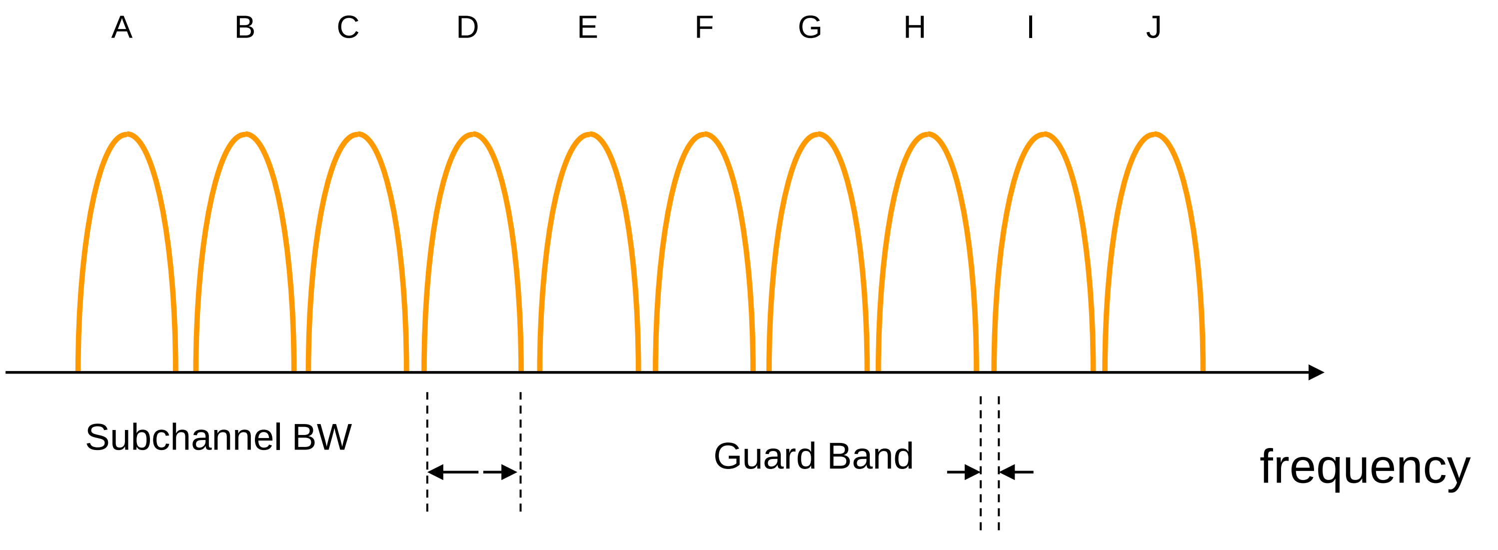
OFDM: Introduction and Foundations

Outline of this Talk

- **What** is OFDM? (basic definition)
- **Why** OFDM? (motivation)
- **How** can we transmit via OFDM?
- **When/Where** is OFDM used? (history & use)
- OFDM advantages and disadvantages

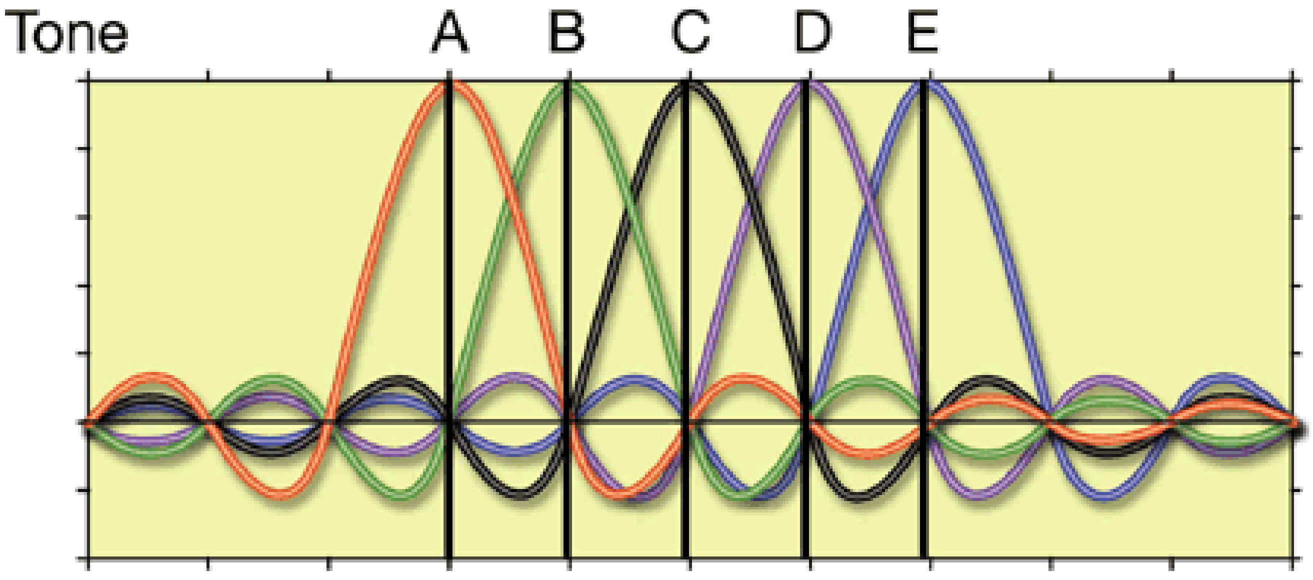
What is OFDM?

- Frequency-Division Multiplexing (FDM)
  - Data sent across various frequency channels.
  - Guard bands used to avoid interference between channels
  - Not very spectrally efficient.
  - Examples are AM radio, and analog TV transmission



What is OFDM?

- Orthogonal Frequency-Division Multiplexing

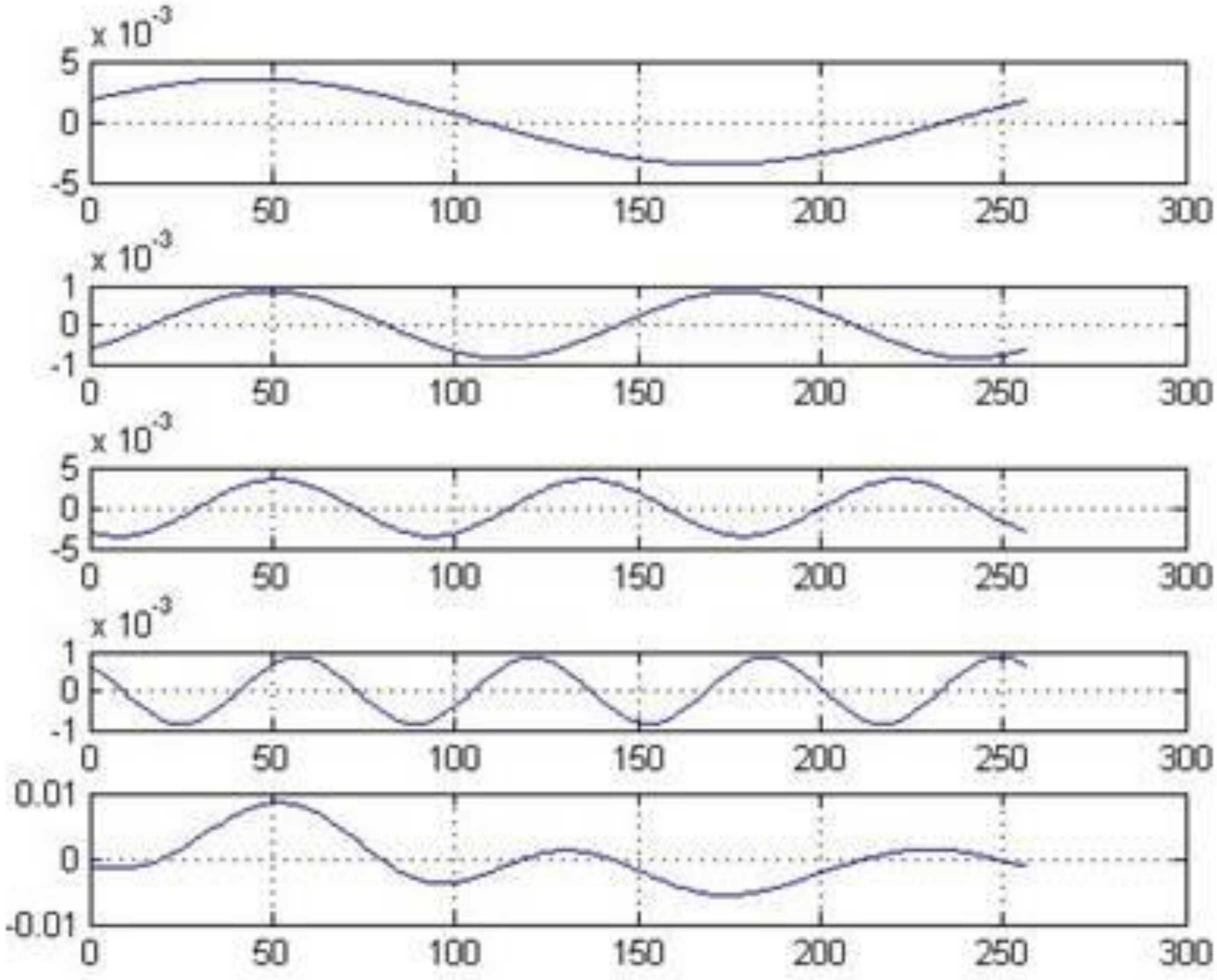


- FDM where carriers are appropriately spaced to insure orthogonality. Notice the overlap!!
- Spectrally efficient!!

