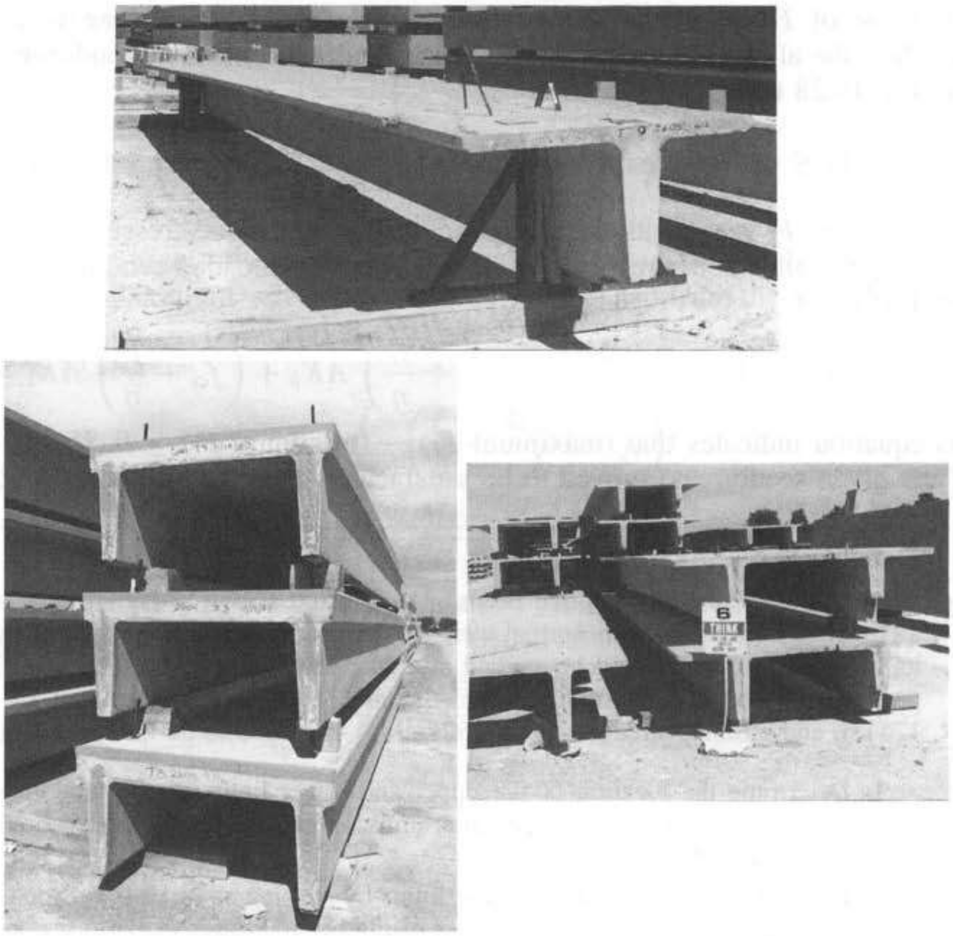


Precast, prestressed concrete sections: (*a*) single T-, (*b*) double T- and (*c*) U-sections.

2. Estimate prestress losses, given $F_o = 175\,\text{ksi}$.

a. Assume elastic loss is 4%, or $0.04 \times 175 = 7\,\text{ksi}$.

b. Loss due to shrinkage is $0.0003E_s = 0.0003 \times 29{,}000 = 8.7\,\text{ksi}$.

c. Loss due to creep of concrete: A good first estimate of creep loss is 1.67 times the elastic loss.

$$1.67 \times 7 = 11.7 \text{ ksi}$$

d. Loss due to relaxation of steel is 4%:

$$0.04 \times 175 = 7 \text{ ksi}$$

Time-dependent losses are $8.7 + 11.7 + 7 = 27.4\,\text{ksi}$.

$$\text{Percentage} = \frac{27.4}{175} = 15.7\%$$

e. The total loss is $27.4 + 7$ (elastic loss) $= 34.4\,\text{ksi}$. The percentage of total loss is

$$34.4/175 = 19.7\%$$

f. Prestress stresses are

$$F_i = 175 - 7 = 168 \text{ ksi} \qquad \text{(at transfer)}$$

$$F = 175 - 34.4 = 140.6 \text{ ksi}$$

$$F = \eta F_i$$

$$\eta = 1 - \text{time} - \text{dependent losses ratio}$$

$$= \frac{140.6}{168} = 0.837$$

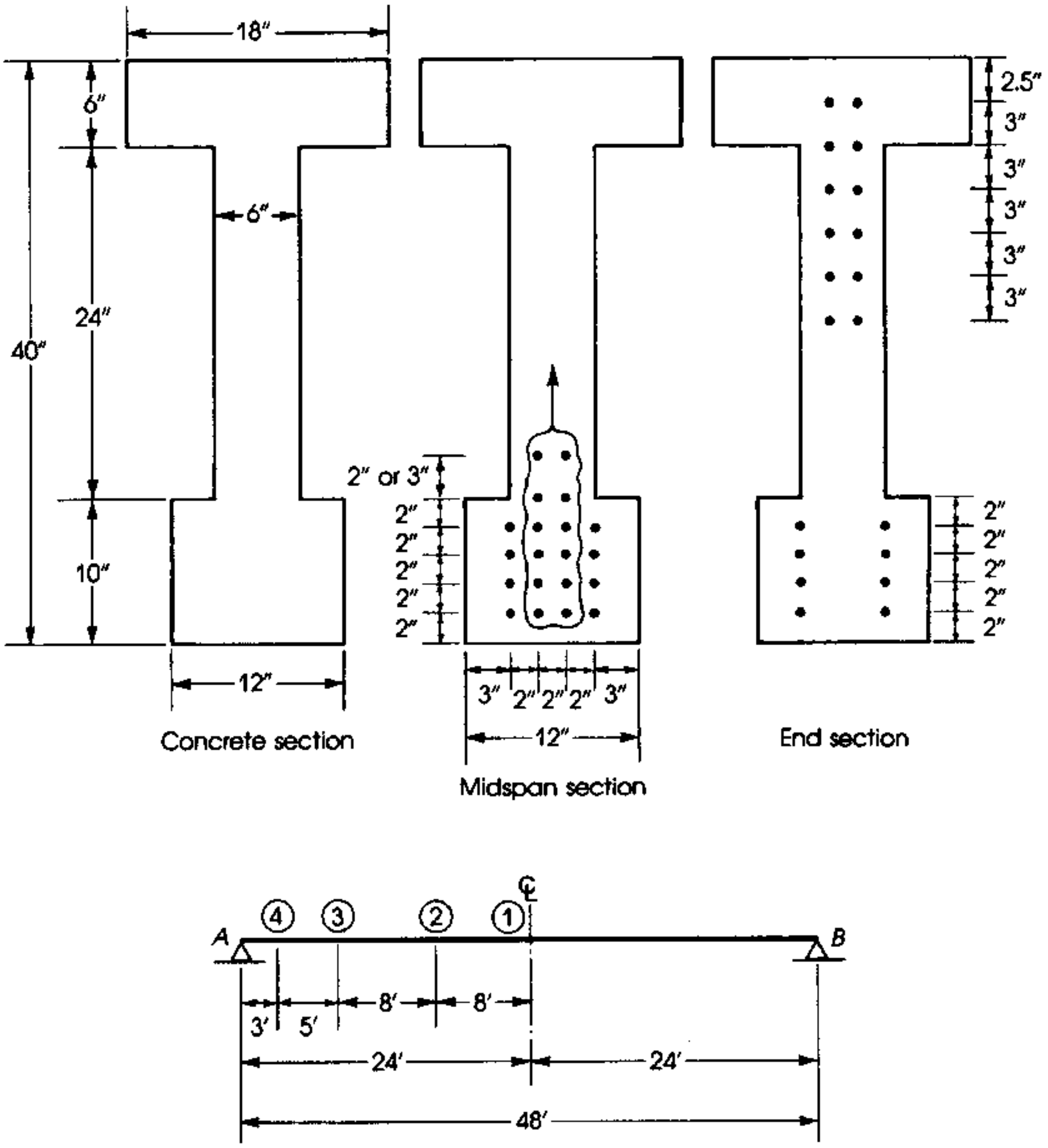


Figure 19.6 (a) Example 19.4.

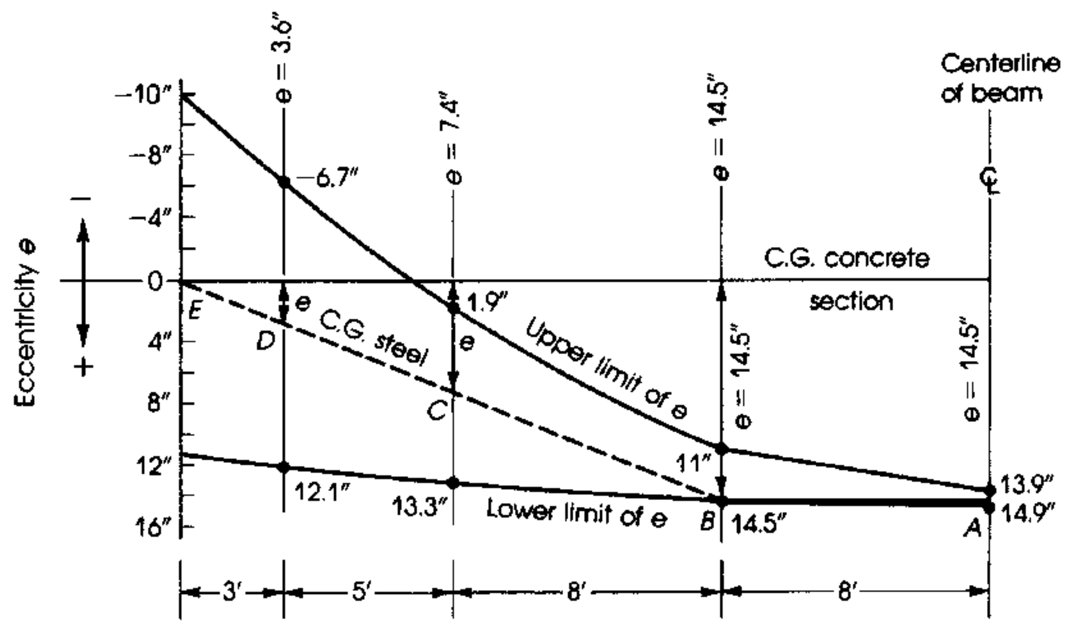


Figure 19.6 (b) Example 19.4: tendon profile.

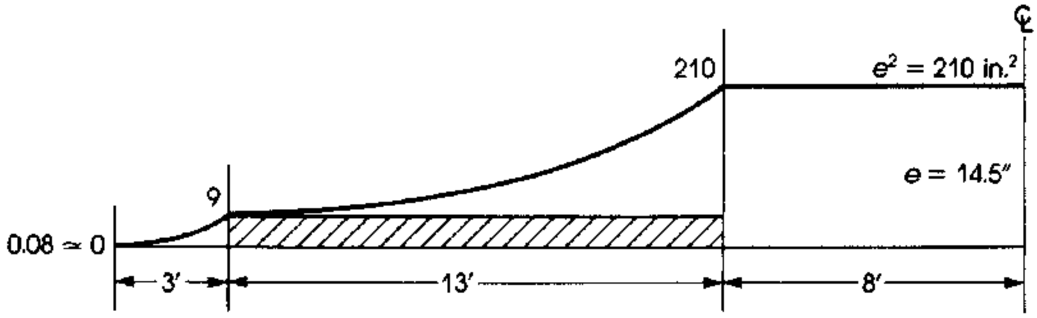


Figure 19.6 (c) Example 19.4: average e^2.

3. Limits of the eccentricity, e, at midspan section: Calculate the allowable stresses and moments. At transfer, $f'_{ci} = 4000$ psi, $f_{ci} = 0.6 \times 4000 = 2400$ psi, and $f_{ti} = 3\sqrt{f'_c} = 190$ psi. At service load, $f'_c = 5000$ psi, $f_c = 0.45f'_c = 2250$ psi, and $f_t = 6\sqrt{f'_c} = 424$ psi.

$$\text{Self-weight of beam} = \frac{372}{144} \times 150 = 388 \text{ lb/ft}$$

$$M_D \text{ (self-weight)} = \frac{0.388}{8}(48)^2 \times 12 = 1341 \text{ K·in.}$$

$$M_a \text{ (additional load and live load)} = \frac{w_a L^2}{8}$$

$$= \frac{(0.9 + 1.1)}{8}(48)^2 \times 12 = 6912 \text{ K·in.}$$

$$\text{Total moment } (M_T) = M_D + M_a = 8253 \text{ K·in.}$$

$$F_i = \text{stress at transfer} \times \text{area of prestressing steel}$$

The area of 20 tendons, $\frac{7}{16}$ in. in diameter, is $20 \times 0.1089 = 2.178\,\text{in.}^2$

$$F_i = 2.178 \times 168 = 365.9 \text{ K}$$

$$F = 2.178 \times 140.6 = 306.2 \text{ K}$$

a. Consider the section at midspan.
Top fibers, *unloaded* condition:

$$e \le K_b + \frac{M_D}{F_i} + \frac{f_{ti} A K_b}{F_i}$$

$$\le 9.4 + \frac{1341}{365.9} + \frac{0.190(372)(9.4)}{365.9} \le 14.9 \text{ in.} \qquad (19.20)$$

Bottom fibers, *unloaded* condition:

$$e \le -K_t + \frac{M_D}{F_i} + \frac{f_{ci} A K_t}{F_i}$$

$$\le -8.6 + \frac{1341}{365.9} + \frac{2.4(372)(8.6)}{365.9} \le 16.1 \text{ in.} \qquad (19.22)$$

Maximum $e = 14.9$ in. controls.

Top fibers, *loaded* condition:

$$e \ge K_b + \frac{M_T}{F} - \frac{f_c A K_b}{F}$$

$$\ge 9.4 + \frac{8253}{306.2} - \frac{0.424(372)(8.6)}{306.2} \ge 10.7 \text{ in.} \qquad (19.24)$$

Bottom fibers, *loaded* condition:

$$e \ge -K_t + \frac{M_T}{F} - \frac{f_t A K_t}{F}$$

$$\ge -8.6 + \frac{8253}{306.2} - \frac{0.424(372)(8.6)}{306.2} \ge 13.9 \text{ in.} \qquad (19.26)$$

Minimum $e = 13.9$ in. controls.

b. Consider a section 8 ft from the midspan (section 2, Fig. 19.6a):

$$M_D \text{ (self-weight)} = R_A(16) - \frac{w_D}{2} \times (16)^2$$

$$= 0.388(24)(16) - \frac{0.388}{2}(16)^2 = 99.3 \text{ K·ft} = 1192 \text{ K·in.}$$

$$M_a = 2(24)(16) - \frac{2}{2}(16)^2 = 512 \text{ K·ft} = 6144 \text{ K·in.}$$

$$M_T = 6144 + 1192 = 7336 \text{ K·in.}$$

Top fibers, *unloaded* condition:

$$e \le 9.4 + \frac{1192}{365.9} + \frac{0.190(372)(9.4)}{365.9} \le 14.5 \text{ in.}$$

Bottom fibers, *unloaded* condition:

$$e \le -8.6 + \frac{1192}{365.9} + \frac{2.4(372)(8.6)}{365.9} \le 15.6 \text{ in.}$$

Maximum $e = 14.5$ in. controls.

