000
		NET:	**$1,368,000**

Choice: No expansion

CHANCE EVENT	PROBABILITY (1)	PRESENT VALUE YIELD (2)	DISCOUNTED EXPECTED VALUE (1) x (2)
HIGH AVERAGE DEMAND	0.86	$1,800,000	$1,548,000
LOW AVERAGE DEMAND	0.14	2,300,000	322,000
		TOTAL:	1,870,000
		LESS INVESTMENT:	0
		NET:	**$1,870,000**

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Now we want to go through the same procedure used in Exhibit V when we obtained expected values, only this time using the discounted yield figures and obtaining a discounted expected value. The results are shown in Part B of Exhibit VII. Since the discounted expected value of the no-expansion alternative is higher, *that* figure becomes the position value of Decision #2 this time.

Having done this, we go back to work through Decision #1 again, repeating the same analytical procedure as before only with discounting. The calculations are shown in Exhibit VIII. Note that the Decision #2 position value is treated at the time of Decision #1 as if it were a lump sum received at the end of the two years.

Exhibit VIII: Analysis of Decision #1

Choice: Build big plant

CHANCE EVENT	PROBABILITY (1)	YIELD	DISCOUNTED VALUE OF YIELD (2)	DISCOUNTED EXPECTED YIELD (1) X (2)
HIGH AVERAGE DEMAND	0.60	$1,000,000/yr, 10 yrs	$6,700,000	$4,020,000
HIGH INITIAL, LOW AVERAGE DEMAND	0.10	1,000,000/yr, 2 yrs 100,000/yr, 8 yrs	2,400,000	240,000
LOW AVERAGE DEMAND	0.30	100,000/yr, 10 yrs	700,000	210,000
			TOTAL:	4,470,000
			LESS INVESTMENT:	3,000,000
			NET:	**1,470,000**

Choice: Build small plant

CHANCE EVENT	PROBABILITY (1)	YIELD	DISCOUNTED VALUE OF YIELD (2)	DISCOUNTED EXPECTED YIELD (1) X (2)
HIGH INITIAL DEMAND	0.70	$450,000/yr, 2 yrs	$860,000	$600,000
		Decision #2 value, 1,870,000 at end of 2 yrs	1,530,000	1,070,000
LOW INITIAL DEMAND	0.30	400,000/yr, 10 yrs	2,690,000	810,000
			TOTAL:	2,480,000
			LESS INVESTMENT:	1,300,000
			NET:	**1,180,000**

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The large-plant alternative is again the preferred one on the basis of discounted expected cash flow. But the margin of difference over the small-plant alternative ($290,000) is smaller than it was without discounting.

Uncertainty Alternatives

In illustrating the decision-tree concept, I have treated uncertainty alternatives as if they were discrete, well-defined possibilities. For my examples I have made use of uncertain situations depending basically on a single variable, such as the level of demand or the success or failure of a development project. I have sought to avoid unnecessary complication while putting emphasis on the key interrelationships among the present decision, future choices, and the intervening uncertainties.

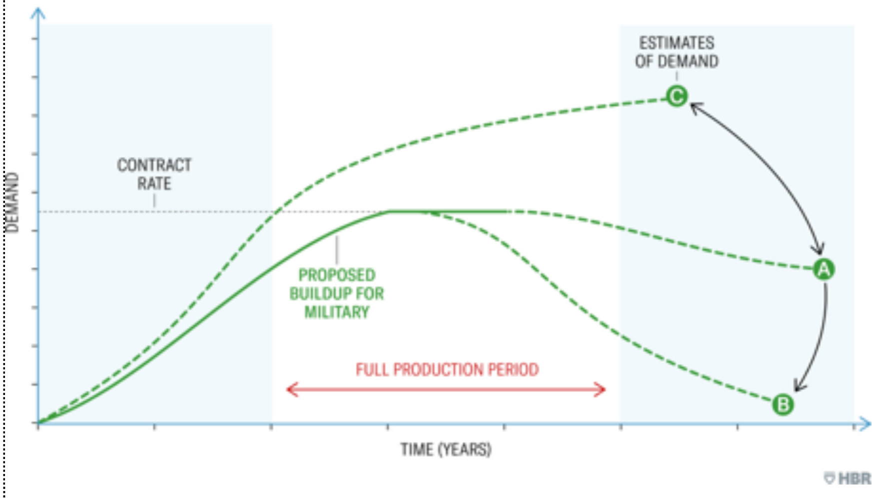
In many cases, the uncertain elements do take the form of discrete, single-variable alternatives. In others, however, the possibilities for cash flow during a stage may range through a whole spectrum and may depend on a number of independent or partially related variables subject to chance influences—cost, demand, yield, economic climate, and so forth. In these cases, we have found that the range of variability or the likelihood of the cash flow falling in a given range during a stage can be calculated readily from knowledge of the key variables and the uncertainties surrounding them. Then the range of cash-flow possibilities during the stage can be broken down into two, three, or more "subsets," which can be used as discrete chance alternatives.

...

Peter F. Drucker has succinctly expressed the relation between present planning and future events: "Long-range planning does not deal with future decisions. It deals with the futurity of present decisions." Today's decision should be made in light of the anticipated effect it and the outcome of uncertain events will have on future values and decisions. Since today's decision sets the stage for tomorrow's decision, today's decision must balance economy with flexibility; it must balance the need to capitalize on profit opportunities that may exist with the capacity to react to future circumstances and needs.

Appendix

Figure A



For readers interested in further examples of decision-tree structure, I shall describe in this appendix two ...

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The unique feature of the decision tree is that it allows management to combine analytical techniques such as discounted cash flow and present value methods with a clear portrayal of the impact of future decision alternatives and events. Using the decision tree, management can consider various courses of action with greater ease and clarity. The interactions between present decision alternatives, uncertain events, and future choices and their results become more visible.

Surely the decision-tree concept does not offer final answers to managements making investment decisions in the face of uncertainty. We have not reached that stage, and perhaps we never will. Nevertheless, the concept is valuable for illustrating the structure of investment decisions, and it can likewise provide excellent help in the evaluation of capital investment *opportunities*.

A version of this article appeared in the July 1964 issue of *Harvard Business Review*.

JM

John F. Magee was chairman of the board of directors of Arthur D. Little, Inc.