What's So Great About Orthogonality?

- Transmit waveform for a given subchannel is orthogonal to that of the remaining subchannels.
- Same concept of CDMA signals having orthogonal spreading codes.
- At the receiver, an individual subchannel's data can be demodulated without interference from the others.
- Allows the receiver, in principle, to deal with each subchannel separately.

Orthogonal OFDM Signals



Why OFDM?

- Next-generation systems are demanding higher and higher bit rates.
  - i.e., requires more and more bandwidth (wider channel)
  - Wider channel increasingly less likely to be flat. (spectrally shaped)
  - Spectrally-shaped channels are the frequency-domain equivalent to time-domain dispersive channels.

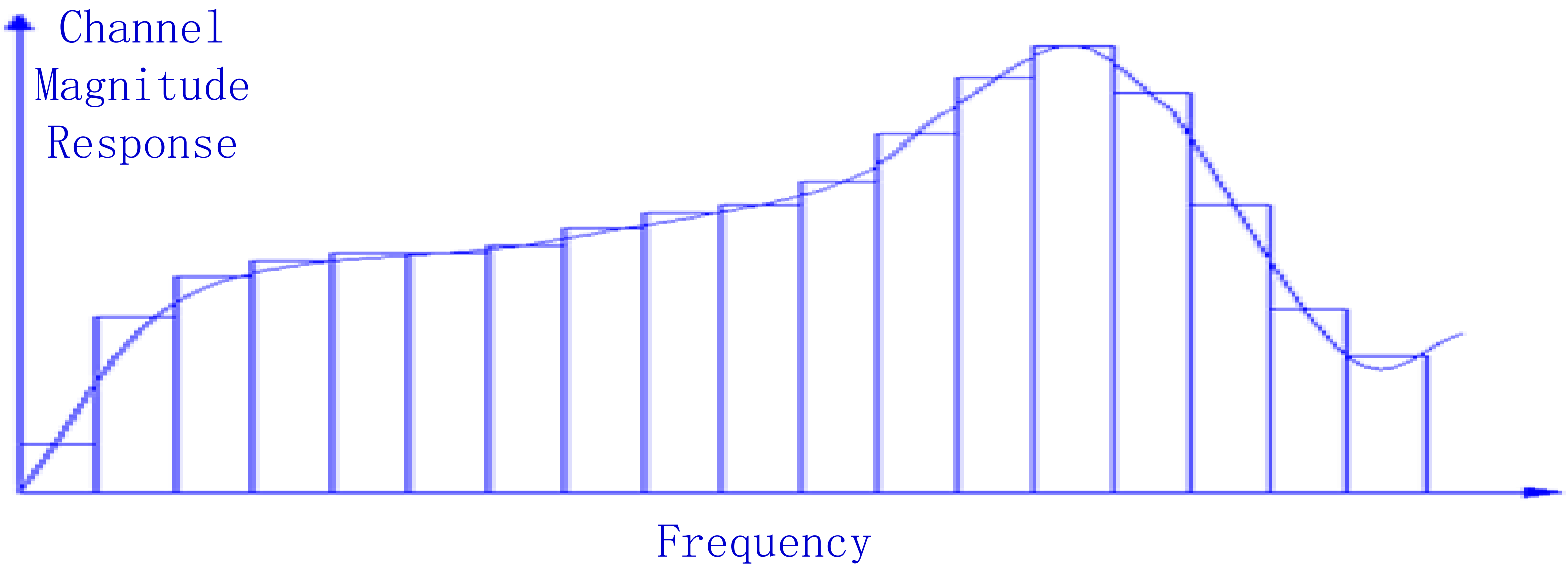
- Multipath propagation effects in wireless channels limit the increase of such rates.
  - These effects cause Intersymbol Interference (ISI).
  - Smearing of multiple adjacent data symbols with each other which increase the bit-error rate.

Why OFDM?

- Narrowband signals are less sensitive to ISI and frequency-selective fading.
  - “Flat channels” have no ISI (flatness in the frequency response).
  - They become additive white Gaussian noise (AWGN) channels.
  - Only effect on data symbols is white noise and a complex scaling (magnitude scaling and phase rotation)
- Solution: Transmit a wideband signal with many narrowband sub-bands! (Multicarrier System)

Multicarrier Concept

- Single-carrier system: transmit bits over one carrier.
- Multicarrier system: transmit bits over N subcarriers.
  - Divide channel into many subchannels (orthogonal for OFDM)
  - Large $N \longrightarrow$ approximately independent AWGN subchannels.



Why OFDM? A Simple Answer

- In theory, data transmitted over a given OFDM subchannel can be demodulated without interference from other sub-bands due to orthogonality.

- In theory, each subchannel can be individually equalized with a simple complex scalar multiplication.
  - High-rate single-carrier systems require very complicated adaptive equalizers whose performance can degrade with faster and faster data rates.

How is OFDM Implemented?

- Difficult to use analog hardware to modulate data onto many subchannels.

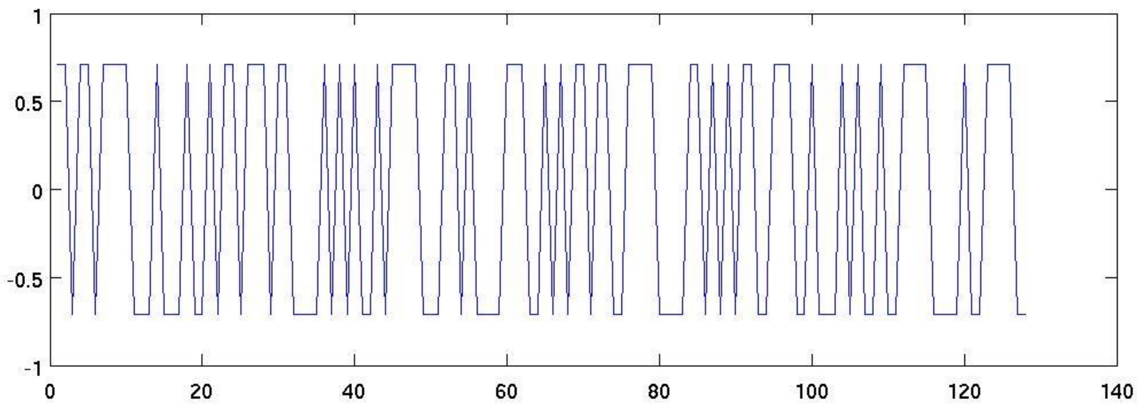
- Weinstein and Ebert (1971) discovered that a digital complex-baseband OFDM signal can be formed using the discrete Fourier transform (DFT).
  - DFT is an orthogonal transformation.
  - Time domain ←→ Discrete frequency domain.

- Better yet, let's use FFTs (which are the same, but implemented more efficiently!)

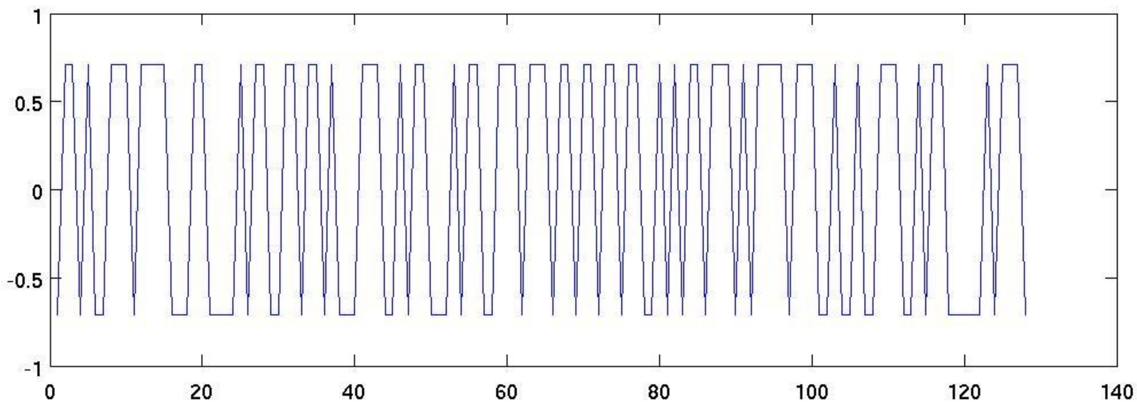
How is OFDM Implemented?

