



CARRERA DE ECONOMÍA ESTADÍSTICA INFERENCIAL DISTRIBUCIÓN PROBABILIDADES CONTINUAS

**UNIVERSIDAD
NACIONAL DE
CHIMBORAZO**

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CONCEPTOS
BÁSICOS DE DISTRIBUCIÓN
PROBABILIDADES CONTINUAS
DISTRIBUCIÓN NORMAL


$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Donde:

X es la variable aleatoria

Parámetros:

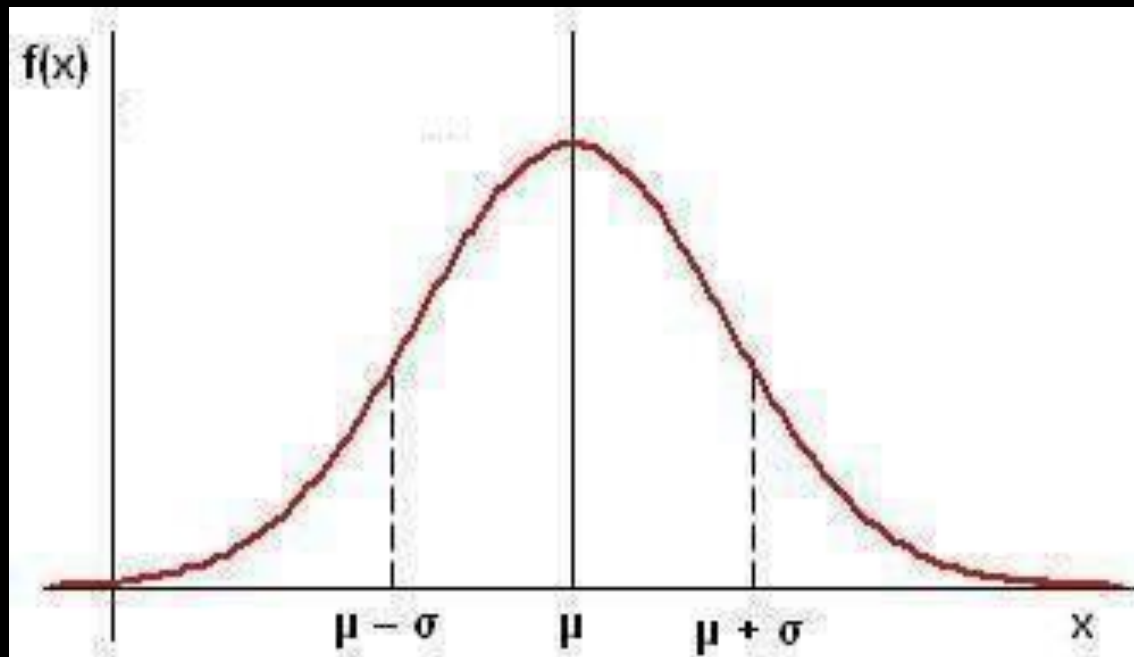
μ es el $E(x)$ - Media

σ es la desviación estándar


$$x \sim N(\mu, \sigma^2)$$

Distribución Normal - Características

- La función es asintótica al eje x. Cuando la función tiende a cero, los valores de la variable tienden a infinito (positivo y negativo)
- El valor máximo es cuando $f(x)=f(\mu)$
- La función es simétrica alrededor de la media (μ)
- Los puntos de inflexión son $\mu-\sigma$ y $\mu+\sigma$



Distribución Normal – Tipificación de variables

Para facilitar los cálculos y que puedan emplearse los valores de la tabla de la distribución normal, se transforma la variable (tipifica) para que la media sea 0 y la varianza 1.

Para la tipificación se resta la media y se divide entre la desviación

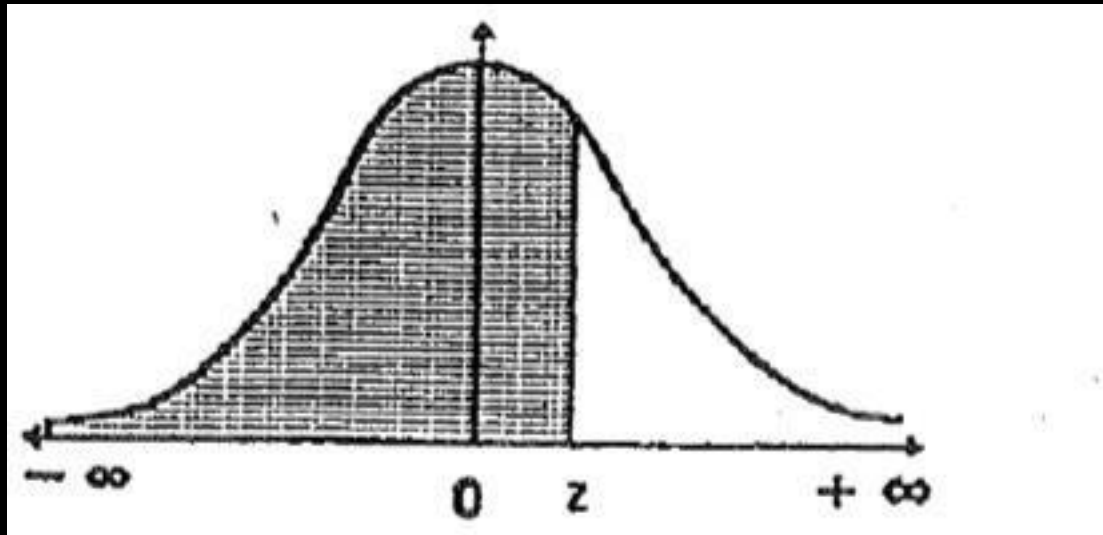
$$x \sim N(\mu, \sigma^2)$$

$$y = \frac{x - \mu}{\sigma} \rightarrow y \sim N(0, 1)$$

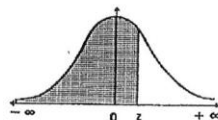
Distribución Normal – TABLA

La tabla indica la probabilidad acumulada hasta un determinado valor que haya sido tipificado