Top fibers, *loaded* condition:

$$e \geq 9.4 + \frac{7336}{306.2} - \frac{2.25(372)(9.4)}{306.2} \geq 7.7 \text{ in.}$$

Bottom fibers, *loaded* condition:

$$e \geq -8.6 + \frac{7336}{306.2} - \frac{0.424(372)(8.6)}{306.2} \geq 11.0 \text{ in.}$$

Minimum $e = 11.0$ in. controls.

c. Consider a section 16 ft from midspan (section 3, Fig. 19.6a): M_D (self-weight) = 745 K·in., $M_a = 3840$ K·in., and $M_T = 4585$ K·in.

- Top fibers, unloaded condition, $e \leq 13.3$ in. (max) controls.
- Bottom fibers, unloaded condition, $e \leq 14.4$ in.
- Top fibers, loaded condition, $e \geq -1.3$ in.
- Bottom fibers, loaded condition, $e \geq 1.9$ in. (min) controls.

d. Consider a section 3 ft from the end (anchorage length): $M_D = 314$ K·in., $M_a = 1620$ K·in., and $M_T = 1934$ K·in.

- Top fibers, unloaded condition, $e \leq 12.1$ in. (max) controls.
- Bottom fibers, unloaded condition, $e \leq 13.3$ in.
- Top fibers, loaded condition, $e \geq -10$ in.
- Bottom fibers, loaded condition, $e \geq -6.7$ in. (min) controls.

4. The tendon profile is shown in Fig. 19.6b. The eccentricity chosen at midspan is $e = 14.5$ in., which is adequate for section B at 8 ft from midspan. The centroid of the prestressing steel is horizontal between A and B and then harped linearly between B and the end section at E. The eccentricities at sections C and D are calculated by establishing the slope of line BE, which is 14.5/16 = 0.91 in/ft. The eccentricity at C is 7.25 in. and at D it is 2.72 in. The tendon profile chosen satisfies the upper and lower limits of the eccentricity at all sections.

Harping of tendons is performed as follows:

a. Place the 20 tendons ($\frac{7}{16}$ diameter) within the middle third of the beam at spacings of 2 in., as shown in Fig. 19.6a. To calculate the actual eccentricity at midspan section, take moments for the number of tendons about the base line of the section:

$$\text{Distance from base} = \tfrac{1}{20}(16 \times 5 + 4 \times 11) = 6.2 \text{ in.}$$

$$e \text{ (midspan)} = y_b - 6.2 \text{ in.}$$

$$= 20.8 - 6.2 = 14.6 \text{ in.}$$

which is close to the 14.5 in. assumed. If the top two tendons are placed at 3 in. from the row below them, then the distance from the base becomes $\frac{1}{20}(16 \times 5 + 2 \times 10 + 2 \times 13) =$ 6.3 in. The eccentricity becomes $20.8 - 6.3 = 14.5$ in., which is equal to the assumed eccentricity. Practically, all tendons may be left at 2 in. spacing by neglecting the difference of 0.1 in.

b. Harp the central 12 tendons only. The distribution of tendons at the end section is shown in Fig. 19.6a. To check the eccentricity of tendons, take moments about the centroid of the concrete section for the 12 tendons at top and the eight tendons left at bottom:

$$e = \tfrac{1}{20}(8 \times 14.5 - 12 \times 9.2) = 0.28 \text{ in.}$$

This value of e is small and adequate. The actual eccentricity at 3 ft from the end section is

$$e = \tfrac{3}{16}(14.5 - 0.28) + 0.28 = 2.95 \text{ in.} \qquad (3 \text{ in.})$$

The actual eccentricity at 8 ft from the end section is

$$e = \tfrac{1}{2}(14.5 - 0.28) + 0.28 = 7.4 \text{ in.}$$

5. Limited values of F_i: The value of F_i used in the preceding calculations is $F_i = 365.9\,\text{K}$. Check minimum F_i by Eq. 19.31:

$$\text{Min. } F_i = \frac{1}{(K_b + K_t)}\left[\left(\frac{1}{\eta} - 1\right)M_D + \frac{M_L}{\eta} - \frac{(f_t A K_t)}{\eta}\right) - (f_{ti} A K_b)$$

$$= \frac{1}{(9.4 + 8.6)}\left[\left(\frac{1}{0.8423} - 1\right)1341 + \frac{6912}{0.843}\right.$$

$$\left. - \frac{(0.424 \times 372 \times 8.6)}{0.843}(0.19 \times 372 \times 9.4)\right] = 343.1 \text{ K}$$

which is less than the F_i used. Check maximum F_i using Eq. 19.32:

$$\text{Max. } F_i = \frac{1}{(K_b + K_t)}\left[\left(1 - \frac{1}{\eta}\right)M_D - \frac{M_L}{\eta} + \frac{(f_c A K_b)}{\eta} + (f_{ci} A K_t)\right]$$

$$= \frac{1}{18}\left[\left(1 - \frac{1}{0.843}\right)1341 - \frac{6912}{0.843} + \frac{(2.25 \times 3.72 \times 9.4)}{0.843} + (2.4 \times 3.72 \times 8.6)\right]$$

$$= 475.7 \text{ K}$$

which is greater than the F_i used. Therefore, the critical section at midspan is adequate.

6. Check prestress losses, recalling that $F_o = 175\,\text{ksi}$ and $A_{ps} = 2.178\,\text{in.}^2$

$$\text{Total } F_o = 2.178 \times 175 = 381 \text{ K}$$

$$E_c = 4000 \text{ ksi}$$

$$n = \frac{E_s}{E_c} = \frac{29}{4.0} = 7.25$$

n can be assumed to be 7.

$$M_D \text{ at midspan} = 1341 \text{ K·in.}$$

$$F_i = \frac{F_o + nA_{ps}f_c(\text{D.L.}) \times \frac{2}{3}}{1 + (nA_{ps})\left(\frac{1}{A} + \frac{e^2}{I}\right)} \qquad (19.5)$$

The value of f_c due to the distributed dead load is multiplied by $\frac{2}{3}$ to reflect the parabolic variation of the dead load stress along the span, giving a better approximation of F_i.

a. Determine the average value of e^2, as adopted in the beam. The curve representing e^2 is shown in Fig. 19.6c:

$$\text{Average } e^2 = \tfrac{1}{24}\left[\left(\tfrac{1}{3} \times 3 \times 9\right) + (9 \times 13) + \left(\tfrac{1}{3} \times 13 \times 201\right) + (210 \times 8)\right]$$

$$= 111.5 \text{ in.}^2$$

$$e = 10.56 \text{ in.}$$

The area of a parabola is one-third the area of its rectangle.

b. Stress due to dead load at the level of the tendons is

$$f_c(\text{D.L.}) = \frac{1341 \times 10.56}{66{,}862} = 0.212 \text{ ksi}$$

Therefore,

$$F_i = \frac{381 + 7(2.178) \times 0.212 \times 2/3}{1 + (7 \times 2.178)\left(\frac{1}{372} + \frac{111.5}{66{,}862}\right)} = 358 \text{ K}$$

Elastic loss is $381 - 358 = 23\,\text{K} = 6.1\%$. This value is greater than the assumed elastic loss of 4%.

$$\text{Elastic loss per unit steel area} = \frac{23}{2.178} = 10.6\ \text{ksi}$$

$$F_i \text{ per unit steel area} = \frac{358}{2.178} = 164.4\ \text{ksi}$$

c. Time-dependent losses:

$$\text{Loss due to shrinkage} = 8.7\ \text{ksi (as before)}$$

Loss due to creep:

$$\text{Elastic strain} = \frac{F_i}{A_c E_c} = \frac{358}{372 \times 4000} = 0.240 \times 10^{-3}$$

$$\Delta f_s = C_c(\varepsilon_{cr} E_s)$$

Let $C_c = 1.5$. Then

$$\Delta f_s = 1.5(0.24 \times 10^{-3} \times 29{,}000) = 10.4\ \text{ksi}$$

$$\text{Percent loss} = \frac{10.4}{164.4} = 6.3\%$$

Loss due to relaxation of steel is 7 ksi (as before). Time-dependent losses equal 8.7 + 10.4 + 7 = 26.1 ksi, for a percentage loss of 26.1/164.4 = 15.8%, which is very close to the previously estimated value of 15.7%.

$$F = \eta F_i = (1 - 0.158)F_i = 0.842F_i$$

$$\eta = 0.842$$

19.5 DESIGN OF FLEXURAL MEMBERS

19.5.1 General

The previous section emphasized that the stresses at the top and bottom fibers of the critical sections of a prestressed concrete member must not exceed the allowable stresses for all cases or stages of loading. In addition to these conditions, a prestressed concrete member must be designed with an adequate factor of safety against failure. The ACI Code requires that the moment due to the factored service loads, M_u, must not exceed ϕM_n, the flexural strength of the designed cross-section.

For the case of a tension-controlled, prestressed concrete beam, failure begins when the steel stress exceeds the yield strength of steel used in the concrete section. The high-tensile prestressing steel will not exhibit a definite yield point, such as that of the ordinary mild steel bars used in reinforced concrete. But under additional increments of load, the strain in the steel increases at an accelerated rate, and failure occurs when the maximum compressive strain in the concrete reaches a value of 0.003 (Fig. 19.9).

The limits for reinforcement of prestressed concrete flexural members according to the ACI Code, Section 18.8, is based on the net tensile strain for tension-controlled, transition, or compression-controlled sections in accordance with the ACI Code, Section 10.3, as was explained here in this textbook, Section 3.5. The strength reduction factor, ϕ, was given earlier in Section 3.7 of this textbook based on the ACI Code, Section 9.3.

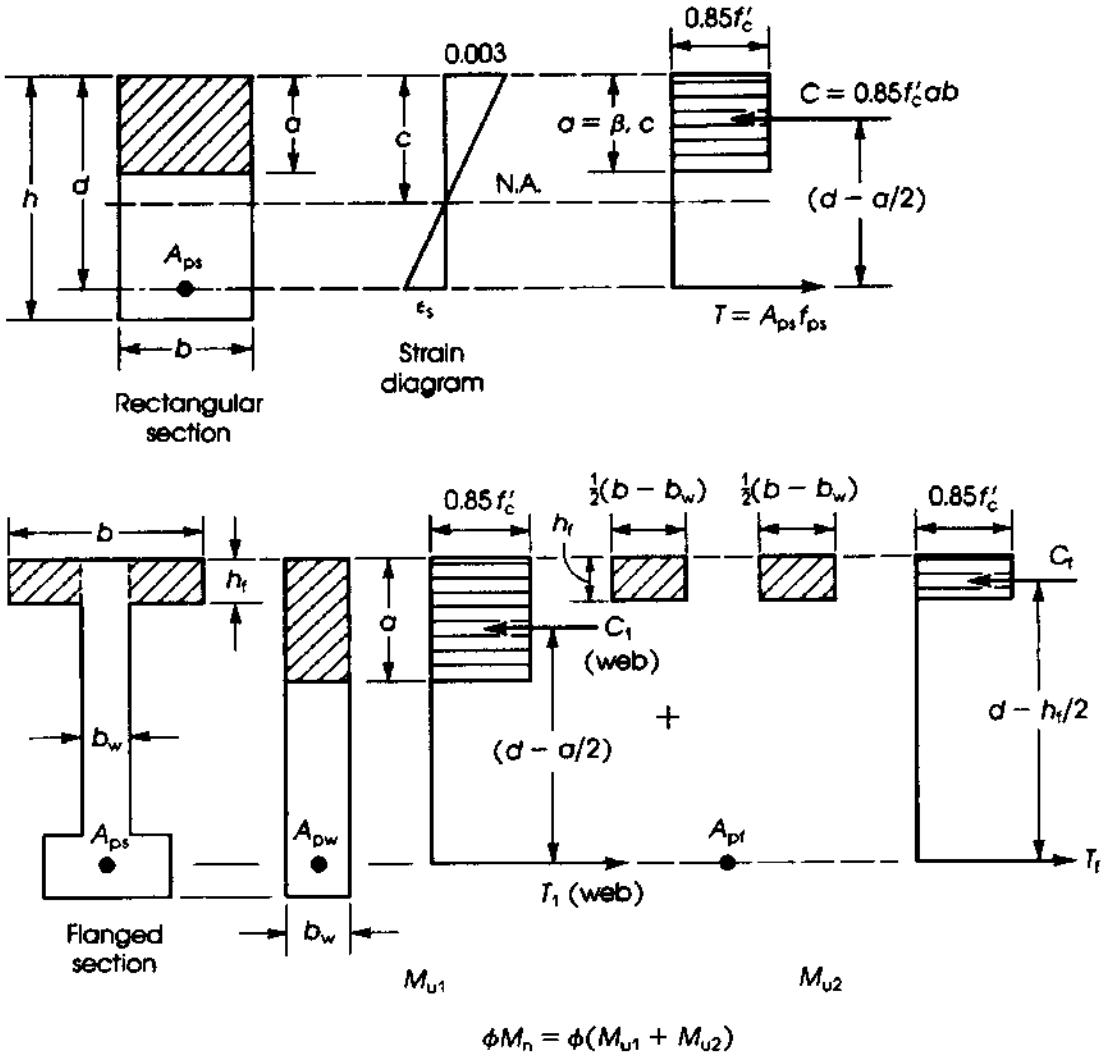


Figure 19.7 Factored moment capacity of prestressed concrete beams.