- (1) Let's take N data symbols and allocate one to each of N sub-bands.
  - Can be BPSK, QPSK, or M-QAM
- Consider these symbols to be in the ``frequency symbol domain''.
- (2) Apply an inverse FFT (IFFT) to obtain a length-N digital time-domain signal.
  - This is our digital complex-baseband OFDM signal.
- (3) Pass this signal through a D/A (or DAC) to form an analog signal.
- (4) Modulate the analog signal to a carrier frequency

Example: Frequency Symbol Domain

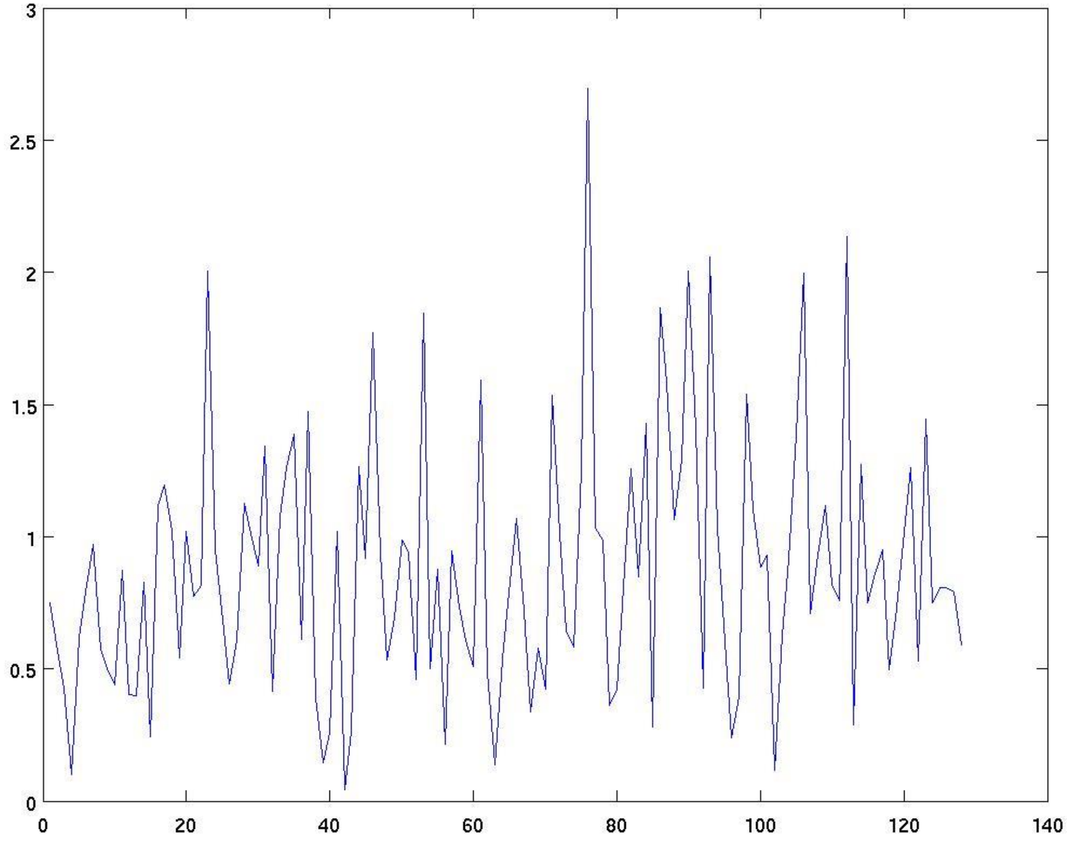


Real part
Of QPSK
symbols



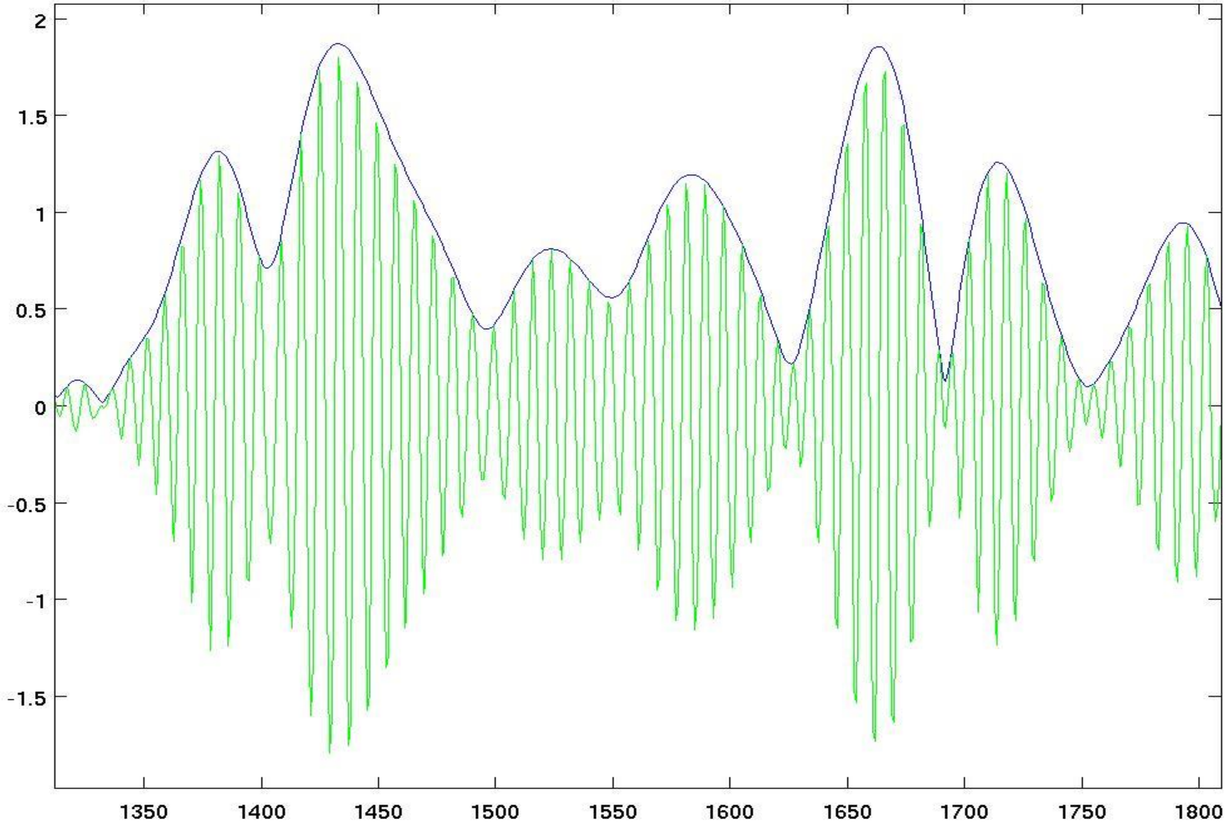
Imag part
Of QPSK
symbols

Example: Magnitude in Time Domain



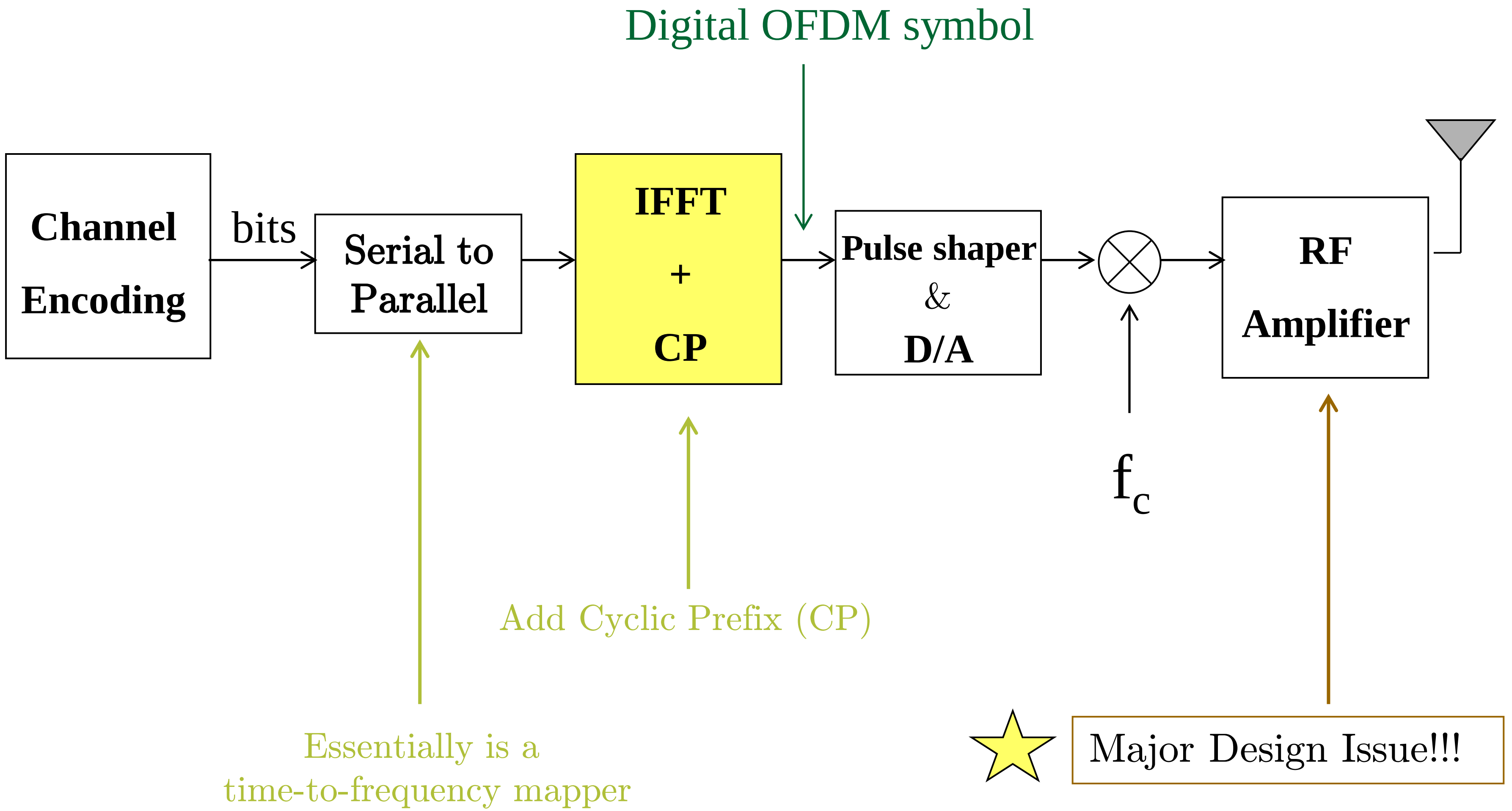
Magnitude of N-point time-domain signal after IFFT operation