$P\ y \leq y_o$ Probabilidad acumulada hasta y_o
 $y \sim N(0,1)$



FUNCIÓN DE DISTRIBUCIÓN NORMAL $N(0,1)$

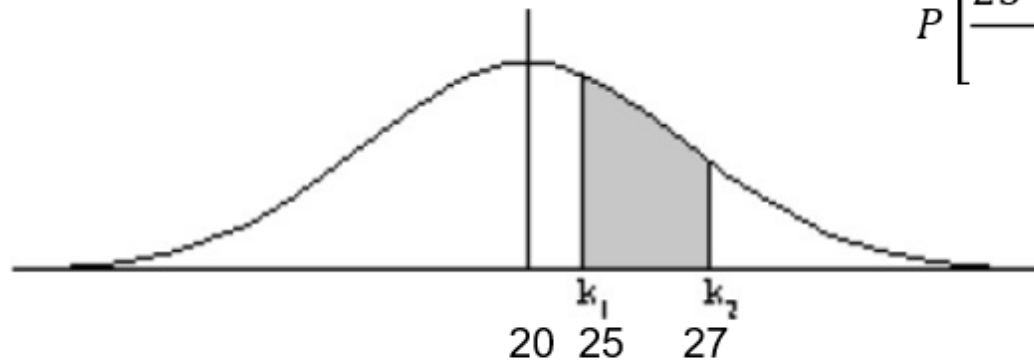


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990
3.1	0.9990	0.9990	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992
3.2	0.9993	0.9993	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995
3.3	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996
3.4	0.9996	0.9996	0.9996	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
3.5	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.0	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

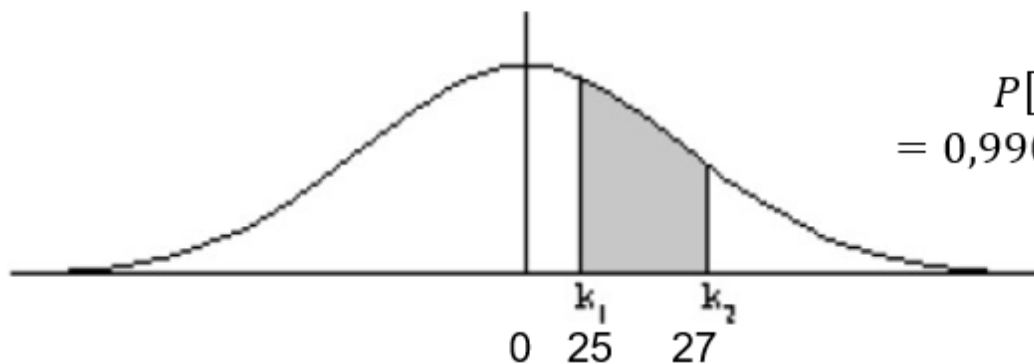
Nota: En el interior de la tabla se da la probabilidad de que la variable aleatoria Z , con distribución $N(0,1)$, esté por debajo del valor z .

Distribución Normal – EJERCICIOS

$$P[25 \leq x \leq 27]; x \sim N(20, 9)$$



$$P\left[\frac{25 - 20}{3} \leq y \leq \frac{27 - 20}{3}\right]$$
$$P\left[\frac{5}{3} \leq y \leq \frac{7}{3}\right]$$



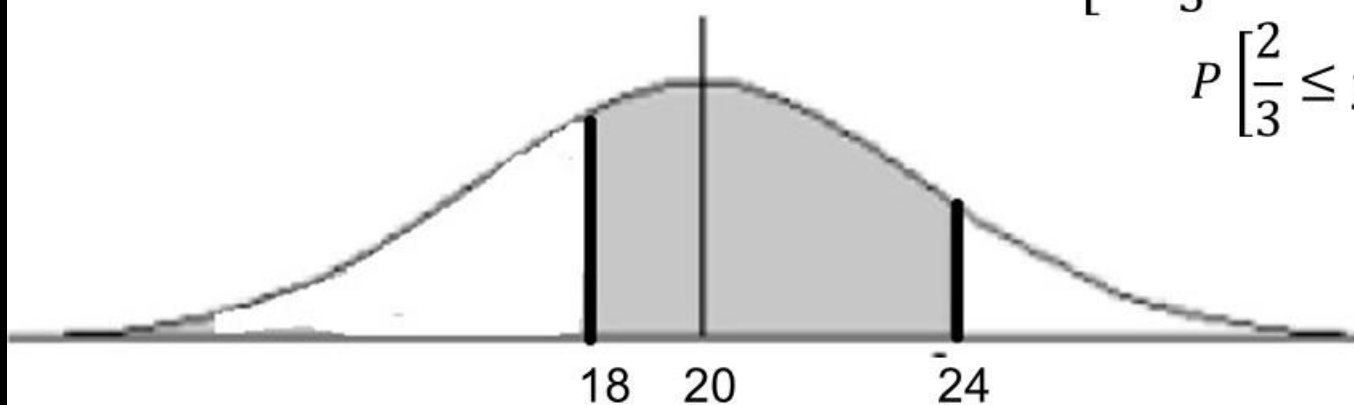
$$P[1,67 \leq y \leq 2,33]$$
$$= 0,9901 - 0,9225 = 0,0376$$

Distribución Normal – EJERCICIOS

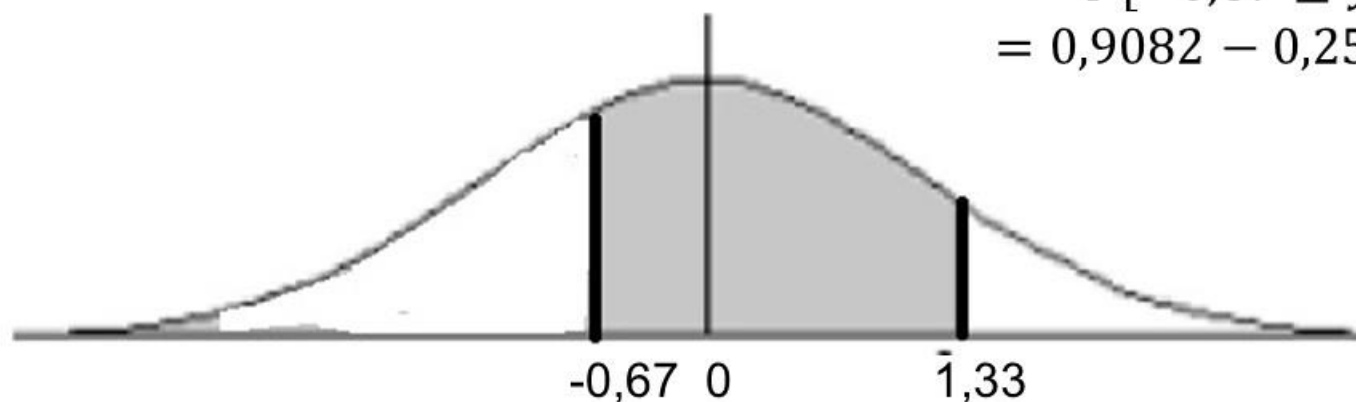
$$P[18 \leq x \leq 24]; x \sim N(20, 9)$$