19.5.2 Rectangular Sections

The nominal moment capacity of a rectangular section may be determined as follows (refer to Fig. 19.7):

$$M_n = C\left(d - \frac{a}{2}\right) = T\left(d - \frac{a}{2}\right) \tag{19.34}$$

where $T = A_{ps} f_s$ and $C = 0.85\ f'_c ab$. For $C = T$,

$$a = \frac{A_{ps} f_{ps}}{0.85 f'_c b} = \frac{\rho_p\ f_{ps}}{0.85 f'_c} d \tag{19.35}$$

where the prestressing steel ratio is $\rho_p = A_{ps}/bd$, and A_{ps} and f_{ps} refer to the area and tensile stress of the prestressing steel. Let

$$\omega_p = \rho_p \left(\frac{f_{ps}}{f'_c}\right) \leq 0.32\,\beta_1$$

Then

$$a = \frac{\omega_p}{0.85} d \tag{19.36}$$

The quantity ω_p is a direct measure of the force in the tendon. To ensure a tension-controlled behavior, the ACI Code, Section 18.8.1, specifies that ω_p must not exceed $0.32\beta_1$, which corresponds to a net tensile strain, ε_t, of 0.005. Note that the value of $\beta_1 = 0.85$ for $f'_c \leq 4$ ksi and reduces by 0.05 for each 1 ksi greater than 4 ksi (ACI Code, Section 10.2.7.3). M_n can also be written as follows:

$$M_n = A_{ps} f_{ps} \left(d - \frac{a}{2}\right)$$

$$M_n = A_{ps} f_{ps} d \left(1 - \frac{\rho_p f_{ps}}{1.7 f'_c}\right) \tag{19.37}$$

$$M_n = A_{ps} f_{ps} d \left(1 - \frac{\omega_p}{1.7}\right) \tag{19.38}$$

and $M_u = \phi M_n$.

In the preceding equations, f_{ps} indicates the stress in the prestressing steel at failure. The actual value of f_{ps} may not be easily determined. Therefore, the ACI Code, Section 18.7.2, permits f_{ps} to be evaluated as follows (all stresses are in psi). For *bonded* tendons,

$$f_{ps} = f_{pu} \left[1 - \frac{\gamma_p}{\beta_1} \left(\rho_p \times \frac{f_{pu}}{f'_c}\right)\right] \tag{19.39}$$

For *unbonded* tendons in members with a span-to-depth ratio less than or equal to 35,

$$f_{ps} = \left(f_{se} + 10{,}000 + \frac{f'_c}{100\rho_p}\right) \leq f_{py} \tag{19.40}$$

provided that $f_{se} \geq 0.5\ f_{pu}$ and that f_{ps} for unbonded tendons does not exceed either f_{py} or $f_{se} + 60{,}000$ psi. For *unbonded* tendons in members with a span-to-depth ratio greater than 35,

$$f_{ps} = \left(f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p}\right) \tag{19.41}$$

but not greater than f_{py} or $f_{se} + 30{,}000$ psi, where

γ_p = factor for the type of prestressing tendon

= 0.55 for f_{py}/f_{pu} not less than 0.8

= 0.4 for f_{py}/f_{pu} not less than 0.85

= 0.28 for f_{py}/f_{pu} not less than 0.9

f_{pu} = specified tensile strength of prestressing steel

f_{se} = effective stress in prestressing steel after all losses

f_{py} = specified yield strength of prestressing steel

In the event that $\omega_p > 0.32\beta_1$, a compression-controlled, prestressed concrete beam may develop. To ensure a ductile failure, ω_p is limited to a maximum value of $0.32\beta_1$. For

$\omega_p = 0.32\beta_1$, $a = 0.377\beta_1 d$ (from Eq. 19.36). Substituting this value of a in Eq. 19.38,

$$M_n = A_{ps} f_{ps} d \left(1 - \frac{0.32\beta_1}{1.7}\right)$$
$$= (\rho_p bd) f_{ps} d(1 - 0.188\beta_1)$$
$$= \omega_p f'_c (1 - 0.188\beta_1) bd^2$$
$$= (0.32\beta_1 - 0.06\beta_1^2) f'_c bd^2 \quad (19.42)$$

for $f'_c = 5$ ksi, $\beta_1 = 0.8$. Then

$$M_n = 0.22 f'_c bd^2 = 1.09bd^2$$

Similarly, for $f'_c = 4$ ksi, $M_n = 0.915bd^2$, and for $f'_c = 6$ ksi, $M_n = 1.238bd^2$.

19.5.3 Flanged Sections

For flanged sections (T- or I-sections), if the stress block depth a lies within the flange, it will be treated as a rectangular section. If a lies within the web, then the web may be treated as a rectangular section using the web width, b_w, and the excess flange width $(b - b_w)$ will be treated similarly to that of reinforced concrete T-sections discussed in Chapters 3 and 4. The design moment strength of a flanged section can be calculated as follows (see Fig. 19.7)

$$M_n = M_{n_1} \text{ (moment strength of the web)} + M_{n_2} \text{ (moment strength of excess flange)}$$

$$M_n = A_{pw} f_{ps} \left(d_p - \frac{a}{2}\right) + A_{pf} f_{ps} \left(d_p - \frac{h_f}{2}\right) \quad (19.43)$$

$$M_u = \phi M_n \quad \text{and} \quad a = \frac{A_{pw} f_{ps}}{0.85 f'_c b_w}$$

where

$A_{pw} = A_{ps} - A_{pf}$
$A_{pf} = [0.85 f'_c (b - b_w) hf]/f_{ps}$
h_f = thickness of the flange

Note that the total prestressed steel, A_{ps}, is divided into two parts, A_{pw} and A_{pf}, developing the web and flange moment capacity. For flanged sections, the reinforcement index, ω_{pw}, must not exceed $0.32\beta_1$ for tension-controlled sections, where

$$\omega_{pw} = \left(\frac{A_{pw}}{b_w d}\right)\left(\frac{f_{ps}}{f'_c}\right) = \text{prestressed web steel ratio} \times \left(\frac{f_{ps}}{f'_c}\right)$$

19.5.4 Nonprestressed Reinforcement

In some cases, nonprestressed reinforcing bars (A_s) are placed in the tension zone of a prestressed concrete flexural member together with the prestressing steel (A_{ps}) to increase the moment strength of the beam. In this case, the total steel (A_{ps} and A_s) is considered in the moment analysis. For rectangular sections containing prestressed and nonprestressed steel, the design moment strength, ϕM_n, may be computed as follows:

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2}\right) + A_s f_y \left(d - \frac{a}{2}\right) \quad (19.44)$$

where

$$a = \frac{A_{ps} f_{ps} + A_s f_y}{0.85 f_c' b}$$

Also, d_p and d are the distances from extreme compression fibers to the centroid of the prestressed and nonprestressed steels, respectively. For flanged sections,

$$M_n = A_{pw} f_{ps} \left(d_p - \frac{a}{2}\right) + A_s f_y \left(d - \frac{a}{2}\right) + A_{pf} f_{ps} \left(d_p - \frac{h_f}{2}\right) \tag{19.45}$$

where

$$A_{pw} = A_{ps} - A_{pf}$$

$$a = \frac{A_{ps} f_{ps} + A_s f_y}{0.85 f_c' b_w}$$

For rectangular sections with compression reinforcement, and taking moments about the force C,

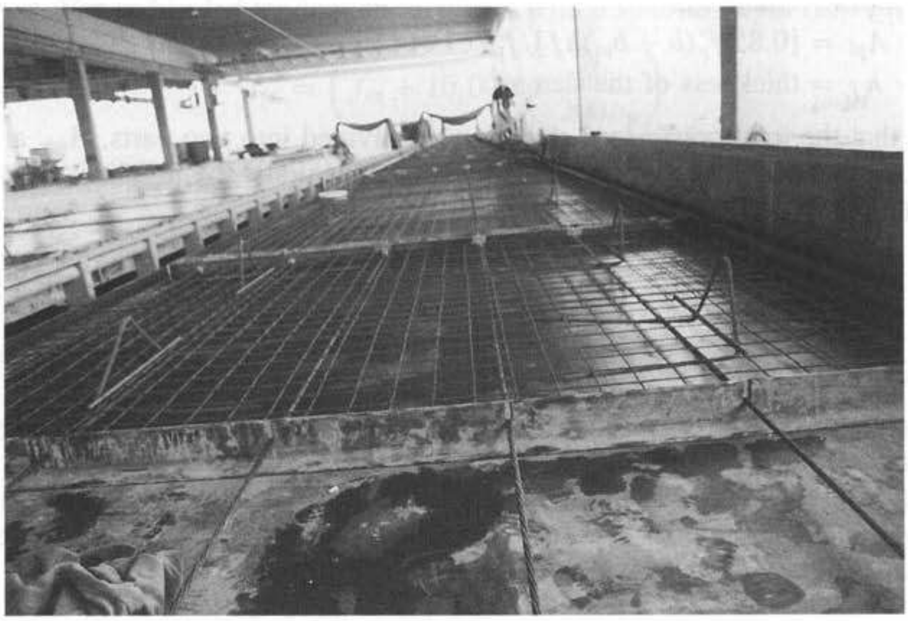
$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2}\right) + A_s f_y \left(d - \frac{a}{2}\right) + A_s' f_y \left(\frac{a}{2} - d'\right) \tag{19.46}$$

where

$$a = \frac{A_{ps} f_{ps} + A_s f_y - A_s' f_y}{0.85 f_c' b}$$

This equation is valid only if compression steel yields. The condition for compression steel to yield is

$$\left(\frac{A_{ps} f_{ps} + A_s f_y - A_s' f_y}{bd}\right) \geq 0.85 \beta_1 \frac{f_c' d'}{d} \left(\frac{87}{87 - f_y}\right)$$



Prestressed concrete beds for slabs and wall panels.

If this condition is not met, then compression steel does not yield. In this case, A'_s may be neglected (let $A'_s = 0$), or alternatively, the stress in A'_s may be determined by general analysis, as explained in Chapter 3.

When prestressed and nonprestressed reinforcement are used in the same section, Eq. 19.39 should read as follows:

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\left(\rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')\right)\right] \tag{19.47}$$

(ACI Code, Eq. 18.3). If any compression reinforcement is taken into account when calculating f_{ps}, the term

$$\rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')$$

must be greater than or equal to 0.17 and d' must be less than or equal to $0.15d_p$, where d, d', and d_p are the distances from the extreme compression fibers to the centroid of the nonprestressed tension steel, compression steel, and prestressed reinforcement, respectively,

γ_p = factor for type of prestressing tendon

$= 0.55$ for $\dfrac{f_{py}}{f_{pu}}$ not less than 0.8

$= 0.40$ for $\dfrac{f_{py}}{f_{pu}}$ not less than 0.85

$= 0.28$ for $\dfrac{f_{py}}{f_{pu}}$ not less than 0.90

$\beta_1 = 0.85$ for $f'_c \leq 4$ ksi less 0.05 for each 1 ksi increase in f'_c, but $\beta_1 \geq 0.65$.

1. For rectangular sections, the ACI Code, Section 18.8, limits the reinforcement ratio as follows ($\varepsilon_t \geq 0.005$ for tension-controlled sections):

$$\omega_p + \frac{d}{d_p}\omega \leq 0.32\beta_1$$

where

$$\omega_p = \rho_p\left(\frac{f_{ps}}{f'_c}\right) \quad \text{and} \quad \rho_p = \frac{A_{ps}}{bd} \quad \text{(prestressed steel)}$$

$$\omega = \rho\left(\frac{f_y}{f'_c}\right) \quad \text{and} \quad \rho = \frac{A_s}{bd} \quad \text{(nonprestressed steel)}$$

2. If ordinary reinforcing bars A'_s are used in the compression zone, then the condition becomes

$$\omega_p + \frac{d}{d_p}(\omega - \omega') \leq 0.32\beta_1$$

where $\omega' = \rho'(f_y/f'_c)$ and $\rho' = A'_s/bd$. This reinforcement limitation is necessary to ensure a plastic failure of underreinforced concrete beams.

3. For flanged sections, the steel area required to develop the strength of the web (A_{pw}) is used to check the reinforcement index.

$$\omega_{pw}(\text{web}) = \rho_{pw}\left(\frac{f_{ps}}{f'_c}\right) \leq 0.32\beta_1$$

where

$$\rho_{pw} = \frac{A_{pw}}{b_w d_d}$$

If nonprestressed reinforcement is used, then the reinforcement limitations are

$$\omega_{pw} + \frac{d}{d_{pw}}(\omega_w - \omega'_w) \leq 0.32\beta_1$$

where

$$\omega_w \quad \text{and} \quad \omega'_w = \frac{A_s}{b_w d}\left(\frac{f_y}{f'_c}\right) \quad \text{and} \quad \frac{A'_s}{b_w d}\left(\frac{f_y}{f'_c}\right)$$

respectively. When compression steel A'_s is not used, then $\omega'_w = 0$. The preceding reinforcement conditions must be met in the analysis and design of partially prestressed concrete members.

For class C of prestressed concrete flexural members, where $f_t > 12\sqrt{f'_c}$ (cracked section), crack control provisions should be used as explained in Section 6.7 of this textbook. When using Eq. 6.18 for the maximum spacing s, the ACI Code, Section 18.4.4, specifies the following:

a. For tendons, use $\frac{2}{3}$ of the spacing s.

b. For a combination of nonprestressed reinforcement and tendons, use $\frac{5}{6}$ of the spacing s.

c. For tendons, use Δf_{ps} in place of f_s, where Δf_{ps} is the difference between the stress computed in the prestressing tendons at service load based on a cracked section and the decompression stress, f_{dc}, in the prestressing tendons, which may be taken conservatively, to be equal to the effective prestress, f_{se}. Note that Δf_{ps} should not exceed 36 ksi. If it is less than or equal to 20 ksi, the spacing requirement will not apply.

Equation 8.18 can be written as follows:

$$s = \left(\frac{2}{3}\right)\left[15\left(\frac{40}{\Delta f_{ps}}\right) - 2.5C_c\right]$$

19.6 CRACKING MOMENT

Cracks may develop in a prestressed concrete beam when the tensile stress at the extreme fibers of the critical section equals or exceeds the modulus of rupture of concrete, f_r. The value of f_r for normal-weight concrete may be assumed to be equal to $7.5\lambda\sqrt{f'_c}$ where $\lambda = 1.0$. The stress at the bottom fibers of a simply supported beam produced by the prestressing force and the cracking moment is

$$\sigma_b = -\frac{F}{A} - \frac{(Fe)y_b}{I} + \frac{M_{cr}y_b}{I}$$

When $\sigma_b = f_r = 7.5\sqrt{f_c'}$, then the cracking moment is

$$M_{cr} = \frac{I}{y_b}\left(7.5\lambda\sqrt{f_c'} + \frac{F}{A} + \frac{(Fe)y_b}{I}\right) \tag{19.48}$$

The maximum tensile stress after all losses is $7.5\lambda\sqrt{f_c'}$, which represents f_r. In this case, prestressed concrete beams may remain uncracked at service loads. To ensure adequate strength against cracking, the ACI Code, Section 18.8.2, requires that the factored moment of the member ϕM_n be at least 1.2 times the cracking moment, M_{cr}.