Example: Zoomed-in of Analog Signal



Analog signal modulated onto a carrier

OFDM Transmitter



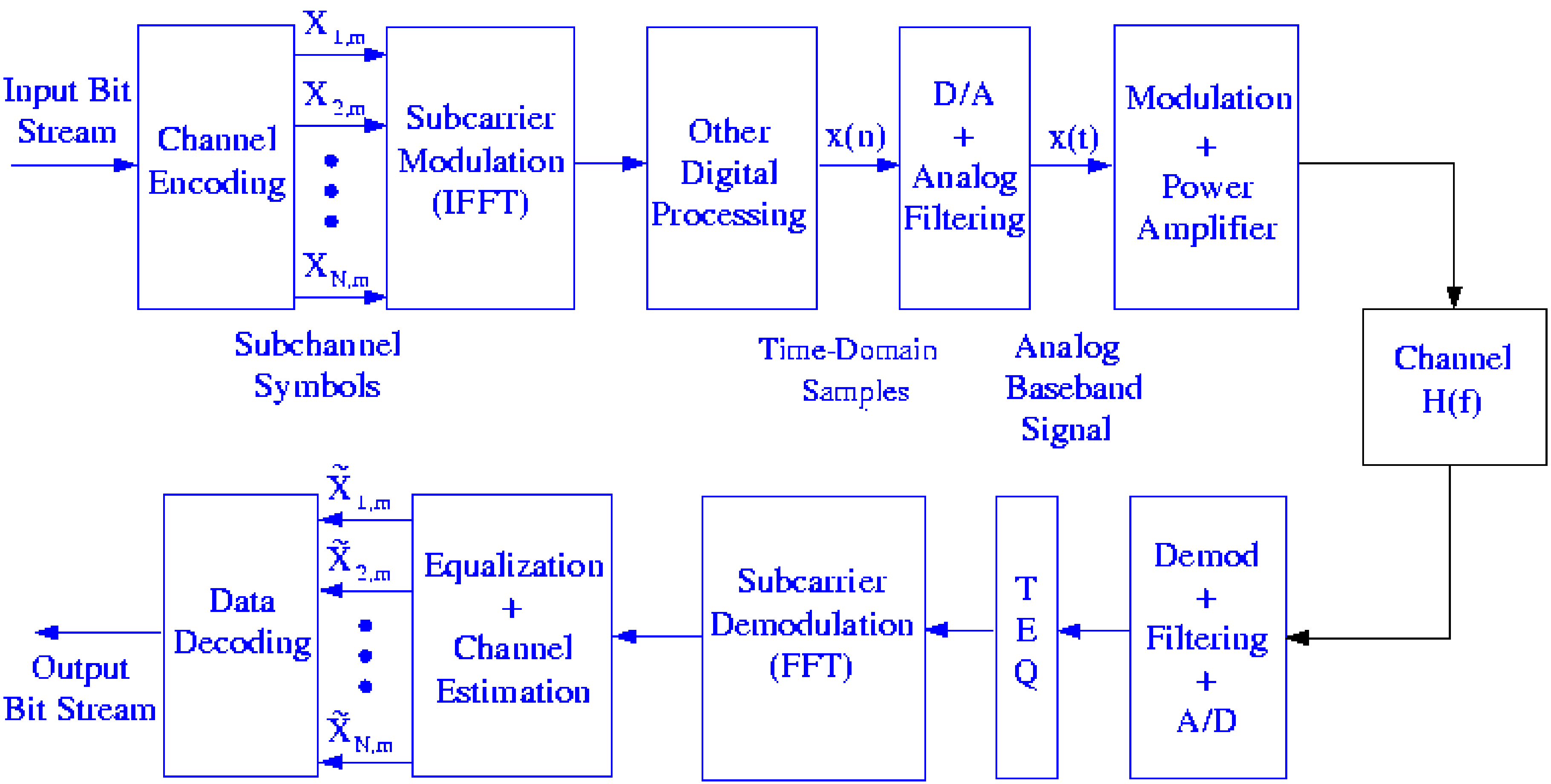
Some OFDM Maths

- OFDM uses a block transformation for modulation.
- Complex (Random) Symbols: $\mathbf{X} = [X_0 \ldots X_{N-1}]^T$
- Time-domain digital signal:

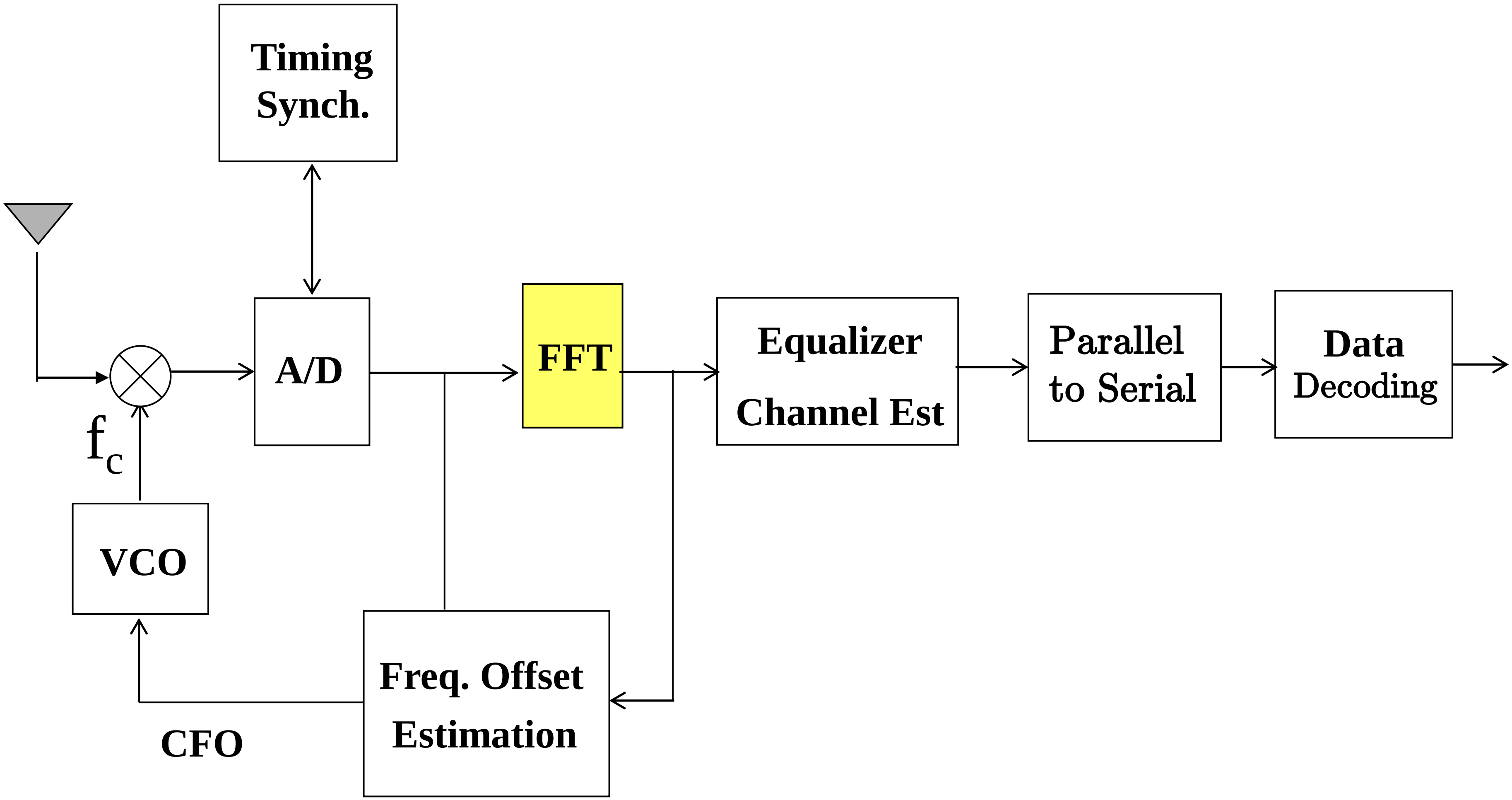
$$x(n) = \mathbf{x} = IFFT(\mathbf{X}) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp^{j2\pi kn/N}$$