$$P\left[\frac{18 - 20}{3} \leq y \leq \frac{24 - 20}{3}\right]$$

$$P\left[\frac{2}{3} \leq y \leq \frac{4}{3}\right]$$



$$P[-0,67 \leq y \leq 1,33]$$
$$= 0,9082 - 0,2514 = 0,6568$$



Distribución Normal – EJERCICIOS

$$P[x \leq 23]; x \sim N(20, 9)$$



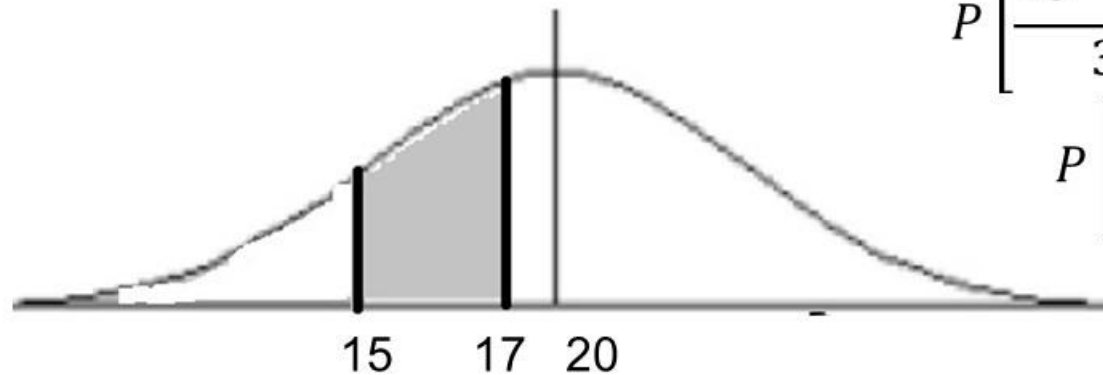
$$P\left[y \leq \frac{23 - 20}{3}\right]$$
$$P\left[y \leq \frac{3}{3}\right]$$

$$P[y \leq 1] = 0,8413$$

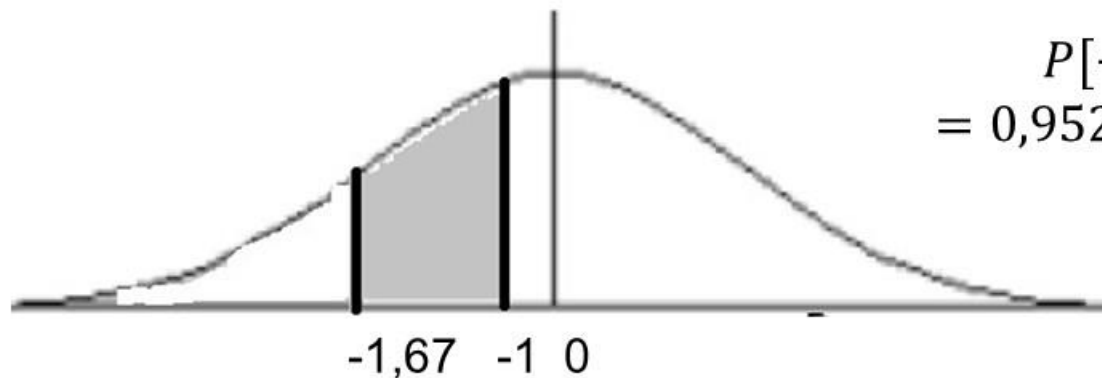


Distribución Normal – EJERCICIOS

$$P[15 \leq x \leq 17]; x \sim N(20, 9)$$



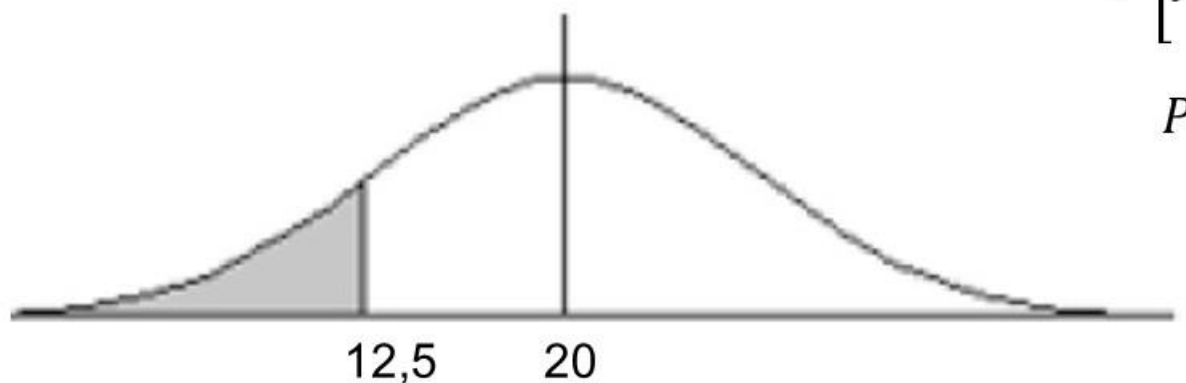
$$P\left[\frac{15 - 20}{3} \leq y \leq \frac{17 - 20}{3}\right]$$
$$P\left[-\frac{5}{3} \leq y \leq -\frac{3}{3}\right]$$



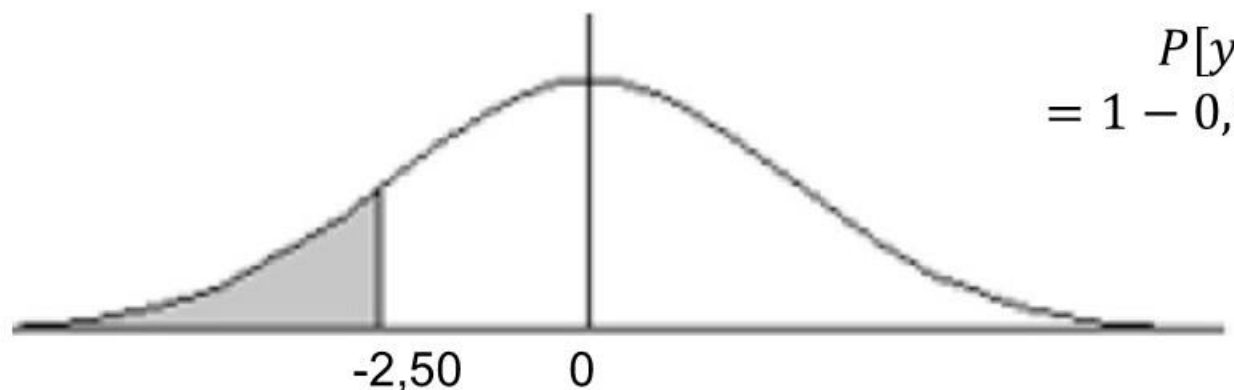
$$P[-1,67 \leq y \leq -1]$$
$$= 0,9525 - 0,8413 = 0,1112$$