Example 19.5

For the beam of Example 19.4, check the design strength and cracking moment against the ACI Code requirements.

Solution

1. Check if the stress block depth a lies within the flange.

$$a = \frac{A_{ps}\,f_{ps}}{0.85 f_c' b} \tag{19.35}$$

$$A_{ps}\left(\text{of 20 tendons } \frac{7}{16} \text{ in. in diameter}\right) = 2.178 \text{ in.}^2$$

Let $f_{py}/f_{pu} = 0.85$, $\rho_p = 0.4$, and $\gamma_p/\beta_1 = 0.4/0.8 = 0.5$. For bonded tensions,

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p \times \frac{f_{pu}}{f_c'}\right) \tag{19.39}$$

$$d = 40 - 6.3 = 33.7 \text{ in.}$$

$$\rho_p = \frac{A_{ps}}{bd} = \frac{2.178}{18 \times 33.7} = 0.00359$$

Given $f_{pu} = 250$ ksi,

$$f_{ps} = 250\left[1 - 0.5(0.00359) \times \frac{250}{5}\right] = 228 \text{ ksi}$$

$$a = \frac{2.178 \times 228}{0.85 \times 5 \times 18} = 6.5 \text{ in.}$$

which is greater than 6 in. Therefore, the section acts as a flanged section.

2. For flanged sections,

$$M_n = A_{pw} f_{ps}\left(d - \frac{a}{2}\right) + A_{pf} f_{ps}\left(d - \frac{h_f}{2}\right)$$

where

$$A_{pw}\text{ (web)} = A_{ps} - A_{pf}\text{ (flange)}$$

$$A_{pf} = \frac{1}{f_{ps}}\left[0.85 f_c'(b - b_w)h_f\right]$$

$$= \frac{1}{228}\left[0.85 \times 5(18 - 6)6\right] = 1.342 \text{ in.}^2$$

$$A_{pw} = 2.178 - 1.342 = 0.836 \text{ in.}^2$$

$$a = \frac{A_{pw} f_{ps}}{0.85 f_c' b_w} = \frac{0.836(228)}{0.85 \times 5 \times 6} = 7.5 \text{ in.}$$

$$M_n = 0.836(228)\left(33.7 - \frac{7.5}{2}\right) + 1.342 \times 228\left(33.7 - \frac{6}{2}\right)$$

$$= 15{,}102 \text{ K·in.} = 1258.5 \text{ K·ft}$$

$$\phi M_n = 0.9(1258.5) = 1132.7 \text{ K·ft}$$

Check the reinforcement index for the flanged section:

$$\rho_{pw} \text{ (web)} = \frac{A_{pw}}{b_w d} = \frac{0.836}{6 \times 33.7} = 0.00413$$

$$\omega_{pw} \text{ (web)} = \rho_{pw} \frac{f_{ps}}{f'_c} \le 0.32\beta_1 = 0.32 \times 0.8 = 0.256$$

($\beta_1 = 0.8$ for $f'_c = 5$ ksi.)

$$\omega_{pw} = 0.00413 \frac{(228)}{5} = 0.188 < 0.256 \qquad (\phi = 0.9)$$

3. Calculate the external factored moment due to dead and live loads.

$$\text{Dead load} = \text{self-weight} + \text{additional dead load}$$

$$= 0.388 + 0.9 = 1.29 \text{ K/ft}$$

$$\text{Live load} = 1.1 \text{ K/ft}$$

$$U = 1.2D + 1.6L$$

$$M_u = \frac{(48)^2}{8}[1.2(1.29) + 1.6(1.1)] = 952.7 \text{ K·ft}$$

This external moment is less than the factored moment capacity of the section of 1132.7 K·ft; therefore, the section is adequate.

4. The cracking moment (Eq. 19.48) is

$$M_{cr} = \frac{I}{y_b}\left(7.5\lambda\sqrt{f'_c} + \frac{F}{A} + (Fe)\frac{y_b}{I}\right)$$

From Example 19.4, $F = 306.2$ K, $A = 372$ in.^{2}, $e = 14.5$ in., $y_b = 20.8$ in., $I = 66{,}862$ in.^{4}, $f'_c = 5$ ksi, and $7.5\lambda\sqrt{f'_c} = 7.55000 = 530$ psi.

$$M_{cr} = \frac{66{,}862}{20.8}\left[0.53 + \frac{306.2}{372} + \frac{(306.2)(14.5)(20.8)}{66{,}862}\right]$$

$$= 8790 \text{ K·in.} = 732.5 \text{ K·ft}$$

Check that $1.2M_{cr} \le \phi M_n$.

$$1.2\,M_{cr} = 1.2\,(732.5) = 879 \text{ K·ft}$$

This value is less than $\phi M_n = 1132.7$ K·ft. Thus, the beam is adequate against cracking.

19.7 DEFLECTION

Deflection of a point in a beam is the total movement of the point, either downward or upward, due to the application of load on that beam. In a simply supported prestressed concrete beam, the prestressing force is usually applied below the centroid of the section, causing an upward deflection called *camber*. The self-weight of the beam and any external gravity loads acting on the beam will cause a downward deflection. The net deflection will be the algebraic sum of both deflections.

In computing deflections, it is important to consider both the short-term, or immediate, deflection and the long-term deflection. To ensure that the structure remains serviceable, the maximum short- and long-term deflections at all critical stages of loading must not exceed the limiting values specified by the ACI Code (see Section 6.3 in this text).

The deflection of a prestressed concrete member may be calculated by standard deflection equations or by the conventional methods given in books on structural analysis. For example, the midspan deflection of a simply supported beam subjected to a uniform gravity load w is equal to $(5wL^4/384EI)$. The modulus of elasticity of concrete is $E_c = 33\omega^{1.5}\sqrt{f'_c} = 57{,}000\sqrt{f'_c}$ for normal-weight concrete.

The moment of inertia of the concrete section I is calculated based on the properties of the gross section for an uncracked beam. This case is appropriate when the maximum tensile stress in the concrete extreme fibers does not exceed the modulus of rupture of concrete, $f_r = 7.5\sqrt{f'_c}$ (class U beams). When the maximum tensile stress based on the properties of the gross section exceeds $7.5\sqrt{f'_c}$, the effective moment of inertia, I_e, based on the cracked and uncracked sections must be used as explained in Chapter 6 (class T and C beams). Typical midspan deflections for simply supported beams due to gravity loads and prestressing forces are shown in Table 19.3.

Example 19.6

For the beam of Example 19.4, calculate the camber at transfer and then calculate the final anticipated immediate deflection at service load.

Solution

1. Deflection at transfer:
   a. Calculate the downward deflection due to dead load at transfer, self-weight in this case. For a simply supported beam subjected to a uniform load,

$$\Delta_D \text{ (dead load)} = \frac{5wL_4}{384EI}$$

   From Example 19.4, $w_D = 388$ lb/ft, $L = 48$ ft, $E_{ci} = 3600$ ksi, and $I = 66{,}862$ in.^{4}

$$\Delta_D = \frac{5(0.388/12)(48 \times 12)^4}{384(3600)(66{,}862)} = 0.192 \text{ in.} \qquad \text{(downward)}$$

   b. Calculate the camber due to the prestressing force. For a simply supported beam harped at one-third points with the eccentricity $e_1 = 14.5$ in. at the middle third and $e_2 = 0$ at the ends,

$$\Delta_p = \frac{23(F_i e_1)L^2}{216E_{ci}I} \qquad \text{(Table 19.3)}$$

$$= \frac{23(365.9 \times 14.5)(48 \times 12)^2}{216(3600)(66{,}862)} = -0.779 \text{ in.} \qquad \text{(upward)}$$

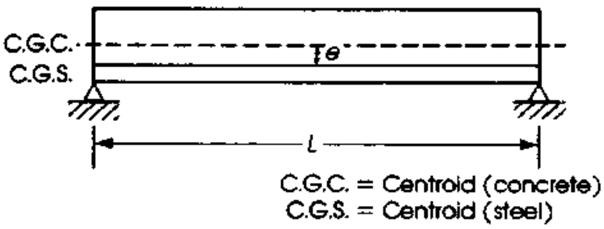
   c. Final camber at transfer is $-0.779 + 0.192 = -0.587$ in. (upward).
2. Deflection at service load: The total uniform service load is $W_T = 0.388 + 0.9 + 1.1 = 2.388$ K/ft, and $E_c = 4000$ ksi. The downward deflection due to W_T is

$$\Delta_w = \frac{5W_T L^4}{384E_c I} = \frac{5(2.388/12)(48 \times 12)^4}{384(4000)(66{,}862)} = +1.067 \text{ in.} \qquad \text{(downward)}$$

Table 19.3 Midspan Deflections of Simply Supported Beams

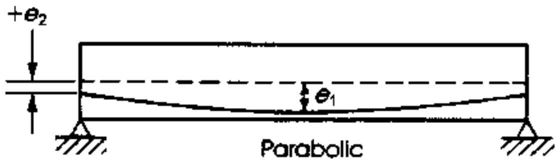
Schematic	**Deflection Equations**

Camber due to prestressing force



$$\Delta = \frac{Fe)L^2}{8\,EI} \quad (1)$$

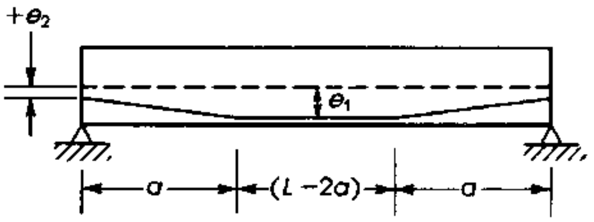
(Horizontal tendons)



$$\Delta = \frac{FL^2}{8\,EI}\left[\frac{5}{6}e_1 + \frac{1}{6}e_2\right] \quad (2)$$

When $e_2 = 0$:

$$\Delta = \frac{5(Fe_1)L^2}{48\,EI} \quad (3)$$



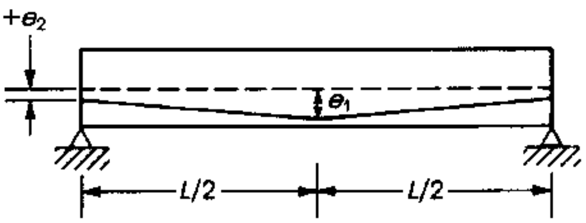
$$\Delta = \frac{FL^2}{8\,EI}\left[e_1 + \frac{4}{3}\left(\frac{a}{L}\right)^2 (e_2 - e_1)\right] \quad (4)$$

When $a = \dfrac{L}{3}$:

$$\Delta = \frac{FL^2}{8\,EI}\left[e_1 + \frac{4}{27}(e_2 - e_1)\right] \quad (5)$$

When $a = \dfrac{L}{3}$ and $e_2 = 0$:

$$\Delta = \frac{23(Fe_1)L^2}{216\,EI} \quad (6)$$

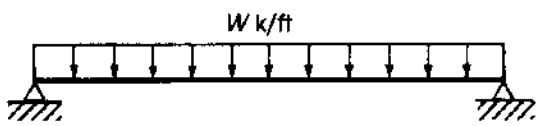


$$\Delta = \frac{FL^2}{24\,EI}[2e_1 + e_2] \quad (7)$$

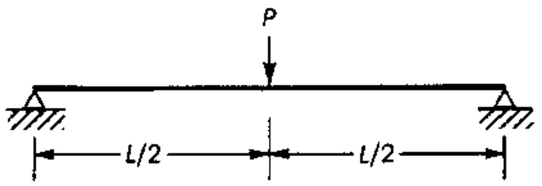
When $e_2 = 0$:

$$\Delta = \frac{(Fe_1)L^2}{12\,EI} \quad (8)$$

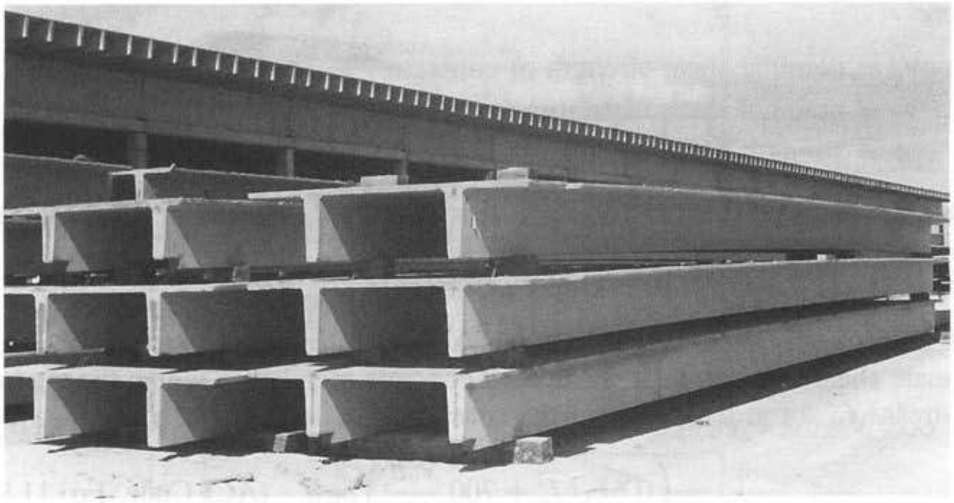
Deflection due to gravity loads



$$\Delta = \frac{5\,wL^4}{384\,EI} \quad (9)$$



$$\Delta = \frac{PL^3}{48\,EI} \quad (10)$$



Upward deflection (camber) in double-T prestressed concrete beams.

The camber due to prestressing force $F = 306.2\,\text{K}$ and $E_c = 4000\,\text{ksi}$ is

$$\Delta_p = \frac{23(306.2 \times 14.5)(48 \times 12)^2}{216(4000)(66{,}862)} = -0.587 \text{ in.} \qquad \text{(upward)}$$

The final immediate deflection at service load is

$$\Delta = \Delta_w - \Delta_p = 1.067 - 0.587 = +0.48 \text{ in.} \qquad \text{(downward)}$$

19.8 DESIGN FOR SHEAR

The design approach to determine the shear reinforcement in a prestressed concrete beam is almost identical to that used for reinforced concrete beams. Shear cracks are assumed to develop at 45° measured from the axis of the beam. In general, two types of shear-related cracks form. One type is due to a combined effect of flexure and shear: The cracks start as flexural cracks and then deviate and propagate at an inclined direction due to the effect of diagonal tension. The second type, web-shear cracking, occurs in beams with narrow webs when the magnitude of principal tensile stress is high in comparison to flexural stress. Stirrups must be used to resist the principal tensile stresses in both cases. The ACI design criteria for shear will be adopted here.