- For large N, $x(n)$ are approximately i.i.d. complex Gaussian with variance σ_x^2.
  - Gaussian distributed due to central limit theorem effect

OFDM Transceiver



OFDM Receiver



Receiver: OFDM Equalization

Received Analog Signal:

$$\begin{aligned} y(t) &= x(t) * h(t) + w(t) \\ &\updownarrow \\ Y(f) &= X(f)H(f) + W(f) \end{aligned}$$

Received Digital Signal:

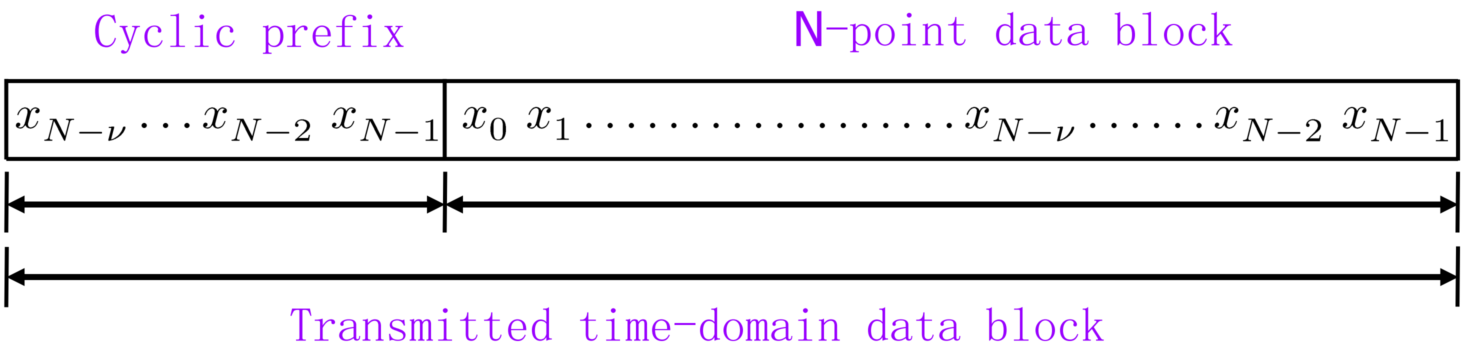
$$y(n) = x(n) * h(n) + w(n)$$

Question: At the output of the FFT, does $Y_k = X_k \cdot H_k$?

OFDM Equalization

- Answer: NO!!! Due to finite FFT block lengths
- FFT-domain multiplication results in time-domain circular convolution.
- Solution: *Force cyclic convolution* by making x(n) *appear* periodic to the channel.
- The Result: adding a cyclic prefix of equal or greater length than the channel impulse response ν.

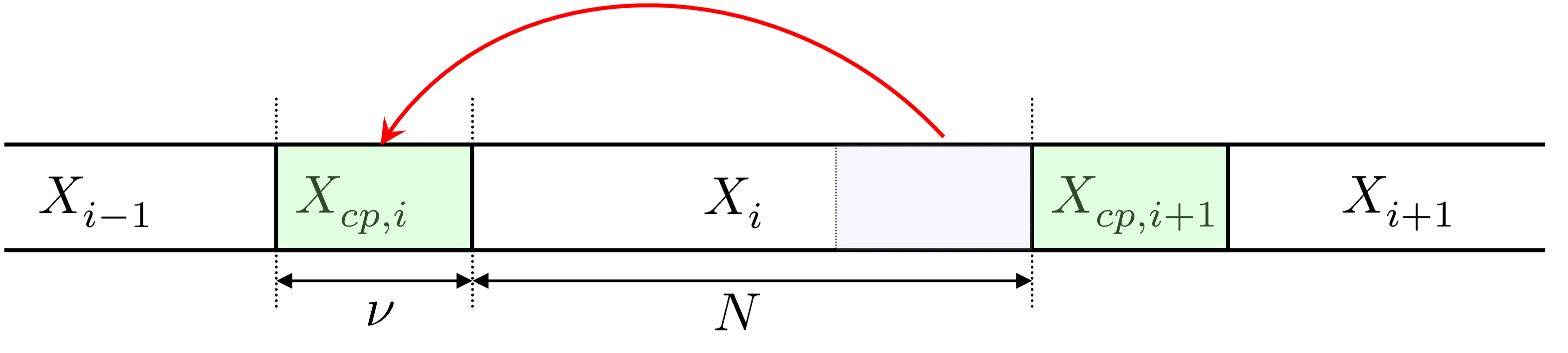
Cyclic Prefix and the FEQ



- Equalization: multiply FFT vectors $\mathbf{X}$ and $\mathbf{H}^{-1}$.
- Cyclic prefix makes signal appear more periodic.
  - Circular convolution mimics linear convolution.
- Also provides a guard interval between blocks.
- Downside: data rate reduced by factor $\frac{N}{N+\nu}$.

OFDM Transmission

- OFDM is a block transform method.
- A “block” consists of a single OFDM symbol and its cyclic prefix.
- A new block follows each previous block, and so on.



When Did OFDM Come About?

- 1966: R. W. Chang proposed OFDM for dispersive fading channels. Patent issued 1970.
- 1971: Weinstein and Ebert first proposed using the DFT for OFDM transmission.
- 1985: Cimini looked at the feasibility of OFDM transmission. Does a proof-of-concept design.
- 1987: Alard and Lasalle propose coded OFDM for digital broadcasting
- 1990s: Standards and implementation of OFDM in
  - Digital Audio Broadcasting (DAB)
  - Asymmetric Digital Subscriber Lines (ADSL)
  - Digital Video Broadcasting (DVB-T)
  - Wireless LAN standards (HIPERLAN2, IEEE 802.11a)
- What took so long for OFDM to come to fruition?
  - FFTs were too expensive to implement pre-1990s. They are now cheap to implement, and OFDM can have less computational complexity than conventional single-carrier systems in some systems.

Recent/Future Use of OFDM

- IEEE 802.11n (MIMO WLAN)
- IEEE 802.16e (WiMAX), WiBRO, 802.20
  - Wireless broadband standards
- DVB-T2 (next-gen digital video broadcasting)
  - DVB-H standardized for handheld devices.
- LTE / 4G mobile communications
- Optical OFDM? (coherent and non-coherent)

Advantages of OFDM

- OFDM is spectrally efficient (remember the overlap!)
- Robust to multipath interference
  - Subchannels are narrowband with essentially zero ISI
  - Simple equalization compared to single-carrier systems.
- Robust to narrowband interference
  - Can always not use (i.e., turn off) any bad subchannels
- Computationally efficient compared to single-carrier.
- Simple exploitation of frequency diversity (COFDM)

Disadvantages of OFDM

- Very sensitive to carrier-frequency offset (CFOs)
  - Causes a loss in orthogonality resulting in inter-channel interference (ICI).

- High peak-to-average power ratio (PAPR).
  - Occasional large peaks require an expensive high-powered amplifier (HPA) for clean (i.e., linear) transmission.
  - Low power efficiency results to handle a large dynamic range.
  - Any nonlinear amplification will destroy orthogonality!
    - Introduces out-of-band distortion, which is a big "No-No"!!