Distribución Normal – EJERCICIOS

$$P[x \leq 12,5]; x \sim N(20,9)$$



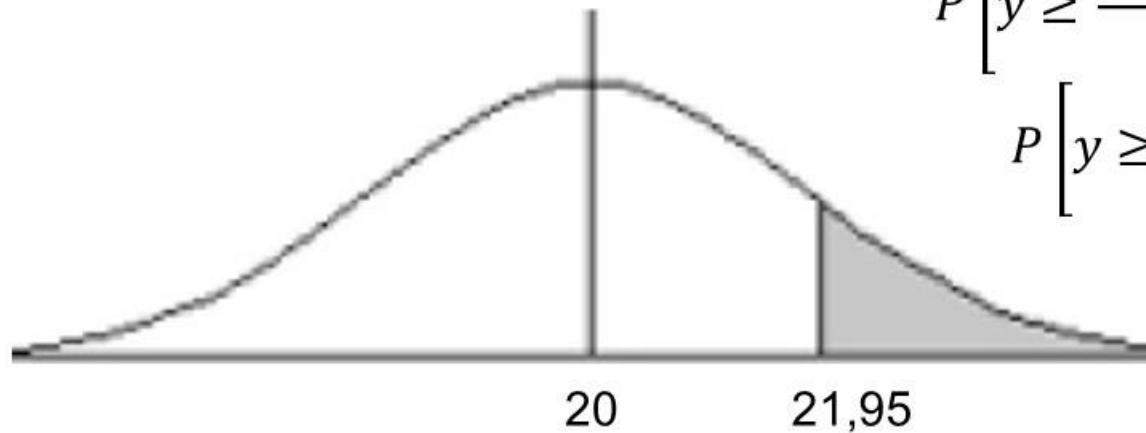
$$P\left[y \leq \frac{12,5 - 20}{3}\right]$$
$$P\left[y \leq -\frac{7,5}{3}\right]$$



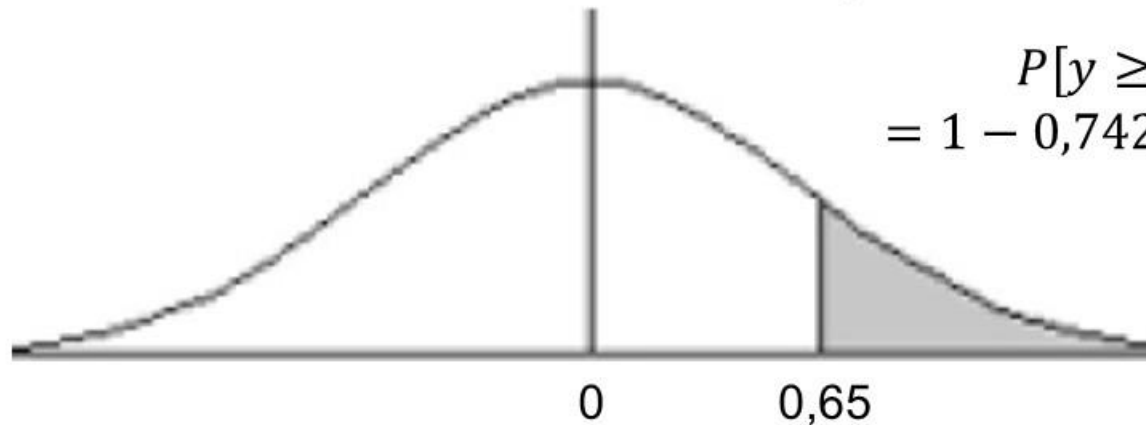
$$P[y \leq -2,50]$$
$$= 1 - 0,9938 = 0,0062$$

Distribución Normal – EJERCICIOS

$$P[x \geq 21,95]; x \sim N(20,9)$$



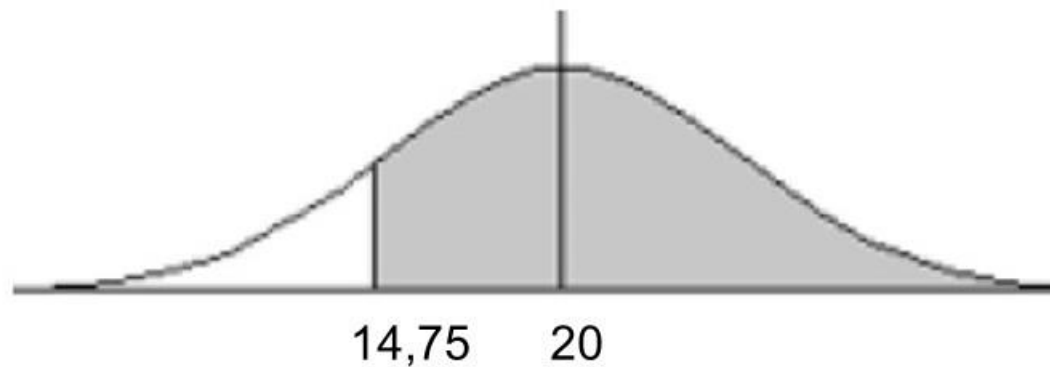
$$P\left[y \geq \frac{21,95 - 20}{3}\right]$$
$$P\left[y \geq \frac{1,95}{3}\right]$$



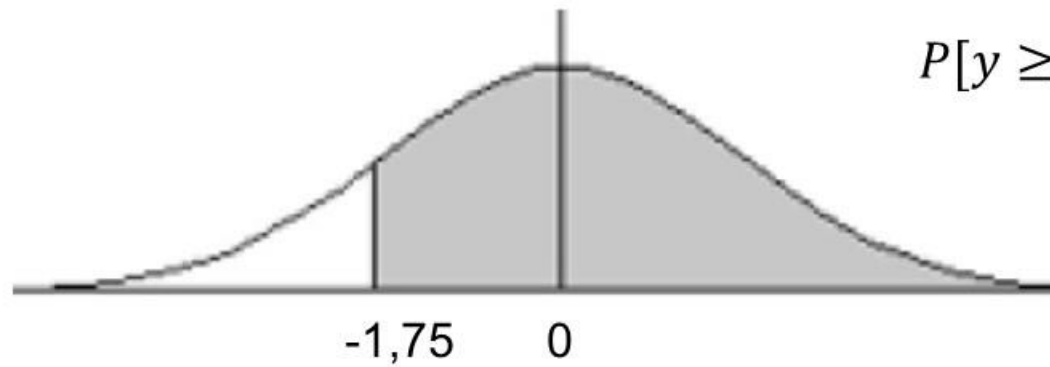
$$P[y \geq 0,65]$$
$$= 1 - 0,7422 = 0,2578$$