19.8.1 Basic Approach

The ACI design approach is based on ultimate strength requirements using the load factors mentioned in Chapter 3. When the factored shear force, V_u, exceeds half the nominal shear strength ($\phi V_c/2$), shear reinforcement must be provided. The required design shear force, V_u, at each section must not exceed the nominal design strength, ϕV_n, of the cross-section based on the combined nominal shear capacity of concrete and web reinforcement:

$$V_u \leq \phi V_u \leq \phi(V_c + V_s) \qquad (19.49)$$

where

V_c = nominal shear strength of concrete
V_s = nominal shear capacity of reinforcement
ϕ = strength reduction factor = 0.75

When the factored shear force, V_u, is less than $\frac{1}{2}\phi V_c$, minimum shear reinforcement is required.

19.8.2 Shear Strength Provided by Concrete

The ACI Code, Section 11.3, presents a simple empirical expression to estimate the nominal ultimate shear capacity of a prestressed concrete member in which the tendons have an effective prestress, f_{se}, of at least 40% of the specified tensile strength, f_{pu}:

$$V_c = \left(0.6\lambda\sqrt{f'_c} + 700\,\frac{V_u d}{M_u}\right) b_w d \quad \text{(ACI Code, Eq. 11.9)} \qquad (19.50)$$

where

V_u and M_u = factored shear and moment at the section under consideration
b_w = width of web
$d\left(\text{in the term } \frac{V_u d}{M_u}\right)$ = the distance from the compression fibers to the centroid of the prestressing steel
d(in V_{ci} or V_{cw} equations) = the larger of the above d or $0.8h$ (ACI Code, Section 11.3.1)

The use of Eq. 19.50 is limited to the following conditions:

1. The quantity $V_u d/M_u \leq 1.0$ (to account for small values of V_u and M_u)
2. $V_c \geq (2\lambda\sqrt{f'_c})b_w d$(minimum V_c)
3. $V_c \leq (5\lambda\sqrt{f'_c})b_w d$(maximum V_c)

The variation of the concrete shear capacity for a simply supported prestressed concrete beam subjected to a uniform load is shown in Fig. 19.8. Note that the maximum shear reinforcement may be required near the supports and near one-fourth of the span where ϕV_s reaches maximum values. In contrast, similar reinforced concrete beams require maximum shear reinforcement (or minimum spacings) only near the support where maximum ϕV_s develops.

The values of V_c calculated by Eq. 19.50 may be conservative sometimes; therefore, the ACI Code, Section 11.3.3, gives an alternative approach for calculating V_c that takes into consideration the additional strength of concrete in the section. In this approach, V_c is taken as the smaller of two calculated values of the concrete shear strength V_{ci} and V_{cw} (Fig. 19.8). Both are explained next.

The shear strength, V_{ci}, is based on the assumption that flexural-shear cracking occurs near the interior extremity of a flexural crack at an approximate distance of $d/2$ from the load point in the direction of decreasing moment. The ACI Code, Section 11.3.3.1 specifies that V_{ci} be computed as follows:

$$V_{ci} = (0.6\lambda\sqrt{f'_c})b_w d + V_d + \frac{V_i M_{cr}}{M_{max}} \qquad (19.51)$$

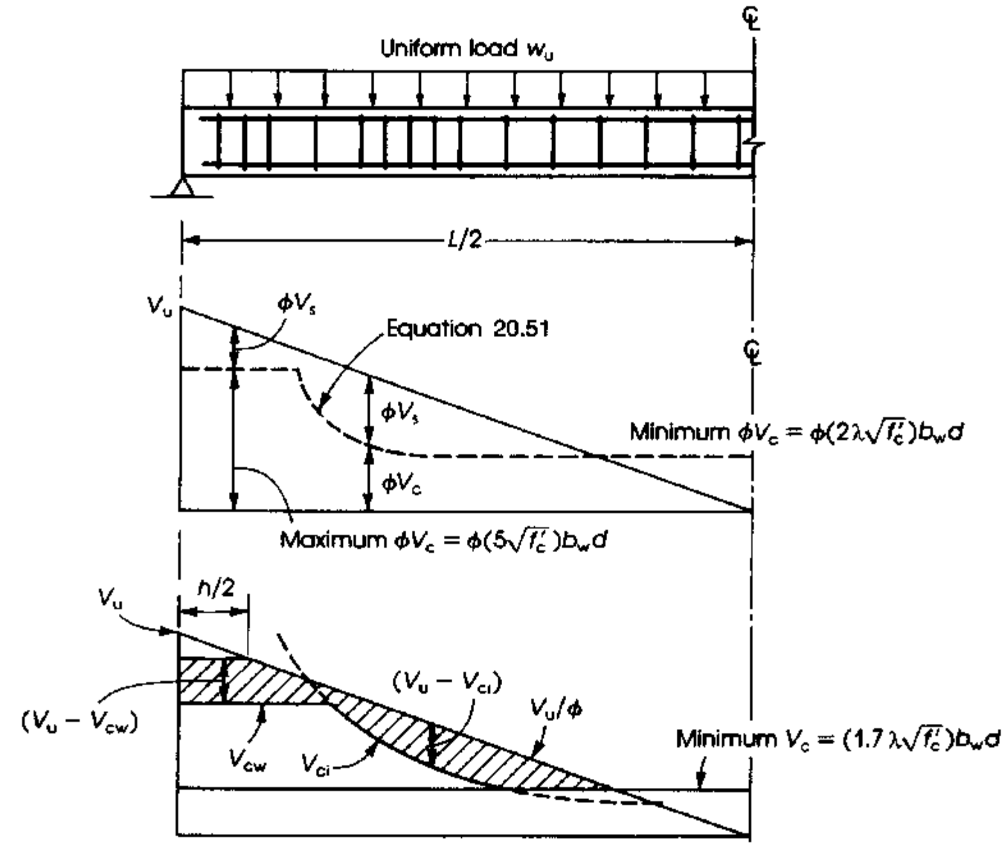


Figure 19.8 Distribution of shear forces along span. The middle diagram shows shear capacity of a simply supported prestressed concrete beam. The bottom diagram shows ACI analysis. (Stirrups are required for shaded areas).

but it is not less than $(1.7\lambda\sqrt{f'_c})b_w d$, where

V_d = shear force at section due to unfactored dead load

V_i = factored shear force at section due to externally applied loads occurring simultaneously with $M_{\max}$

$M_{\max}$ = maximum factored moment at the section due to externally applied loads

M_{cr} = cracking moment

The cracking moment can be determined from the following expression:

$$M_{cr} = \frac{I}{y_t}(6\lambda\sqrt{f'_c} + f_{pe} - f_d) \quad \text{(ACI Code, Eq. 11.11)} \tag{19.52}$$

where

I = moment of inertia of the section resisting external factored loads

y_t = distance from the centroidal axis of the gross section neglecting reinforcement to the extreme fiber in tension

f_{pe} = compressive strength at the extreme fibers of the concrete section due to the effective prestress force after all losses

f_d = stress due to the unfactored dead load at the extreme fiber, where tensile stress is caused by external loads

λ = modification factor for concrete

The web-shear strength, V_{cw}, is based on shear cracking in a beam that has not cracked by flexure. Such cracks develop near the supports of beams with narrow webs. The ACI Code, Section 11.3.3.2, specifies that V_{cw} be computed as follows:

$$V_{cw} = (3.5\lambda\sqrt{f_c'} + 0.3 f_{pc})b_w d + V_p \quad (19.53)$$

where

V_p = vertical component of the effective prestress force at the section considered

f_{pc} = compressive stress (psi) in the concrete (after allowance for prestress losses) at the centroid of the section resisting the applied loads or at the junction of the web and flange when the centroid lies within the flange

Alternatively, V_{cw} may be determined as the shear force that produces a principal tensile stress of $4\lambda\sqrt{f_c'}$ at the centroidal axis of the member or at the intersection of the flange and web when the centroid lies within the flange. The equation for the principal stresses may be expressed as follows:

$$f_t = 4\lambda\sqrt{f_c'} = \sqrt{v_{cw}^2 + \left(\frac{f_{pc}}{2}\right)^2} - \frac{f_{pc}}{2}$$

or

$$V_{cw} = f_t\left(\sqrt{\frac{1 + f_{pc}}{f_t}}\right) b_w d \quad (19.54)$$

where $f_t = 4\lambda\sqrt{f_c'}$. When applying Eqs. 19.51 and 19.53 or 19.54, the value of d is taken as the distance between the compression fibers and the centroid of the prestressing tendons but is not less than $0.8h$.

The critical section for maximum shear is to be taken at $h/2$ from the face of the support. The same shear reinforcement must be used at sections between the support and the section at $h/2$.

19.8.3 Shear Reinforcement

The value of V_s must be calculated to determine the required area of shear reinforcement.

$$V_u = \phi(V_c + V_s) \quad (19.49)$$

$$V_s = \frac{1}{\phi}(V_u - \phi V_c) \quad (19.55)$$

For vertical stirrups,

$$V_s = \frac{A_v f_y d}{s} \quad (19.56)$$

and

$$A_v = \frac{V_s s}{f_y d} \quad \text{or} \quad s = \frac{A_v f_y d}{V_s} \quad (19.57)$$

where A_v = area of vertical stirrups and s = spacing of stirrups. Equations for inclined stirrups are the same as those discussed in Chapter 8.

19.8.4 Limitations

1. Maximum spacing, s_{max}, of the stirrups must not exceed $3h/4$ or 24 in. If V_s exceeds $4\sqrt{f'_c}b_w d$, the maximum spacing must be reduced to half the preceding values (ACI Code, Section 11.4.4).
2. Maximum shear, V_s, must not exceed $8\sqrt{f'_c}b_w d$; otherwise, increase the dimensions of the section (ACI Code, Section 11.4.7.9).
3. The minimum shear reinforcement, A_v, required by the ACI Code is

$$A_{v\,\text{min}} = \frac{50 b_w s}{f_y} \leq 0.75\sqrt{f'_c}\left(\frac{b_w s}{f_y}\right) \tag{19.58}$$

When the effective prestress, f_{pe}, is greater than or equal to 0.4 f_{pe}, the minimum A_v is

$$A_v = \frac{A_{ps}}{80} \times \frac{f_{pu}}{f_y} \times \frac{s}{d} \times \frac{d}{b_w} \tag{19.59}$$

The effective depth, d, need not be taken less than $0.8h$. Generally, Eq. 19.59 requires greater minimum shear reinforcement than Eq. 19.58.

Example 19.7

For the beam of Example 19.4, determine the nominal shear strength and the necessary shear reinforcement. Check the sections at $h/2$ and 10 ft from the end of the beam. Use $f_y = 60$ ksi for the shear reinforcement, and a live load = 1.33 K/ft. using normal-weight concrete.

Solution

1. For the section at $h/2$:

$$\frac{h}{2} = \frac{40}{2} = 20 \text{ in.} = 1.67 \text{ ft from the end}$$

2. The factored uniform load on beam is

$$W_u = 1.2(0.388 + 0.9) + 1.6 \times 1.33 = 3.68 \text{ K/ft}$$

$$V_u \text{ at a distance } \frac{h}{2} = 3.68(24 - 1.67) = 82.2 \text{ K}$$

Using the simplified ACI method (Eq. 19.50), determine M_u at section $h/2$.

$$M_u = (3.68 \times 21) \times 1.67 - 3.68\frac{(1.67)^2}{2} = 142.4 \text{ K·ft} = 1708 \text{ K·in.}$$

The value of d at section $h/2$ from the end (Fig. 19.6b) is

$$d = 33.7 \text{ (at midspan)} - \frac{(16 - 1.67)}{16} \times 14.5 = 20.7 \text{ in.}$$

$$\frac{V_u d}{M_u} = \frac{82.2 \times 20.7}{1708} = 0.966 \leq 1.0$$

as required by the ACI Code.

$$V_c = \left(0.6\lambda\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right) b_w d$$

$$= (0.6 \times 1 \times \sqrt{5000} + 700 \times 0.996)6 \times 20.7 = 91{,}800 \text{ lb} = 91.8 \text{ K}$$

$$\text{Minimum } V_c = 2\lambda\sqrt{f'_c}b_w d = 2 \times 1 \times \sqrt{5000} \times 6 \times 20.7 = 17.6 \text{ K}$$

$$\text{Maximum } V_c = 5\lambda\sqrt{f'_c}b_w d = 43.9 \text{ K}$$

The maximum V_c of 43.9 K controls.

3. The alternative approach presented by the ACI Code is that V_c may be taken as the smaller value of V_{ci} and V_{cw}.

a. Based on the flexural-shear cracking strength,

$$V_{ci} = (0.6\lambda\sqrt{f'_c})b_w d + \left(V_d + \frac{V_j M_{cr}}{M_{max}}\right) \tag{19.51}$$

Calculate each item separately:

$$(0.6\lambda\sqrt{f'_c})b_w d = 0.6 \times 1 \times \sqrt{5000} \times 6 \times 20.7 = 5.3 \text{ K}$$

$$V_d = \text{unfactored dead load shear} = 1.288(24 - 1.67) = 28.8 \text{ K}$$

$$M_{max} = \text{maximum factored moment at section (except for weight of beam)}$$

$$\text{Factored load} = 1.2 \times 0.9 + 1.6 \times 1.3 = 3.13 \text{ K/ft}$$

$$M_{max} = 3.13\left[24 \times 1.67 - \frac{(1.67)^2}{2}\right] = 121 \text{ K/ft} = 1453 \text{ K·in.}$$

$$V_i = 3.13(24 - 1.67) = 69.9 \text{ K}$$

$$M_{cr} = \frac{I}{y_t}(6\lambda\sqrt{f'_c} + f_{pe} - f_d)$$

$$I = 66{,}862 \text{ in.}^4 \qquad y_b = 20.8 \text{ in.}$$

$$f_e = \text{compressive stress due to prestressing force}$$

$$= \frac{F}{A} + \frac{Fey_b}{I}$$

$$= \frac{306.2}{372} + \frac{306.2(1.5)(20.8)}{66{,}862} = 0.966 \text{ ksi}$$

$$f_d = \text{dead load stress} = \frac{M_D y_b}{I}$$

$$M_D = (1.288)\left[24 \times 1.67 - \frac{(1.67)^2}{2}\right] = 49.8 \text{ K·ft} = 598 \text{ K·in.}$$

$$f_d = \frac{598 \times 20.8}{66{,}862} = 0.186 \text{ ksi}$$

$$M_{cr} = \frac{66{,}862}{20.8}(6\sqrt{5000} + 966 + 186) = 3871 \text{ K·in.}$$

Therefore,

$$V_{ci} = (5.3) + (28.8) + 69.9\left(\frac{3871}{1453}\right) = 220.3 \text{ K}$$

V_{ci} must not be less than $(1.7\lambda\sqrt{f'_c})b_w d = (1.7 \times 1 \times \sqrt{5000}) \times 6 \times 20.7 = 14.9$ K.

b. Shear strength based on web-shear cracking is

$$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3 f_{pc}) b_w d + V_p$$

$$f_{pc} = \frac{306.2}{372} = 0.823 \text{ ksi} \tag{19.53}$$

$$d = 20.7 \text{ in.} \quad \text{or} \quad 0.8h = 0.8 \times 40 = 32 \text{ in.}$$

Use $d = 32$ in.

$$V_p = 306.2 \times \frac{1}{13.2} = 23.2 \text{ K}$$

where $1/13.2 =$ slope of tendon profile $= 14.5 \text{ in.}/(16 \times 12)$.

$$3.5\lambda\sqrt{f'_c} = 3.5 \times 1 \times \sqrt{5000} = 248 \text{ psi}$$

Therefore,

$$V_{cw} = (0.248 + 0.3 + 0.823) \times 6 \times 32 + 23.2 = 118.2 \text{ K}$$

c. Because $V_{cw} < V_{ci}$ the value $V_{cw} = 118.2$ K represents the nominal shear strength at section $h/2$ from the end of the beam. In most cases, V_{cw} controls at $h/2$ from the support.