- Sensitive to clock frequency and timing offsets

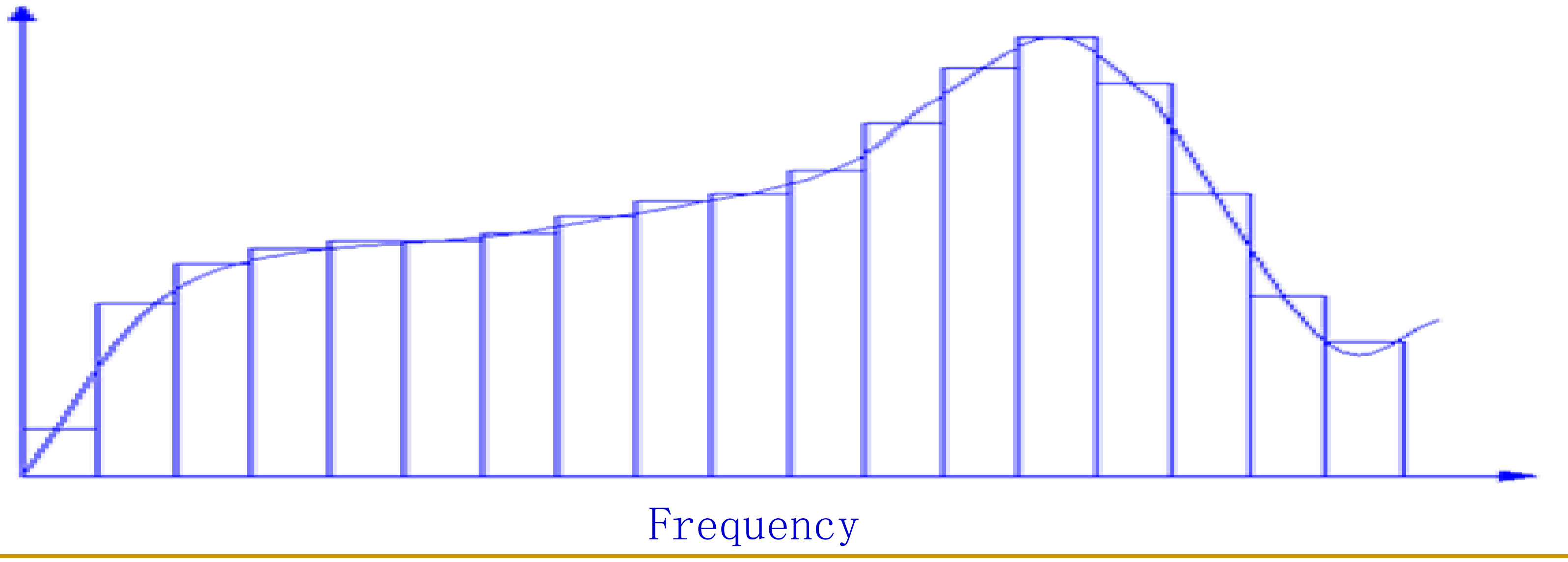
Channel Estimation and Equalization

Channel Estimation

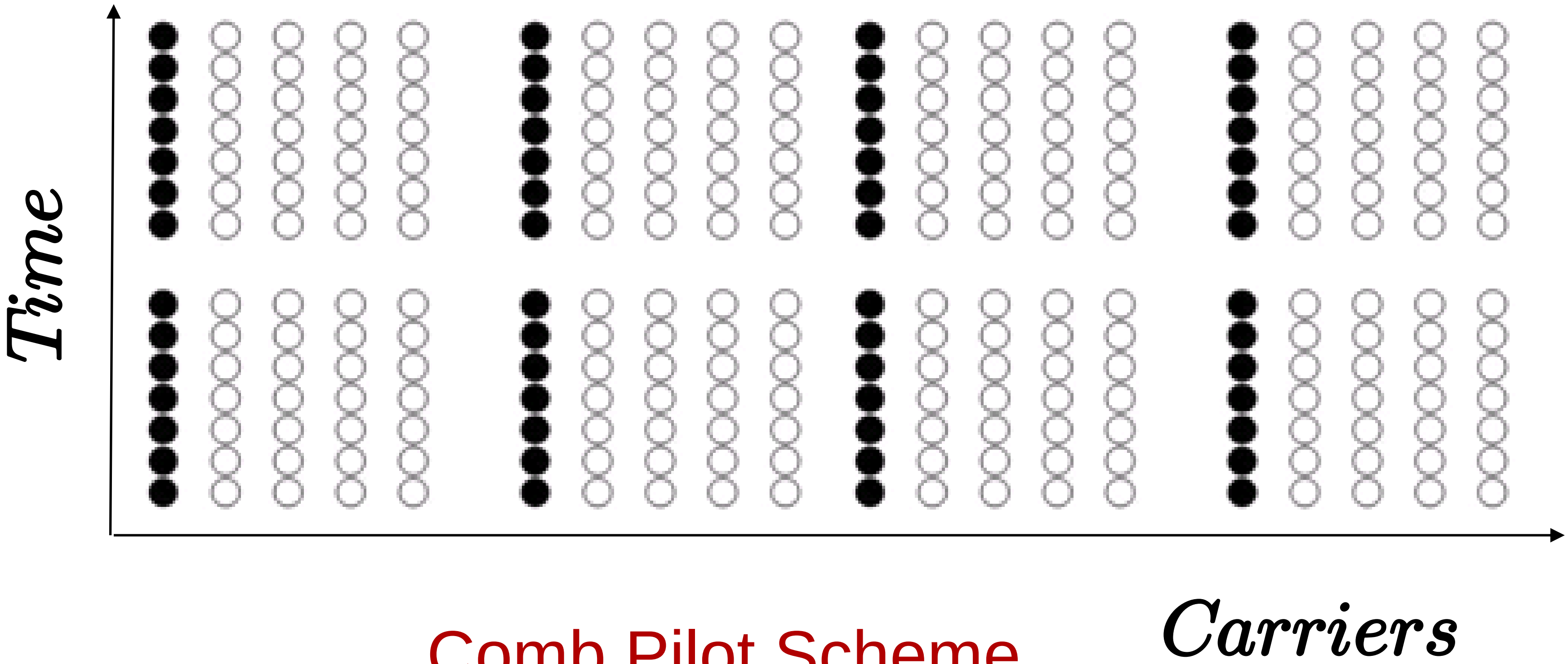
- Need to estimate the complex channel gain in each subchannel in order to equalize.
  - Notice: equalization uses a one-tap complex scalar filter in each subchannel. Very simple and efficient.
- Can estimate each H_k with pilot tones.
- Better idea: utilize bandwidth coherency!!
  - Use pilot tones on some subchannels followed by smart interpolation to estimate remaining ones.

Bandwidth Coherency

- Nearer subchannels have very similar frequency response.
- Farther subchannels have statistically independent frequency responses.



Channel Estimation



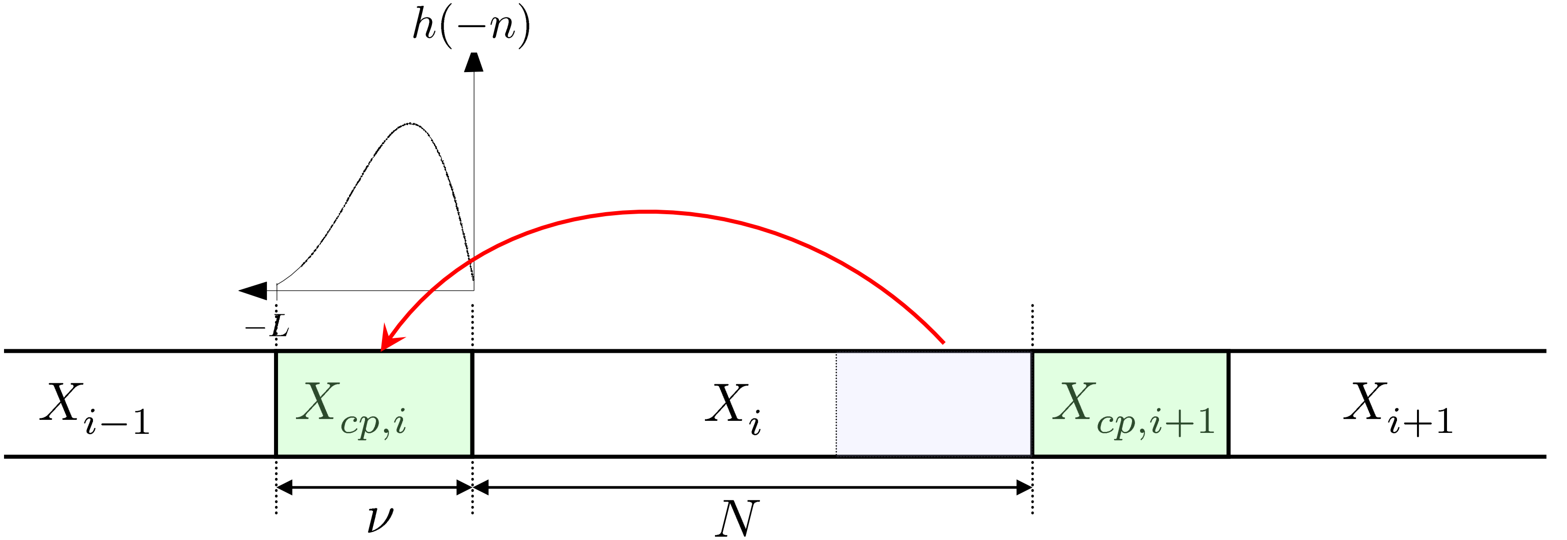
Comb Pilot Scheme

- Interpolate between the pilot tones to get the frequency response of each channel.