Distribución Normal – EJERCICIOS

$$P[x \geq 14,75]; x \sim N(20,9)$$



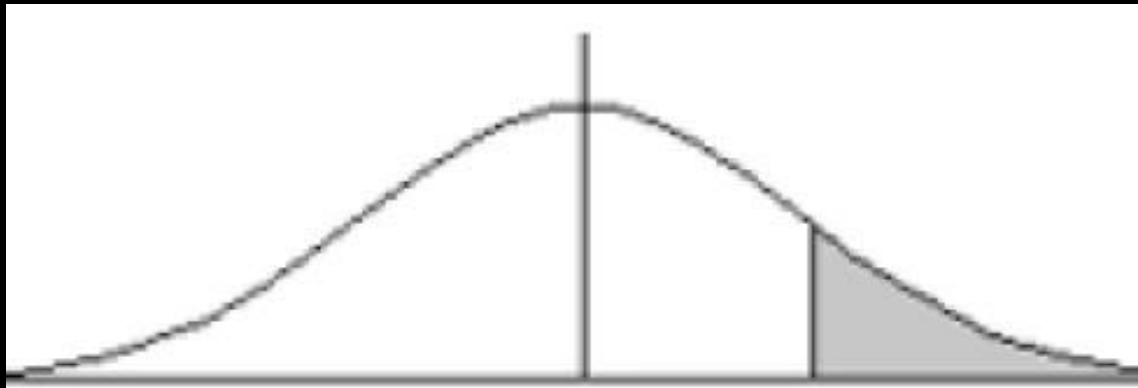
$$\left[y \geq \frac{14,75 - 20}{3} \right]$$
$$P \left[y \geq -\frac{5,25}{3} \right]$$



$$P[y \geq -1,75] = P[y \leq 1,75]$$
$$= 0,9599$$

Distribución Normal – NÚMERO K_α

Es el número de la variable tipificada que tiene a su derecha un área (probabilidad de α)



Números K_α importantes:

$$K_{0,01} = 2,326$$

$$K_{0,025} = 1,96$$

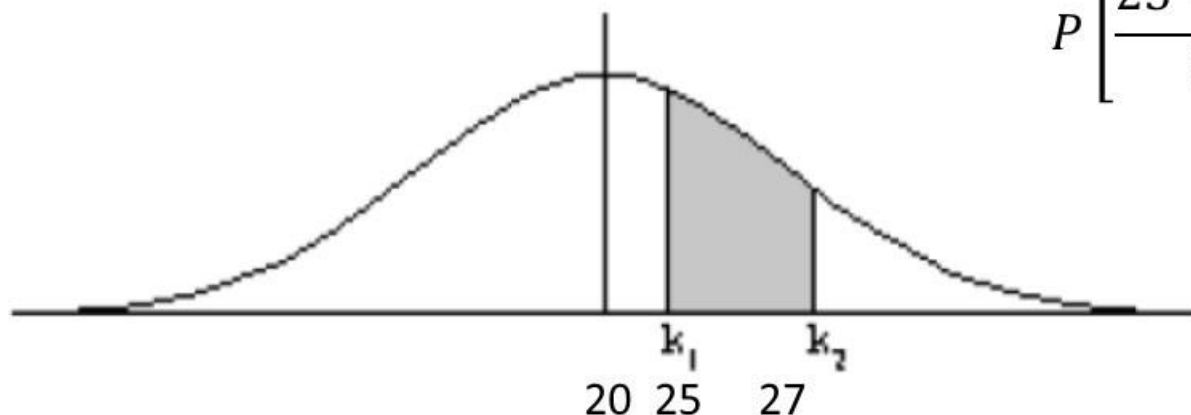
$$K_{0,05} = 1,645$$

EJERCICIOS

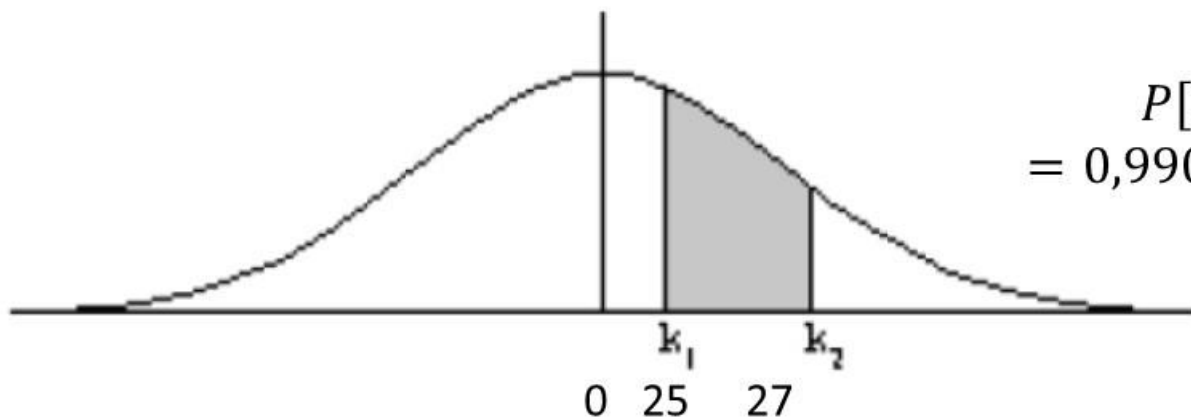


Distribución Normal – EJERCICIOS

$$P[25 \leq x \leq 27]; x \sim N(20, 9)$$



$$P\left[\frac{25 - 20}{3} \leq y \leq \frac{27 - 20}{3}\right]$$
$$P\left[\frac{5}{3} \leq y \leq \frac{7}{3}\right]$$