4. Web reinforcement:

$$V_u = 82.3\text{K} \qquad \phi V_{cw} = 0.75 \times 118.2 = 88.65\text{K}$$

Because $V_u < \phi V_{cw}$, $V_s = 0$; therefore, use minimum stirrups. Use no. 3 stirrups. $A_v = 2 \times 0.11 = 0.22 \text{ in.}^2$ Maximum spacing is the least of

$$s_1 = \frac{3}{4}h = \frac{3}{4} \times 40 = 30 \text{ in.} \qquad s_2 = 24 \text{ in.}$$

Calculate s_3 from the equation of minimum web reinforcement:

$$\text{Min. } A_v = \frac{A_{ps}}{80} \times \frac{f_{pu}}{f_y} \times \frac{s}{d} \times \sqrt{\frac{d}{b_w}}$$

$$0.22 = \frac{2.178}{80} \times \frac{250}{60} \times \frac{s_3}{20.7}\sqrt{\frac{20.7}{6}} \tag{19.59}$$

$$s_3 = 21.6 \text{ in.} \qquad (20 \text{ in.})$$

Also,

$$\text{Min. } A_v = \frac{50 b_w s}{f_y} \leq 0.75\sqrt{f'_c}\left(\frac{b_w s}{f_y}\right), \qquad 0.75\sqrt{f'_c} = 53$$

$$s_4 = \frac{A_v f_y}{53 b_w} = \frac{0.22 \times 60{,}000}{53 \times 6} = 41.5 \text{ in.}$$

$s_{max} = s_3 = 20$ in. controls. Thus, use no. 3 stirrups spaced at 20 in.

5. For the section at 10 ft from the end, the calculation procedure is similar to that for the section at $h/2$. Using the ACI simplified method,

$$V_u = 3.68(24 - 10) = 51.5 \text{ K}$$

$$M_u = 3.68\left[24 \times 10 - \frac{(10)^2}{2}\right] = 699.2 \text{ K·ft} = 8390 \text{ K·in.}$$

$$d = 33.7 \text{ (at midspan)} - \frac{(16 \times 10)}{16} \times 14.5 = 28.3 \text{ in.}$$

$$\frac{V_u d}{M_u} = \frac{515 \times 28.3}{8390} = 0.174 < 1.0$$

$$V_c = (0.6 \times 1 \times \sqrt{5000} + 0.174 \times 700)6 \times 28.3 = 27{,}886 \text{ lb} = 27.9 \text{ K} \qquad \text{(controls)}$$

Minimum $V_c = 17.6\,\text{K}$ and maximum $V_c = 43.9\,\text{K}$.

6. Using the ACI Code equations to compute V_{ci} and V_{cw}, calculate V_{ci} first (which controls at this section):

$$0.6\lambda\sqrt{f_c'}b_w d = 0.6 \times 1 \times \sqrt{5000} \times 6 \times 28.3 = 7.2 \text{ K}$$

$$V_d = 1.288(24 - 10) = 18 \text{ K}$$

$$M_{max} = 3.13\left[24 \times 10 - \frac{(10)^2}{2}\right] = 594.7 \text{ K-ft} = 7136 \text{ K-in.}$$

$$V_i = 3.13(24 - 10) = 43.8 \text{ K}$$

$$f_{pe} = \frac{306.2}{372} + \frac{306.2(9.1)(20.8)}{66{,}862} = 1.69 \text{ ksi}$$

$$M_D = 1.288\left[24 \times 10 - \frac{(10)^2}{2}\right] = 244.7 \text{ K-ft} = 2937 \text{ K-in.}$$

$$f_d = \frac{2937 \times 20.8}{66{,}862} = 0.914 \text{ ksi} \qquad M_{cr} = 3858 \text{ K-in.}$$

Therefore,

$$V_{ci} = 7.2 + 18 + \frac{43.8(3858)}{7136} = 48.9 \text{ K}$$

$$V_{ci\,min} = (1.7 \times 1 \times \sqrt{5000})6 \times 28.3 = 20.4 \text{ K}$$

Thus the minimum is met. Then calculate V_{cw}:

$$f_{pc} = 0.893 \text{ ksi} \qquad V_p = 23.2 \text{ K} \qquad \text{(as before)}$$

$$d = 28.3 \text{ in.} \quad \text{or} \quad 0.8h = 32 \text{ in.}$$

Use $d = 32\,\text{in.}$

$$V_{cw} = (3.5\lambda\sqrt{f_c'} + 0.3 f_{pc})b_w d + V_p$$

$$= (0.248 + 0.3 \times 0.823)6 \times 32 + 23.2 = 118.2 \text{ K}$$

This value of V_{cw} is not critical. At about span/4, the critical shear value is V_{ci} (Fig. 19.8).

7. To calculate web reinforcement,

$$V_u = 51.5 \text{ K} \quad \phi V_{ci} = 0.75 \times 48.9 = 36.7 \text{ K}$$

$$V_u = \phi(V_c + V_s)$$

$$V_s = \frac{1}{0.75}(51.5 - 36.7) = 19.7 \text{ K}$$

Use no. 3 stirrups; $A_v = 0.22\,\text{in}^2$. Check maximum spacing: $s_{max} = 18\,\text{in.}$ (as before).

$$\text{Required } A_v = \frac{V_s s}{f_y d} = \frac{19.7 \times 18}{60 \times 28.3} = 0.209 \text{ in.}^2$$

A_v used is $0.22\,\text{in.}^2 > 0.209\,\text{in}^2$. Therefore, use no. 3 stirrups spaced at 14 in.

19.9 PRELIMINARY DESIGN OF PRESTRESSED CONCRETE FLEXURAL MEMBERS

19.9.1 Shapes and Dimensions

The detailed design of prestressed concrete members often involves a considerable amount of computation. A good guess at the dimensions of the section can result in a savings of time and effort. Hence it is important to ensure, by preliminary design, that the dimensions are reasonable before starting the detailed design.

At the preliminary design stage, some data are usually available to help choose proper dimensions. For example, the bending moments due to the applied external loads, the permissible stresses, and the data for assessing the losses are already known or calculated.

The shape of the cross-section of a prestressed concrete member may be a rectangular, T-, I-, or box section. The total depth of the section, h, may be limited by headroom considerations or may not be specified. Given the freedom of selection, an empirical practical choice of dimensions for a preliminary design is as follows (Fig. 19.9):

1. Total depth of section is $h = \frac{1}{20}$ to $\frac{1}{30}$ of the span L; for heavy loading $h = L/20$ and for light loading $h = L/30$ or $h = 2\sqrt{M_D + M_L}$, where M is in K·ft.
2. The depth of top flange is $h_f = h/8$ to $h/6$.
3. The width of top flange is $b \geq 2h/5$.
4. The thickness of the web is $b_w \geq 4\,\text{in}$. Usually b_w is taken as $h/30 + 4\,\text{in}$.
5. b_w and t are chosen to accommodate and uniformly distribute the prestressing tendons, keeping appropriate concrete cover protection.
6. The approximate area of the concrete section required is

$$A_c(\text{ft}^2) = \frac{M_D + M_t}{30h}$$

where $M_D + M_L$ are in K·ft and h is in ft. In SI units,

$$A_c(\text{m}^2) = \frac{M_D + M_L}{1450h} \quad (M_D + M_L \text{in kN} \cdot \text{m and } h \text{ in m})$$

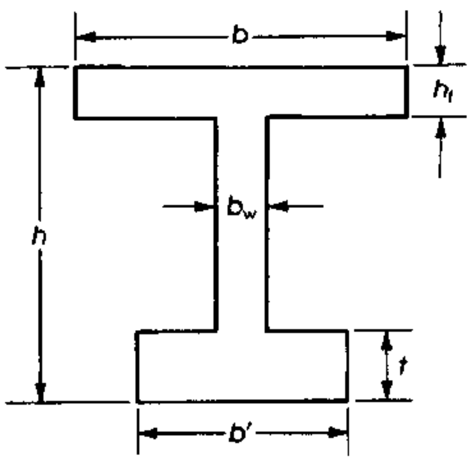


Figure 19.9 Proportioning prestressed concrete sections.

For practical and economical design of prestressed concrete beams and floor slabs, the precast concrete industry has introduced a large number of standardized shapes and dimensions from which the designer can choose an adequate member. Tables of standard sections are available in the PCI Design Handbook [3]. AASHTO [23] has also presented standard girders to be used in bridge construction (Table 19.4).

19.9.2 Prestressing Force and Steel Area

Once the shape, depth, and other dimensions of the cross-section have been selected, approximate values of the prestressing force and the area of the prestressing steel, A_{ps}, can be determined.

From the internal couple concept, the total moment, M_T, due to the service dead and live loads is equal to the tension force, T, times the moment arm, jd.

$$M_T = T(jd) = C(jd)$$

$$M_T = A_{ps} f_{se}(jd) \quad A_{ps} = \frac{M_T}{f_{se}(jd)}$$

where A_{ps} is the area of the prestressing steel and f_{se} is the effective prestressing stress after all losses. The value of the moment arm, jd, varies from $0.4h$ to $0.8h$, with a practical range of $0.6h$ to $0.7h$. An average value of 0.65 may be used. Therefore,

$$A_{ps} = \frac{M_T}{(0.65h) f_{se}} \tag{19.60}$$

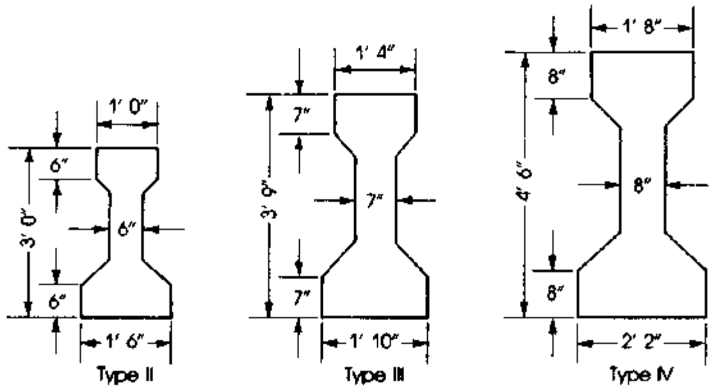
and the prestressing force is

$$F = T = A_{ps} f_{se} = \frac{M_T}{0.65h} \tag{19.61}$$

The prestressing force at transfer is $F_i = F/\eta$, where η is the factor of time-dependent losses.

Table 19.4 AASHTO Girders, Normal-Weight Concrete 25

Designation	A (in.^{2})	I (in.^{4})	y_b (in.)	Z_b (in.^{3})	Z_t (in.^{3})	Weight (lb/ft)
Type II	369	50,979	15.83	3220	2527	384
Type III	560	125,390	20.27	6186	5070	593
Type IV	789	260,741	24.73	10,544	8908	822



The compressive force, C, on the section is equal to the tension force, T:

$$C = T = A_{ps} f_{se}$$

In terms of stresses,

$$\frac{C}{A_c} = \frac{A_{ps} f_{se}}{A_c} = f_{c_1}$$

where f_{c_1} is an assumed uniform stress on the section.

For preliminary design, a triangular stress distribution is assumed with maximum allowable compressive stress, f_{ca}, on one extreme fiber; therefore, the average stress is $0.5 f_{ca} = f_{c_1}$. The allowable compressive stress in concrete is $f_{ca} = 0.45 f'_c$. Thus, the required concrete area, A_c, can be estimated from the force, T, as follows:

$$A_c = \frac{T}{f_{c_1}} = \frac{A_{ps} f_{se}}{f_{c_1}} = \frac{A_{ps} f_{se}}{0.5 f_{ca}} = \frac{A_{ps} f_{se}}{0.225 f'_c} \tag{19.62}$$

$$A_c = \frac{T}{0.5 f_{ca}} = \frac{M_T}{(0.65h)(0.5 f_{ca}})) = \frac{M_T}{0.33 f_{ca}} = \frac{M_T}{0.15 f'_c} \tag{19.63}$$

This analysis is based on the design for service loads and not for the factored loads. The eccentricity, e, is measured from the centroid of the section to the centroid of the prestressing steel and can be estimated approximately as follows:

$$e = K_b + \frac{M_D}{F_i} \tag{19.64}$$

where K_b is the lower Kern limit and M_D is the moment due to the service dead load.

19.10 END-BLOCK STRESSES

19.10.1 Pretensioned Members

Much as a specific development length is required in every bar of a reinforced concrete beam, the prestressing force in a prestressed concrete beam must be transferred to the concrete by embedment or end anchorage or a combination thereof. In pretensioned members, the distance over which the effective prestressing force is transferred to the concrete is called the transfer length, l_t. After transfer, the stress in the tendons at the extreme end of the member is equal to 0, whereas the stress at a distance l_t from the end is equal to the effective prestress, f_{pe}. The transfer length, l_t, depends on the size and type of the tendon, surface condition, concrete strength, f'_c, stress, and method of force transfer. A practical estimation of l_t ranges between 50 and 100 times the tendon diameter. For strands, a practical value of l_t is equal to 50 tendon diameters, whereas for single wires, l_t is equal to 100 wire diameters.