Time & Frequency Synchronization

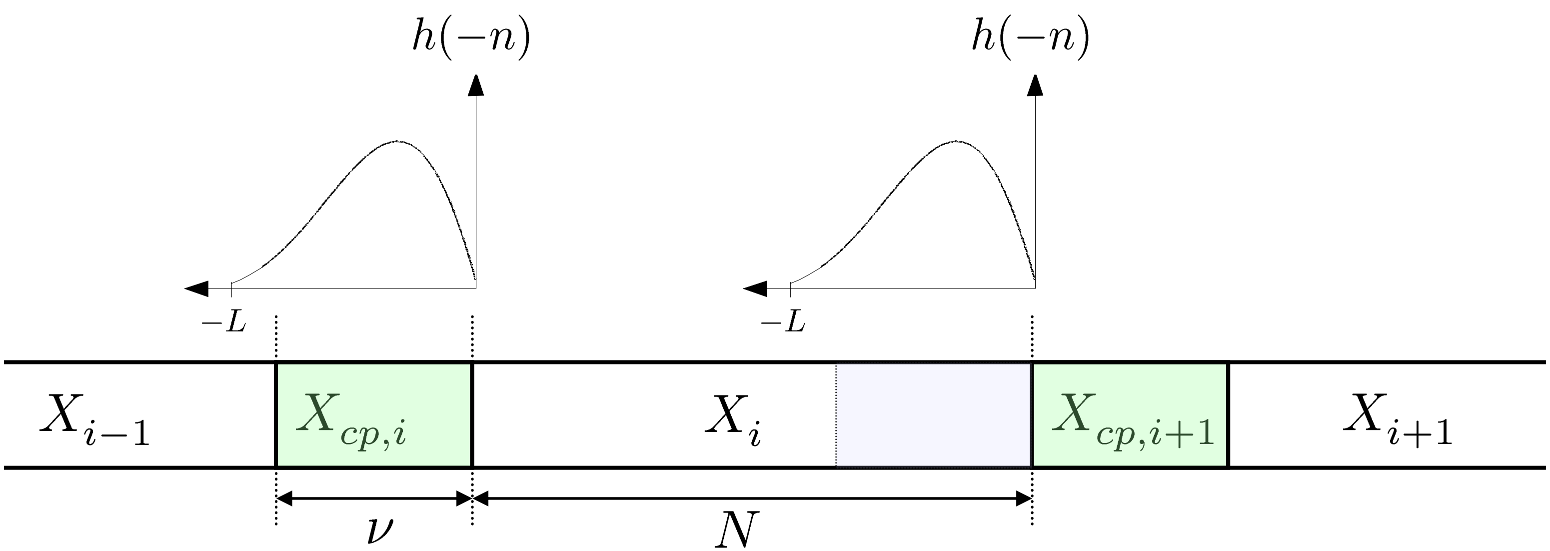
OFDM Time Synchronization

- Must determine when an OFDM block starts.
- Channel dispersion can complicate this.
  - If $L \leq \nu$, perfect equalization is possible.
- Once determined, start time is $N + \nu$ periodic.



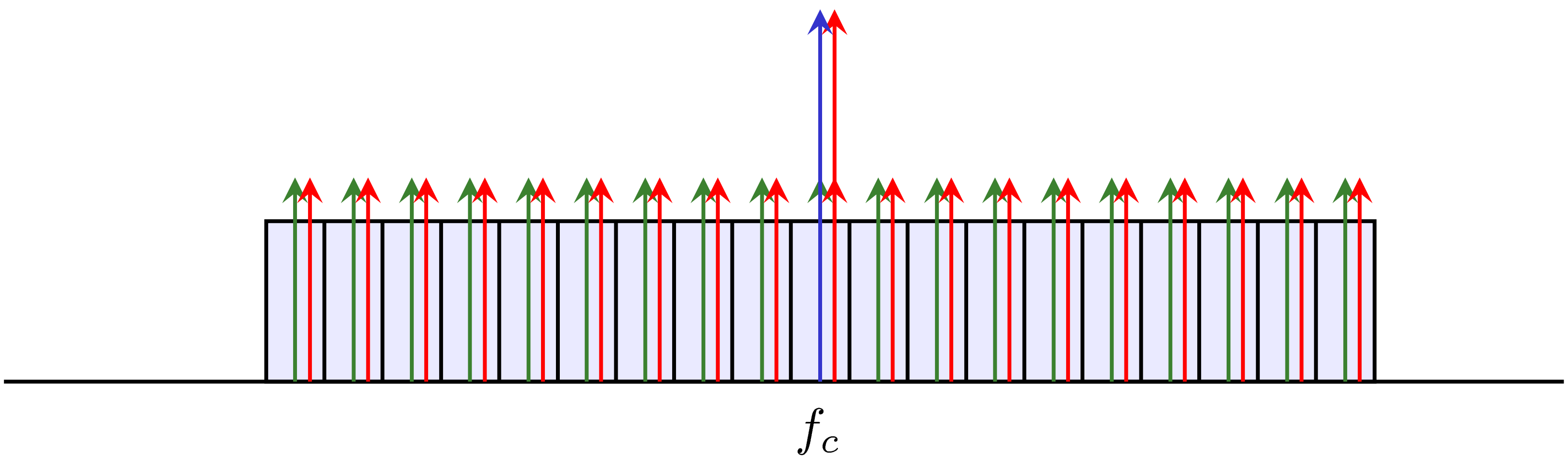
Interference Issues

- If $L > \nu$, perfect equalization is impossible.
  - Interblock interference (IBI) occurs.
- Subchannel orthogonality has been compromised.
  - Interchannel interference (ICI) results.



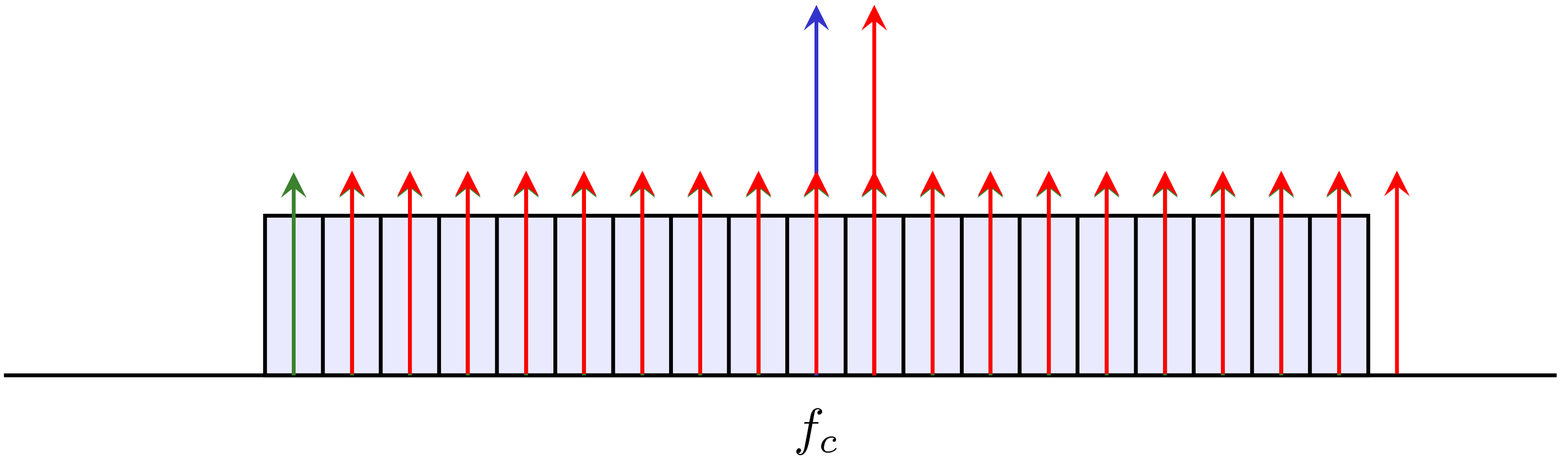
OFDM Carrier-Frequency Recovery

- Recover N subcarriers through one carrier.
- Subchannel bandwidth $\frac{W}{N}$ can be quite small.
- Even slight CFOs can be problematic.



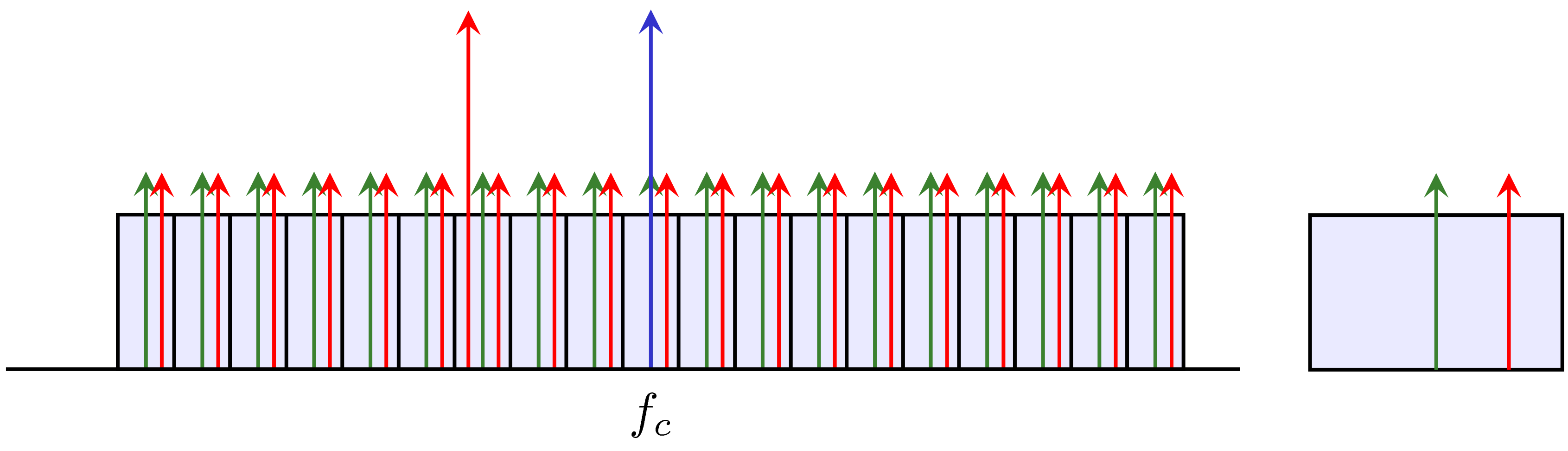
OFDM CFO Sensitivity Example

- Data demodulated into wrong subchannels!!