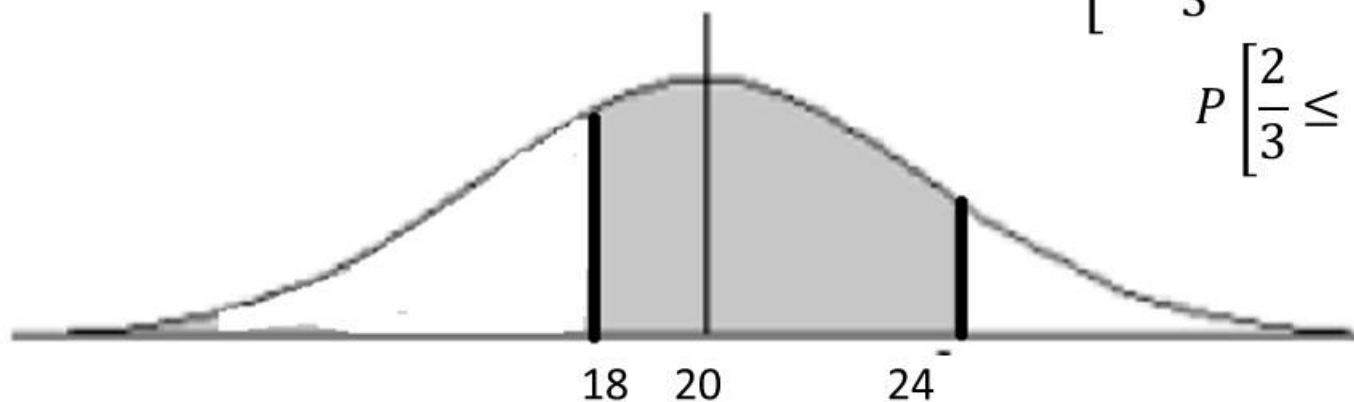
$$P[1,67 \leq y \leq 2,33]$$
$$= 0,9901 - 0,9225 = 0,0376$$

Distribución Normal – EJERCICIOS

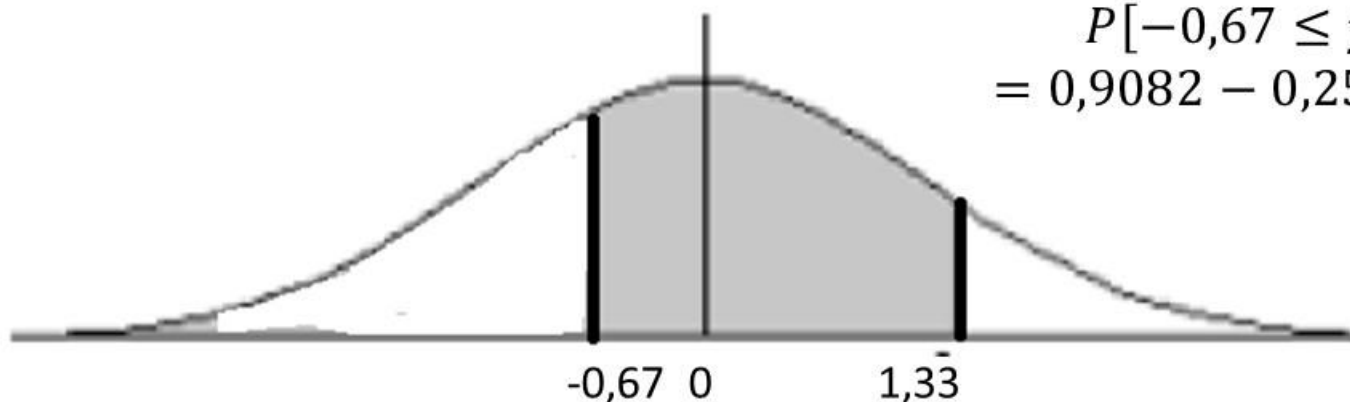
$$P[18 \leq x \leq 24]; x \sim N(20, 9)$$

$$P\left[\frac{18 - 20}{3} \leq y \leq \frac{24 - 20}{3}\right]$$

$$P\left[\frac{2}{3} \leq y \leq \frac{4}{3}\right]$$

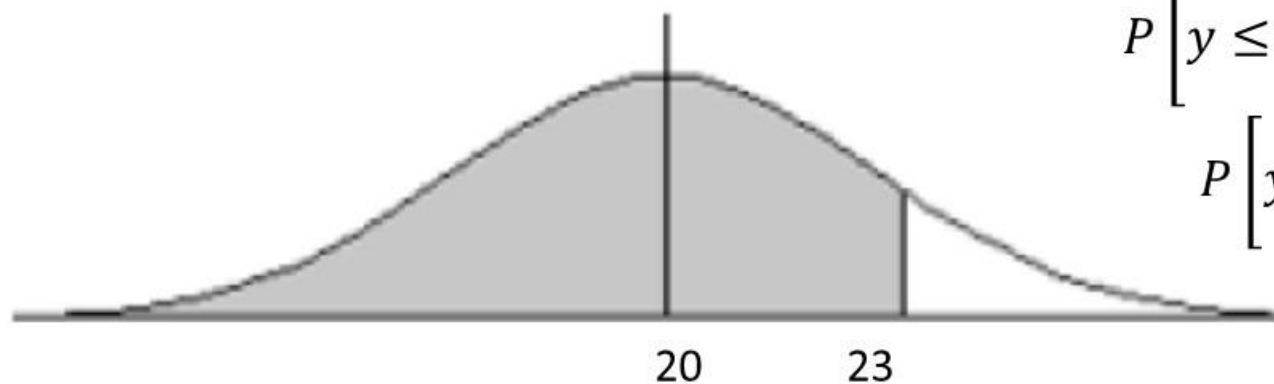


$$P[-0,67 \leq y \leq 1,33]$$
$$= 0,9082 - 0,2514 = 0,6568$$



Distribución Normal – EJERCICIOS

$$P[x \leq 23]; x \sim N(20, 9)$$



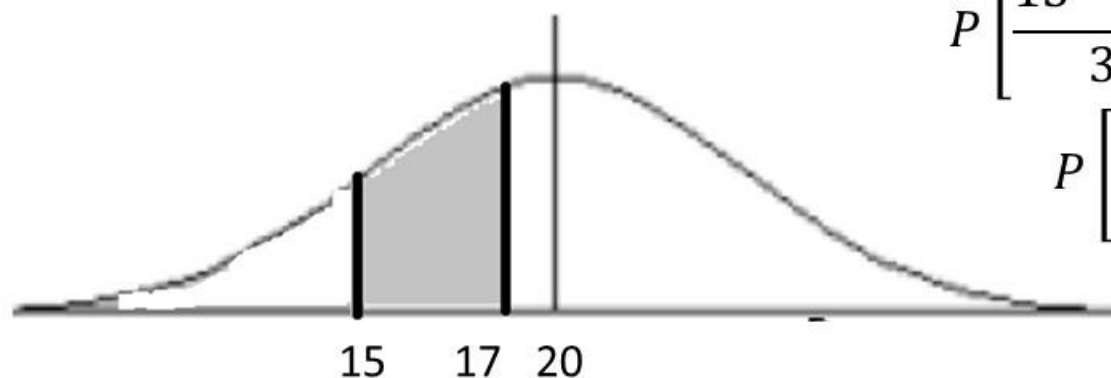
$$P\left[y \leq \frac{23 - 20}{3}\right]$$
$$P\left[y \leq \frac{3}{3}\right]$$