In order that the tension in the prestressing steel develops full ultimate flexural strength, a bond length is required. The purpose is to prevent general slip before the failure of the beam at its full design strength. The development length, l_d, is equal to the bond length plus the transfer length, l_t. Based on established tests, the ACI Code, Section 12.9.1, gives the following expression for computing the development length of three- or seven-wire pretensioning strands:

$$l_d(\text{in.}) = \left(f_{ps} - \frac{2}{3} f_{se}\right) d_b \tag{19.65}$$

where

f_{ps} = stresses in prestressed reinforcement at nominal strength (ksi)
f_{se} = effective stress in prestressed reinforcement after all losses (ksi)
d_b = nominal diameter of wire or strand (in.)

In pretensioned members, high-tensile stresses exist at the end zones, for which special reinforcement must be provided. Such reinforcement in the form of vertical stirrups is uniformly distributed within a distance $h/5$ measured from the end of the beam. The first stirrup is usually placed at 1 to 3 in. from the beam end or as close as possible. It is a common practice to add nominal reinforcement for a distance d measured from the end of the beam. The area of the vertical stirrups, A_v, to be used at the end zone can be calculated approximately from the following expression:

$$A_v = 0.021 \frac{F_i h}{f_{sa} l_t} \tag{19.66}$$

where f_{ss} = allowable stress in the stirrups (usually 20 ksi) and l_t = 50 tendon diameters.

Example 19.8

Determine the necessary stirrup reinforcement required at the end zone of the beam given in Example 19.4.

Solution

$$F_i = 365.9\text{ K} \quad h = 40\text{ in.} \quad f_s = 20\text{ ksi} \quad l_t = 50 \times \frac{7}{16} = 22\text{ in.}$$

Therefore,

$$A_v = 0.021 \times \frac{365.9 \times 40}{20 \times 22} = 0.7\text{ in.}^2$$

$$\frac{h}{2} = \frac{40}{5} = 8\text{ in.}$$

Use four no. 3 closed stirrups within the first 8-in. distance from the support. A_v (provided) = 4 × 0.22 = 0.88 in.^{2}.

19.10.2 Posttensioned Members

In posttensioned concrete members, the prestressing force is transferred from the tendons to the concrete, for both bonded and unbonded tendons, at the ends of the member by special anchorage devices. Within an anchorage zone at the end of the member, very high compressive stresses and transverse tensile stresses develop, as shown in Fig. 19.10. In practice, it is found that the length of the anchorage zone does not exceed the depth of the end of the member; nevertheless, the state of stress within this zone is extremely complex.

The stress distribution due to one tendon within the anchorage zone is shown in Fig. 19.11. At a distance h from the end section, the stress distribution is assumed uniform all over the section. Considering the lines of force (trajectories) as individual elements acting as curved struts, the trajectories tend to deflect laterally toward the centerline of the beam in zone A, inducing compressive stresses. In zone B, the curvature is reversed in direction and the struts deflect outward, inducing tensile stresses. In zone C, struts are approximately straight, inducing uniform stress distribution.

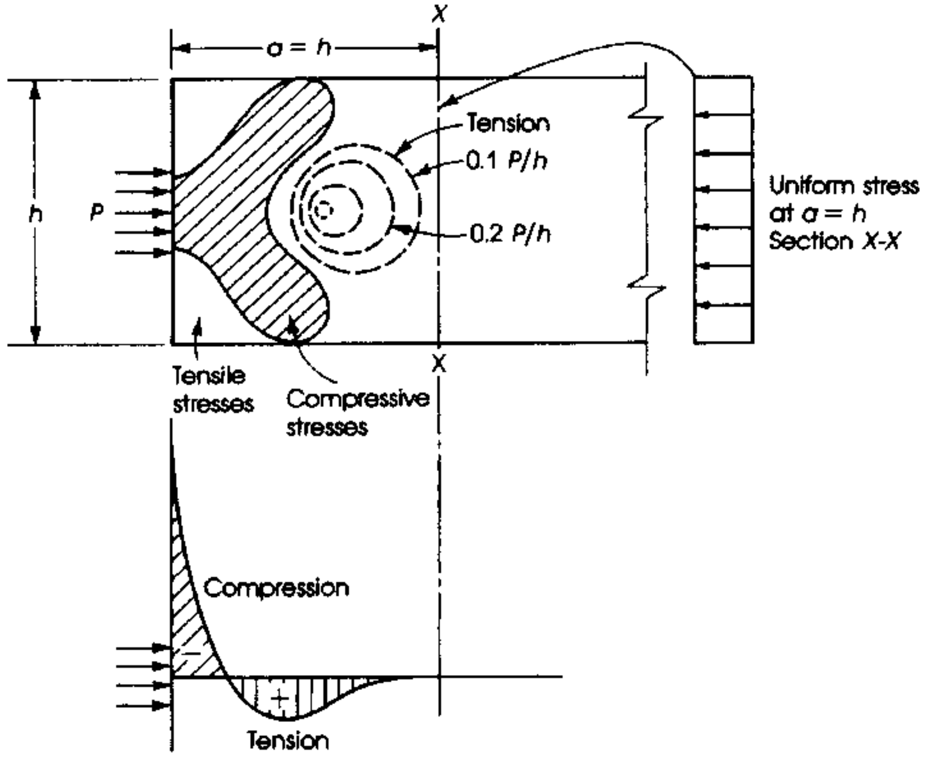


Figure 19.10 Tension and compression zones in a posttensioned member.

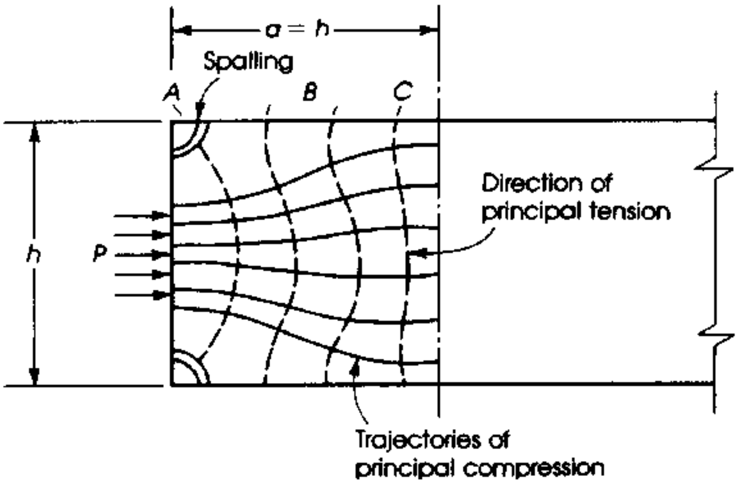


Figure 19.11 Tension and compression trajectories in a posttensioned member.

The reinforcement required for the end anchorage zones of posttensioned members generally consists of a closely spaced grid of vertical and horizontal bars throughout the length of the end block to resist the bursting and tensile stresses. It is a common practice to space the bars not more than 3 in. in each direction and to place the bars not more than 1.5 in. from the inside face of the bearing plate. Approximate design methods are presented in Refs. 24 to 27.

SUMMARY

Section 19.1

The main objective of prestressing is to offset or counteract all or most of the tensile stresses in a structural member produced by external loadings, thus giving some advantages over a reinforced concrete member. A concrete member may be pretensioned or posttensioned. Nonprestressed reinforcement may also be added to the concrete member to increase its ultimate strength.

Section 19.2

1. The allowable stresses in concrete at transfer are

   Maximum compressive stress $= 0.6f'_{ci}$

   Maximum compressive stress at end of simply supported $= 0.7f'_{ci}$

   Maximum tensile stress $= 3\sqrt{f'_{ci}}$

   Maximum tensile stress at end of simply supported $= 6\sqrt{f'_{ci}}$

   The allowable stresses after all losses are $0.45f'_c$ for compression and $6f'_c$ for tension.
2. The allowable stress in a pretensioned tendon at transfer is the smaller of $0.74f_{pu}$ or $0.82f_{py}$. The maximum stress due to the tendon jacking force must not exceed $0.85f_{pu}$ or $0.94f_{py}$; and the maximum stress in a posttensioned tendon after the tendon is anchored is $0.70f_{pu}$.

Section 19.3

The sources of prestress loss are the elastic shortening, shrinkage, and creep of concrete; relaxation of steel tendons; and friction. An approximate lump sum loss is 35 ksi for pretensioned members and 25 ksi for posttensioned members (friction is not included).

$$\text{Loss due to elastic shortening} = \frac{nF_i}{A_c} \tag{19.1}$$

$$\text{Loss due to shrinkage} = \varepsilon_{sh}E_s \tag{19.6}$$

$$\text{Loss due to creep} = C_c(\varepsilon_c E_s) \tag{19.7}$$

Loss due to relaxation of steel varies between 2.5% and 12%. Loss due to friction in posttensioned members stems from the curvature and wobbling of the tendon.

$$P_{px} = P_{pj}e^{-(kl_{px}+\mu_p\alpha_{px})} \tag{19.10}$$

$$P_{px} = P_{pj}(1 + Kl_{px} + \mu_p\alpha_{px})^{-1} \tag{19.11}$$

Section 19.4

Elastic stresses in a flexural member due to loaded and unloaded conditions are given by Eqs. 19.13 through 19.16. The limiting values of the eccentricity, e, are given by Eqs. 19.20, 19.22, 19.24, and 19.26. The minimum and maximum values of F_i are given by Eqs. 19.31 and 19.32, respectively.

Section 19.5

The nominal moment of a rectangular prestressed concrete member is

$$M_n = T\left(d - \frac{a}{2}\right) = A_{ps} f_{ps} d\left(1 - \frac{\rho_p f_{ps}}{1.7 f'_c}\right) \quad (19.37)$$

The values of f_{ps} are given by Eqs. 19.39 and 19.40. For flanged sections,

$$M_n = A_{pv} f_{ps}\left(d - \frac{a}{2}\right) + A_{pf} f_{ps}\left(d - \frac{h_f}{2}\right) \quad (19.43)$$

If nonprestressed reinforcement is used in the flexural member, then

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - \frac{a}{2}\right) \quad (19.44)$$

where $a = (A_{ps} f_{ps} + A_s f_y)/0.85 f'_c b$. For M_n of flanged and rectangular sections with compression reinforcement, refer to Eqs. 19.46 and 19.47, respectively.

Sections 19.6–19.7

1. The cracking moment is

$$M_{cr} = \frac{I}{y_b}\left[7.5\lambda\sqrt{f'_c} + \frac{F}{A} + \frac{(Fe)y_b}{I}\right] \quad (19.48)$$

2. Midspan deflections of simply supported beams are summarized in Table 19.3.

Section 19.8

$$\text{Shear strength of concrete } (V_c) = \left(0.6\lambda\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right) b_w d \quad (19.50)$$

$$\text{Minimum } V_c = 2\lambda\sqrt{f'_c} b_w d$$

$$\text{Maximum } V_c = 5\lambda\sqrt{f'_c} b_w d$$

The shear strength, V_{ci}, based on flexural shear, is given by Eq. 19.47, and the web-shear strength, V_{cw}, is given by Eq. 19.53:

$$V_s = \frac{1}{\phi}(V_u - \phi V_c) \quad \text{and} \quad A_v = \frac{A_{ps}}{80} \times \frac{f_{pu}}{f_y} \times \frac{S}{d} \times \sqrt{\frac{d}{b_w}} \quad (19.59)$$

Section 19.9

Empirical practical dimensions for the preliminary design of prestressed concrete members are suggested in this section.

Section 19.10

The development length of three- to seven-wire strands is

$$l_d = \left(f_{ps} - \frac{2}{3} f_{se}\right) d_b \quad (19.65)$$

The area of stirrups in an end block is

$$A_v = 0.021 \frac{F_i h}{f_{se}} l_t \tag{19.66}$$

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PROBLEMS

19.1 A 60-ft-span simply supported pretensioned beam has the section shown in Fig. 19.12. The beam is prestressed by a force $F_i = 395$ K at transfer (after the elastic loss). The prestress force after all losses is $F = 320$, f'_{ci}(compressive stress at transfer) = 4 ksi and f'_c(compressive stress after all losses) = 6 ksi. For the midspan section and using the ACI Code allowable stresses, (a) calculate the extreme fiber stresses due to the prestressing force plus dead load and (b) calculate the allowable uniform live load on the beam.

19.2 For the beam of Problem 19.1 (Fig. 19.12), calculate the elastic loss and all time-dependent losses using the following data: $F_i = 405$ K, $A_{ps} = 2.39$ in.^{2} located at 6.5 in. from the base, $f'_{ci} = 4$ ksi, and $f'_c = 6$ ksi. $E_c = 57{,}000\sqrt{f'_c}$, and $E_s = 28{,}000$ ksi. The profile of the tendon is parabolic, and the eccentricity at the supports is 0.

19.3 The cross-section of a 56-ft-span simply supported posttensioned girder that is prestressed by 30 cables $\frac{7}{16}$ in. in diameter (area of one cable is 0.1089) is shown in Fig. 19.13. The cables are made of seven-wire stress-relieved strands. The profile of the cables is parabolic with the centroid of the prestressing steel (C.G.S.) located at 9.6 in. from the base at the midspan section and located at the centroid of the concrete section ($e = 0$) at the ends. Calculate the elastic loss of prestress and all other losses. Given: $f'_{ci} = 4$ ksi, $f'_c = 6$ ksi, $E_c = 57{,}000\sqrt{f'_c}$, $E_s = 28{,}000$ ksi, $f_{pu} =$ 250 ksi, $F_o = 175$ ksi, D. L. = 1.0 K/ft (excluding self-weight), and L. L. = 1.6 K/ft.

19.4 For the girder of Problem 19.3,

a. Determine the location of the upper and lower limits of the tendon profile for the section at midspan and for at least two other sections between midspan and support. (Choose sections at 12 ft, 18 ft, and 25 ft from support.)

b. Check if the parabolic profile satisfies these limits.

19.5 For the girder of Problem 19.3, check the limiting values of the prestressing force at transfer F_i.

19.6 A 64-span simply supported pretensioned girder has the section shown in Fig. 19.14. The loads on the girder consist of a dead load = 1.2 K/ft (excluding its own weight) that will be applied at a

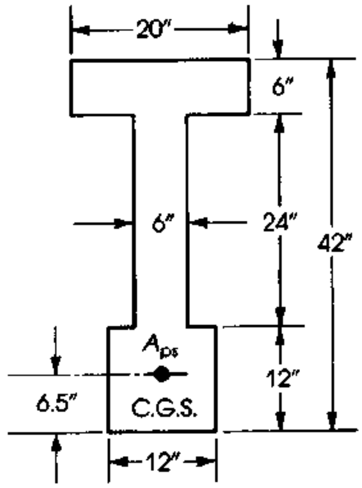


Figure 19.12 Problem 19.1.