OFDM Carrier-Frequency Recovery

- Two-stage carrier-recovery problem.
  - Frequency-acquisition: obtain $|\hat{f}_c - f_c| < \frac{W}{2N}$
  - CFO estimation: "fine tune" the CFO towards zero.

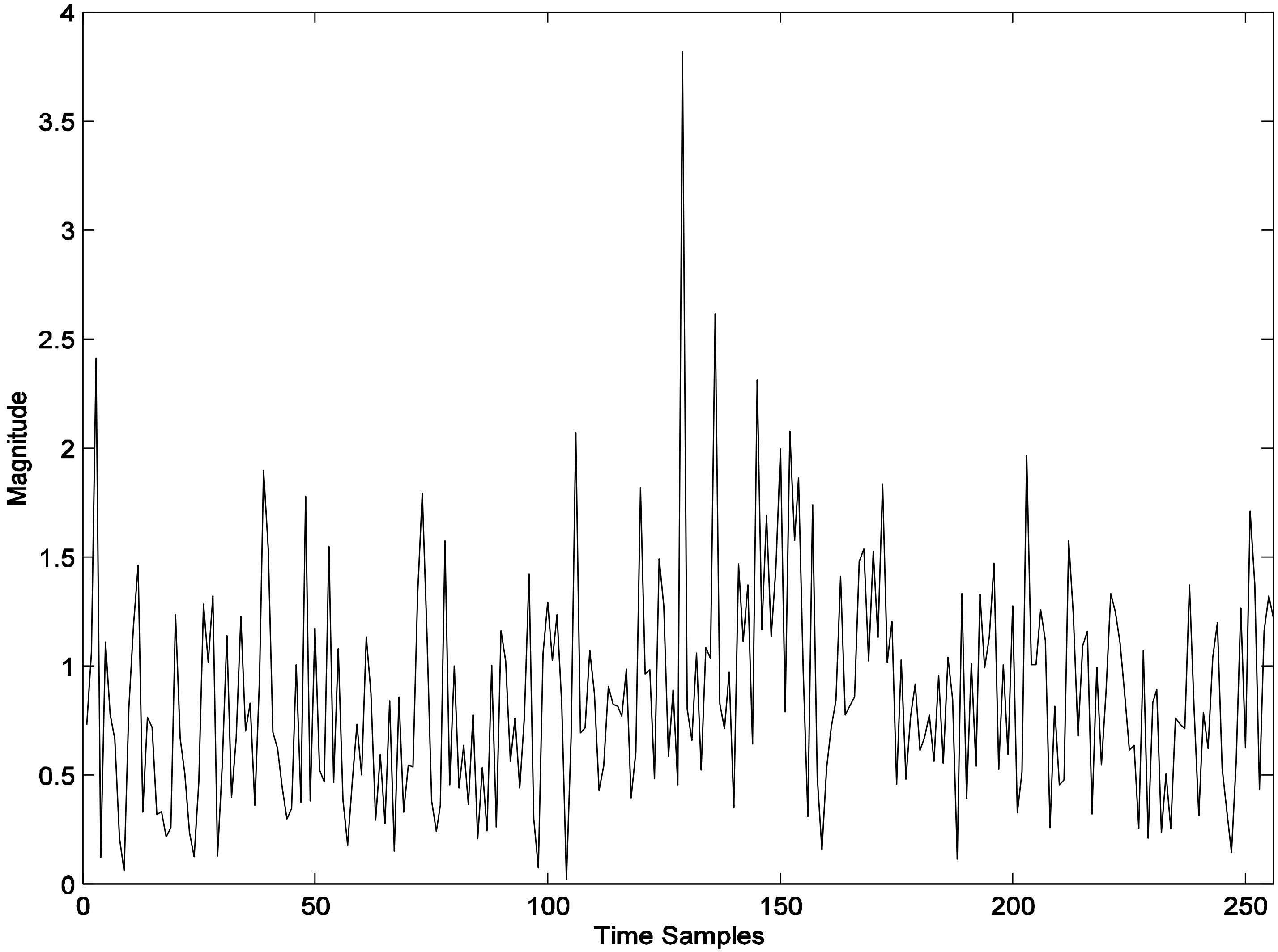


Power Ratio (PAPR or PAR) Problem

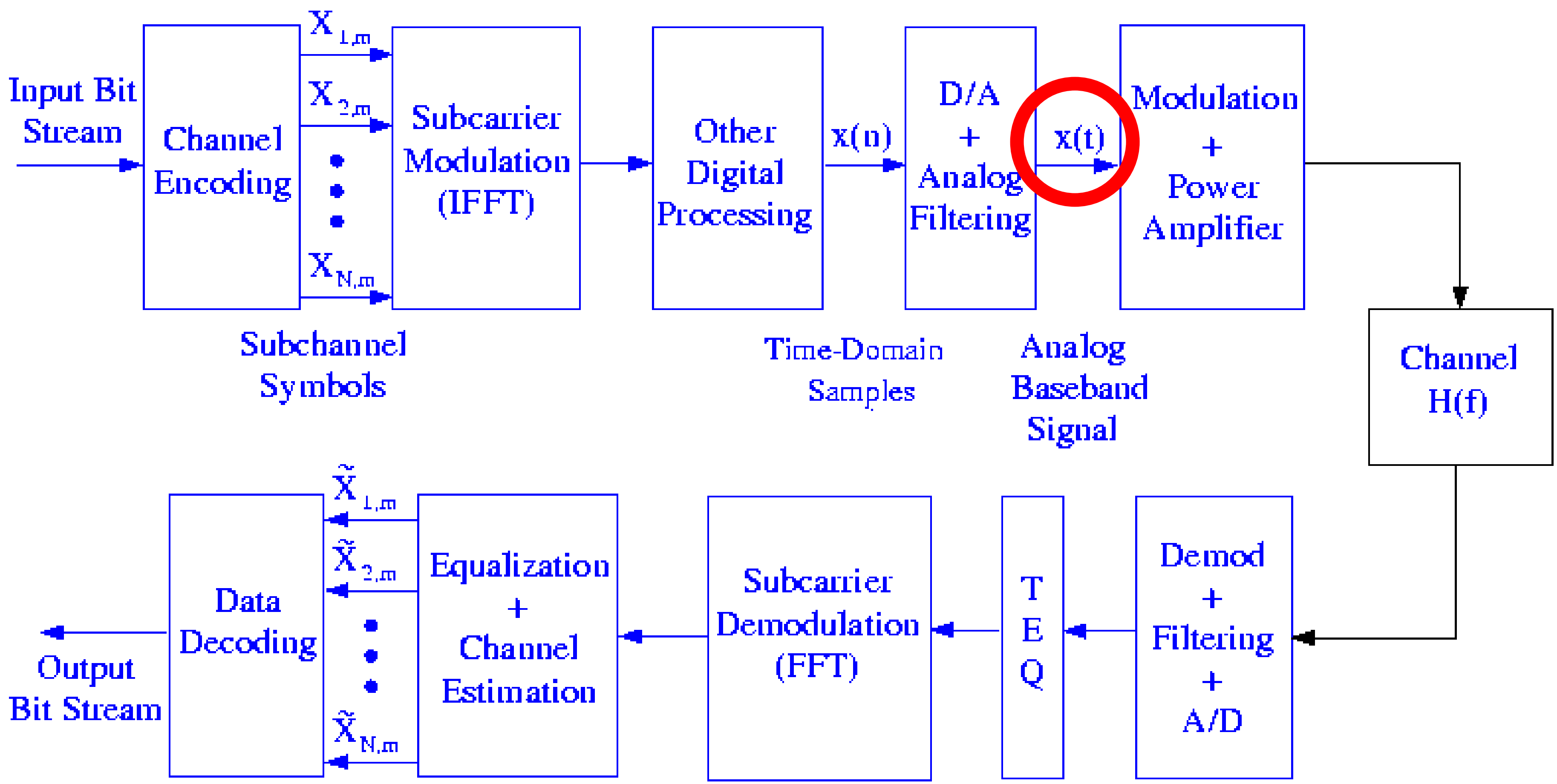
Why does the PAPR Problem Occur?

- Time-domain samples are linear combinations of random variables. If N is large, a central limit theorem effect begins.
- The time-domain samples become approximately Gaussian distributed, and the tails are our “occasional large peaks”.
- A High Powered Amplifier (HPA) essential consume power in relation to their peak power, and not the average power.
  - Creates a HUGE power cost for base stations if an expensive HPA with a large dynamic range is used.
- **NOTICE: This is an <u>ANALOG problem</u>. Viewing PAR results of the digital OFDM signal are not truly indicative of the analog PAR.**

Example Signal: 11.63 dB PAR



OFDM Transceiver



OFDM Resource Allocation

Transmit Optimization with CSI

- If the transmitter has full or partial channel-state information (CSI), resources such as power and data rate can be allocated according to subchannel.
- Optimization problems result such as maximizing rate subject to a total power constraint.
- Transmit optimized OFDM with full CSI can approach channel capacity.
  - Identical concept to water pouring to achieve capacity.