$$P[y \leq 1] = 0,8413$$

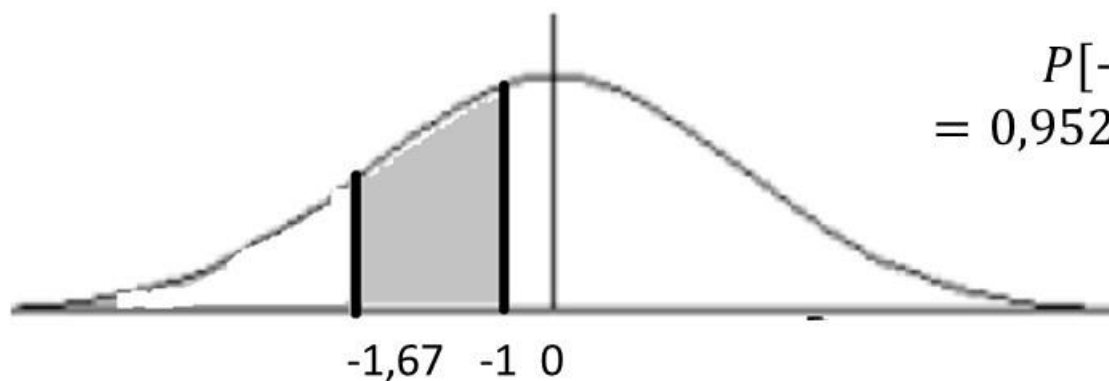


Distribución Normal – EJERCICIOS

$$P[15 \leq x \leq 17]; x \sim N(20, 9)$$



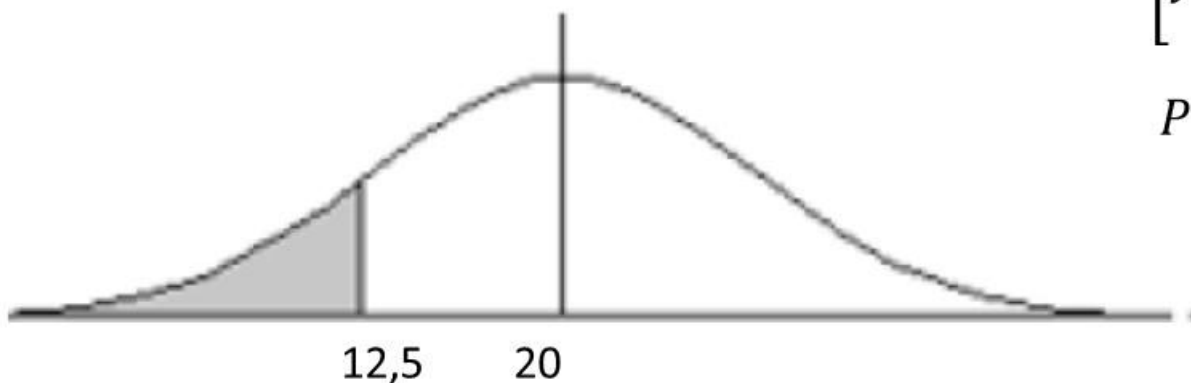
$$P\left[\frac{15 - 20}{3} \leq y \leq \frac{17 - 20}{3}\right]$$
$$P\left[-\frac{5}{3} \leq y \leq -\frac{3}{3}\right]$$



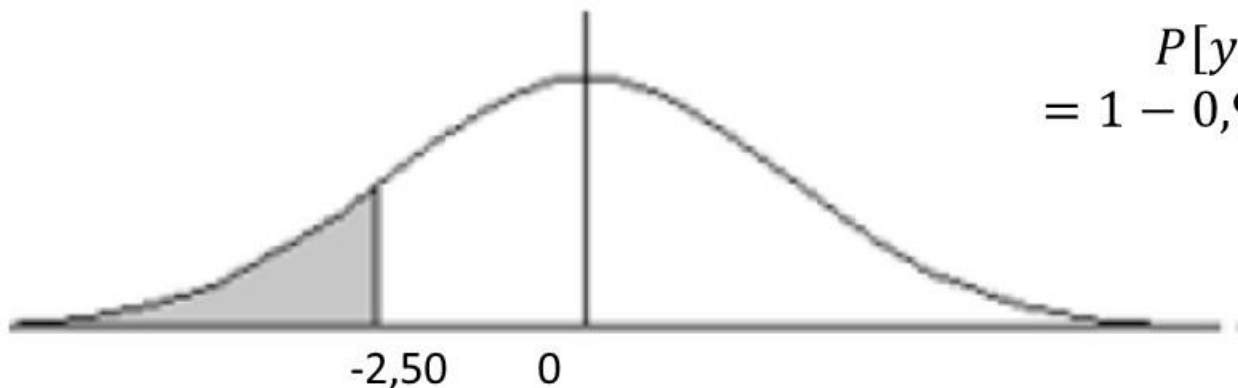
$$P[-1,67 \leq y \leq -1]$$
$$= 0,9525 - 0,8413 = 0,1112$$

Distribución Normal – EJERCICIOS

$$P[x \leq 12,5]; x \sim N(20,9)$$



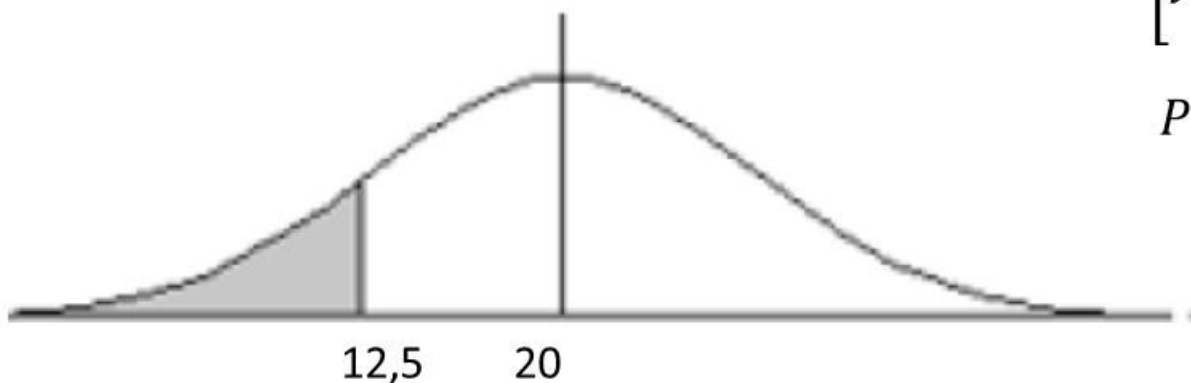
$$P\left[y \leq \frac{12,5 - 20}{3}\right]$$
$$P\left[y \leq -\frac{7,5}{3}\right]$$



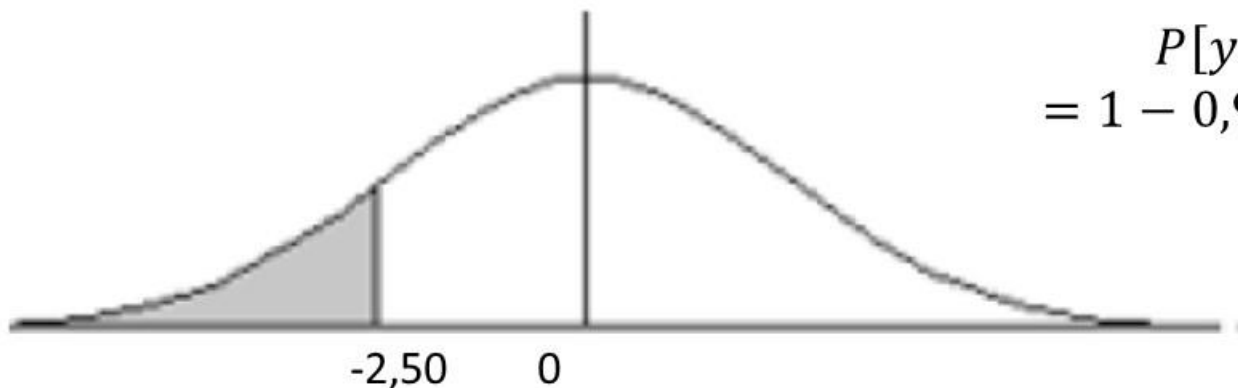
$$P[y \leq -2,50]$$
$$= 1 - 0,9938 = 0,0062$$

Distribución Normal – EJERCICIOS

$$P[x \leq 12,5]; x \sim N(20,9)$$



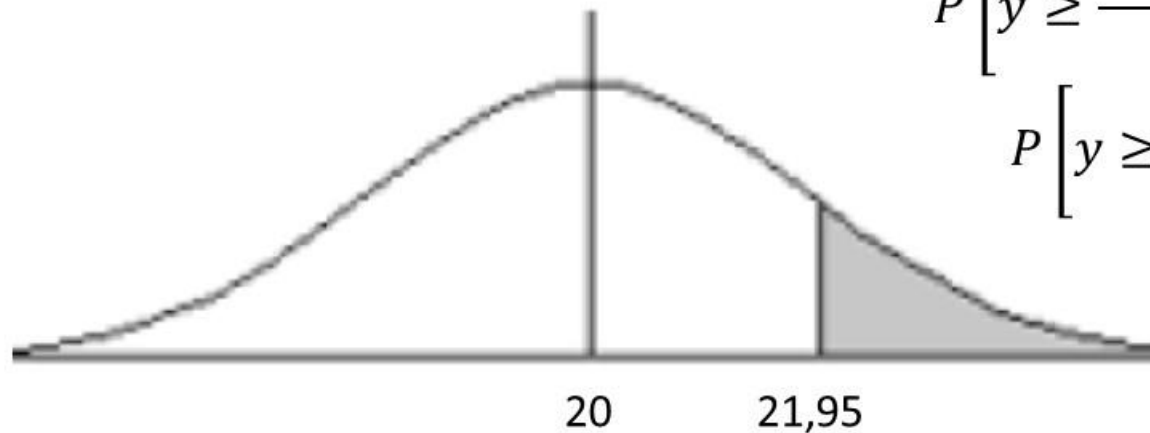
$$P\left[y \leq \frac{12,5 - 20}{3}\right]$$
$$P\left[y \leq -\frac{7,5}{3}\right]$$



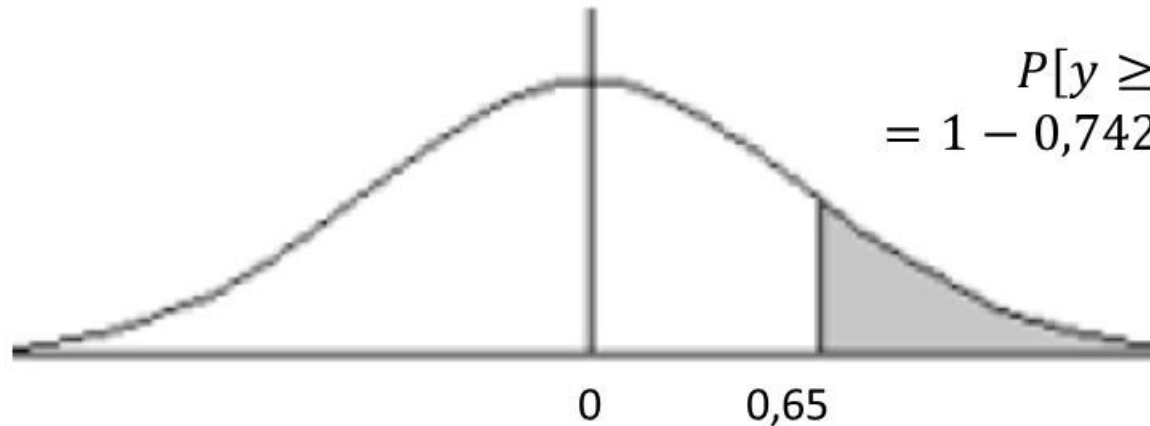
$$P[y \leq -2,50]$$
$$= 1 - 0,9938 = 0,0062$$

Distribución Normal – EJERCICIOS

$$P[x \geq 21,95]; x \sim N(20,9)$$



$$P\left[y \geq \frac{21,95 - 20}{3}\right]$$
$$P\left[y \geq \frac{1,95}{3}\right]$$



$$P[y \geq 0,65]$$
$$= 1 - 0,7422 = 0,2578$$

OTRAS DISTRIBUCIONES CONTINUAS

- Distribución χ^2 (ji- cuadrado) donde la variable es la suma de los cuadrados de variables normales tipificadas. Su distribución tiene n grados de libertad que está vinculado al número de variables que se suman
- Distribución “t-Student”, es una variable que se deriva del cociente entre una normal tipificada y una χ^2

Tiene n grados de libertad que corresponde a la ji-cuadrado



TEOREMA DE LÍMITE CENTRAL

Cuando el número de datos es grande, mayor a 30, la variable tiende a distribuirse normalmente.