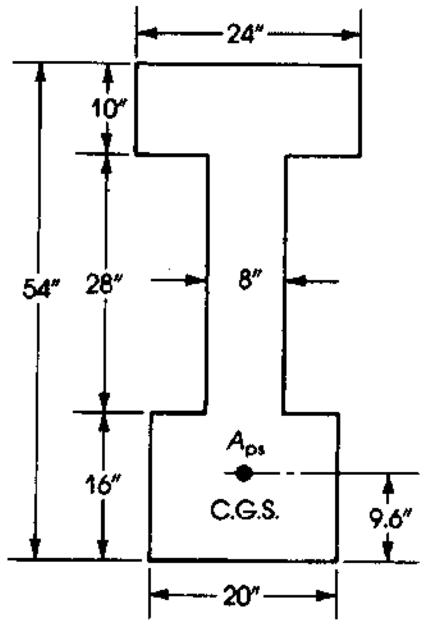


Figure 19.13 Problem 19.3.

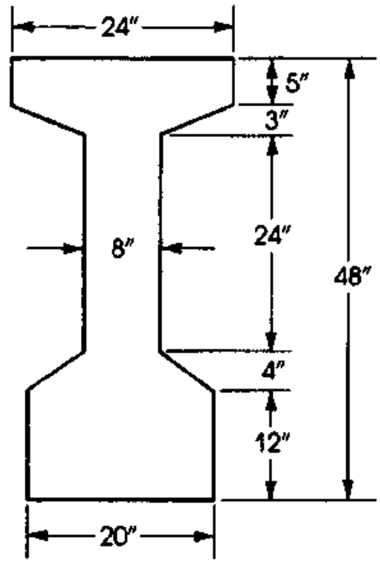


Figure 19.14 Problem 19.6.

later stage and a live load of 0.6 K/ft. The prestressing steel consists of 24 cables $\frac{1}{2}$ in. in diameter (area of one cable = 0.114 in.^{2}), with $E_s = 28{,}000$ ksi, $F_o = 175$ ksi, and $f_{pu} = 250$ ksi. The strands are made of seven-wire stress-relieved steel. The concrete compressive strength at transfer is $f_{ci} = 4$ ksi, and at 28 days, $f'_c = 5$ ksi. The modulus of elasticity is $E_c = 57{,}000\sqrt{f'_c}$. For the beam just described,

a. Determine the upper and lower limits of the tendon profile for the section at midspan and three other sections between the midspan section and the support. (Choose sections at 3 ft, 11 ft, and 22 ft from the support.)

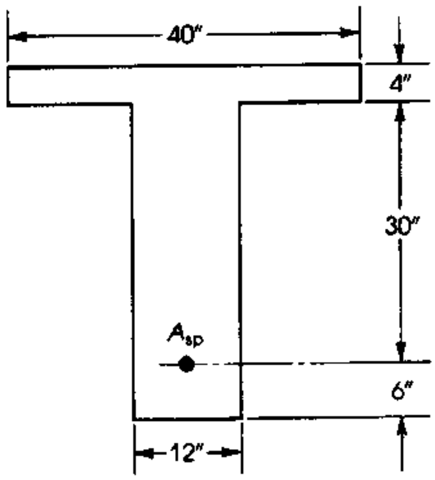


Figure 19.15 Problem 19.10.

b. Locate the tendons to satisfy these limits using straight horizontal tendons within the middle third of the span.

c. Check the limiting values of the prestressing force at transfer.

19.7 For the girder of Problem 19.6,

a. Harp some of the tendons at one-third points, and draw sections at midspan and at the end of the beam showing the distribution of tendons.

b. Revise the prestress losses, taking into consideration the variation of the eccentricity, e, along the beam.

c. Check the factored moment capacity of the section at midspan.

d. Determine the cracking moment.

19.8 For the girder of Problem 19.6,

a. Calculate the camber at transfer.

b. Calculate the immediate deflection at service load.

19.9 For the girder of Problem 19.6, determine the shear capacity of the section and calculate the necessary web reinforcement.

19.10 Determine the nominal moment capacity, M_n, of a pretensioned concrete beam that has the cross-section shown in Fig. 19.15. Given: $f'_c = 5$ ksi, $f_{pu} = 270$ psi, $f_{se} = 160$ ksi, and $A_{se} = 2.88\,\text{in}^2$.

CHAPTER 20

SEISMIC DESIGN OF REINFORCED CONCRETE STRUCTURES

Collapse of frame concrete structures due to an earthquake.

20.1 INTRODUCTION

Ground motions during an earthquake can severely damage the structure. The ground acceleration when transmitted through the structure is amplified, and it is called the response acceleration. The amplified motion can produce forces and displacements that can be larger than the motions the structure can sustain.

Many factors influence the intensity of shaking of the structure such as earthquake magnitude, distance from fault or epicenter, duration of strong shaking, soil conditions of the site, and frequency content of the motion.

A structure should be designed, depending on the type of structure and its function, to have acceptable levels of response generated in an earthquake. Economy of design is achieved by allowing the structure to deform above elastic limit.

20.2 SEISMIC DESIGN CATEGORY

Building Code Requirements for Structural Concrete (ACI 318-08) gives the procedure for design and detailing of structures subjected to earthquake loads but does not address the calculations of seismic forces. In this chapter the International Building Code (IBC 2006) will be utilized for the calculation of seismic forces.

The IBC 2006 section 1613.5.6 defines six seismic design categories (SDC): A, B, C, D, E, and F. It also defines four occupancy categories: I, II, III, and IV. To relate the SDC and the occupancy category, the design spectral response accelerated coefficients S_{DS} and S_{DI} are used. S_{DS} is the design spectral response acceleration coefficient for short periods and S_{D1} is the design response acceleration coefficient for 1-second period. Design spectral response acceleration coefficients are related to severity of the design earthquake ground motions at the site.

A seismic design category will determine which type of lateral force analysis must be performed and which type of lateral-force resisting system must be used.

20.2.1 Determination of Occupancy Category

Buildings shall be assigned an occupancy category according to Table 20.1 as described in IBC 2006 Section 1604.5. The first step is to define the nature of occupancy of the structure according to the occupancy category. The seismic factor, I_E, also called seismic occupancy factor, is also listed in Table 20.1 and will be utilized in a later section.

20.2.2 Determination of Design Spectral Response Acceleration Coefficients

Earthquake ground motion is usually recorded as an acceleration of the ground at a particular location. The acceleration of the ground generates the acceleration of the structure (response acceleration), which produces earthquake forces that act on the structure. Earthquake forces generate deformations, internal forces, and stresses in the structure. If the structure is not properly designed to sustained deformations and forces it will have great damage and may even collapse.

Therefore, the first step to design an earthquake-resistant structure is to determine the maximum possible response accelerations that can occur during the earthquake. It is also important to know that response of the given structure depends on period of vibration and damping characteristics of the structure.

The IBC 2006 Section 1613.5.4 gives a procedure to determine the design response spectrum curve, from which the design response accelerations, S_a, for any given period of vibration T are calculated. One part of this procedure is the determination of the spectral response acceleration coefficients for short periods, S_{DS}, and for a 1-second period, S_{D1}.

To calculate the design acceleration values for short periods, S_{DS} and 1-second periods, S_{D1}, the following equation can be utilized:

$$S_{\mathrm{DS}} = \frac{2}{3} S_{\mathrm{MS}} \qquad (20.1a)$$

$$S_{\mathrm{D1}} = \frac{2}{3} S_{\mathrm{M1}} \qquad (20.1b)$$

where

S_{MS} = mapped maximum considered earthquake spectral response accelerations for short periods adjusted for soil type

S_{M1} = mapped maximum considered earthquake spectral response accelerations for 1-second period adjusted for soil type

S_{MS} and S_{M1} can be determined from

$$S_{\mathrm{MS}} = F_a S_S \qquad (20.2a)$$

$$S_{\mathrm{M1}} = F_v S_1 \qquad (20.2b)$$

where

S_S = mapped maximum considered earthquake spectral response accelerations at short periods determined from Fig. 20.1*a*

S_1 = Mapped maximum considered earthquake spectral response accelerations at 1-second period determined from Fig. 20.1*b*

F_a, F_v = Site coefficients

The values of F_a and F_v are determined from Tables 20.2 and 20.3 and are dependent on the mapped spectral values (S_S and S_1) and the site class as can be determined in Table 20.4.

Table 20.1 Classification of Structures Based on their Nature of Occupancy

Occupancy Category	Nature of Occupancy	Seismic Factor, I_E
I	Buildings and other structures that represent a low hazard to human life in the event of failure including, but not limited to • Agricultural facilities • Certain temporary facilities • Minor storage facilities	1.00
II	Buildings and other structures except those listed in Categories I, III, and IV	1.00
III	Buildings and other structures that represent a substantial hazard to human life in the event of failure including, but not limited to • Buildings and other structures where more than 300 people congregate in one area • Buildings and other structures with elementary school, secondary school, or day care facilities with an occupant load greater than 250 • Buildings and other structures with an occupant load greater than 500 for colleges or adult education facilities. • Health care facilities with an occupant load of 50 or more resident patients but not having surgery or emergency treatment facilities • Jails and detention facilities • Any other occupancy with an occupant load greater than 5000 • Power-generating stations, water treatment for potable water, waste water treatment facilities and other public utility facilities not included in Category IV • Buildings and other structures not included in Category IV containing sufficient quantities of toxic or explosive substances to be dangerous to the public if released	1.25
IV	Buildings and other structures designed as essential facilities including, but not limited to • Hospitals and other health care facilities having surgery or emergency treatment facilities • Fire, rescue, and police stations and emergency vehicle garages • Designed earthquake, hurricane, or other emergency shelters • Designed emergency preparedness, communication, and operation centers and other facilities required for emergency response • Power-generating stations and other public utility facilities required as emergency backup facilities for Category IV structures • Structures containing highly toxic materials as defined by Section 307 of IBC 2006 where the quantity of the material exceeds the maximum allowable quantities of Table 307.7(2) of IBC 2006 • Aviation control towers, air traffic control centers, and emergency aircraft hangers • Buildings and other structures having critical national defense functions • Water treatment facilities required to maintain water pressure for fire suppression	1.50

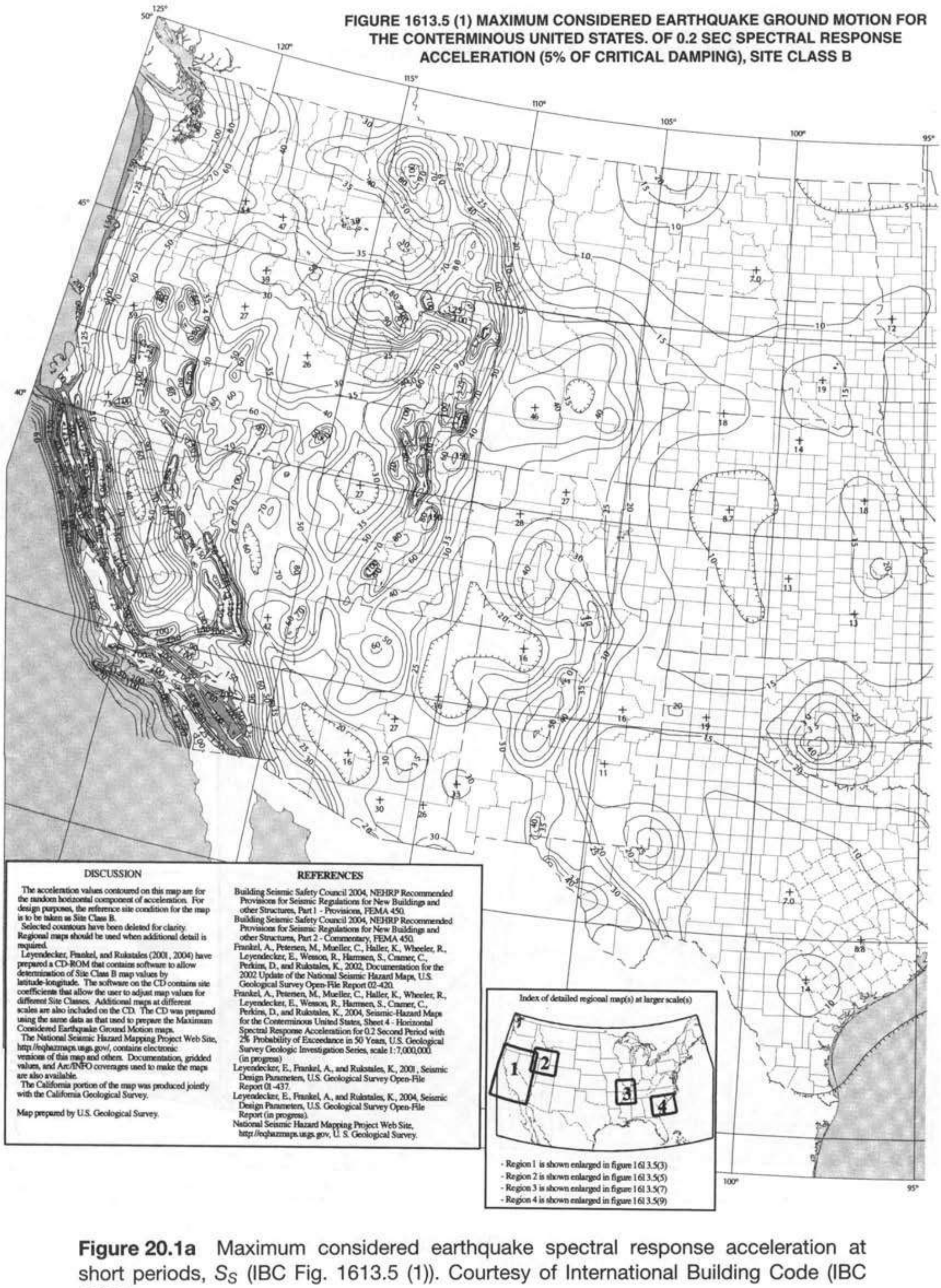


Figure 20.1a Maximum considered earthquake spectral response acceleration at short periods, S_S (IBC Fig. 1613.5 (1)). Courtesy of International Building Code (IBC 2006).

FIGURE 1613.5 (1) (continued) MAXIMUM CONSIDERED EARTHQUAKE GROUND MOTION FOR THE CONTERMINOUS UNITED STATES. OF 0.2 SEC SPECTRAL RESPONSE ACCELERATION (5% OF CRITICAL DAMPING), SITE CLASS B

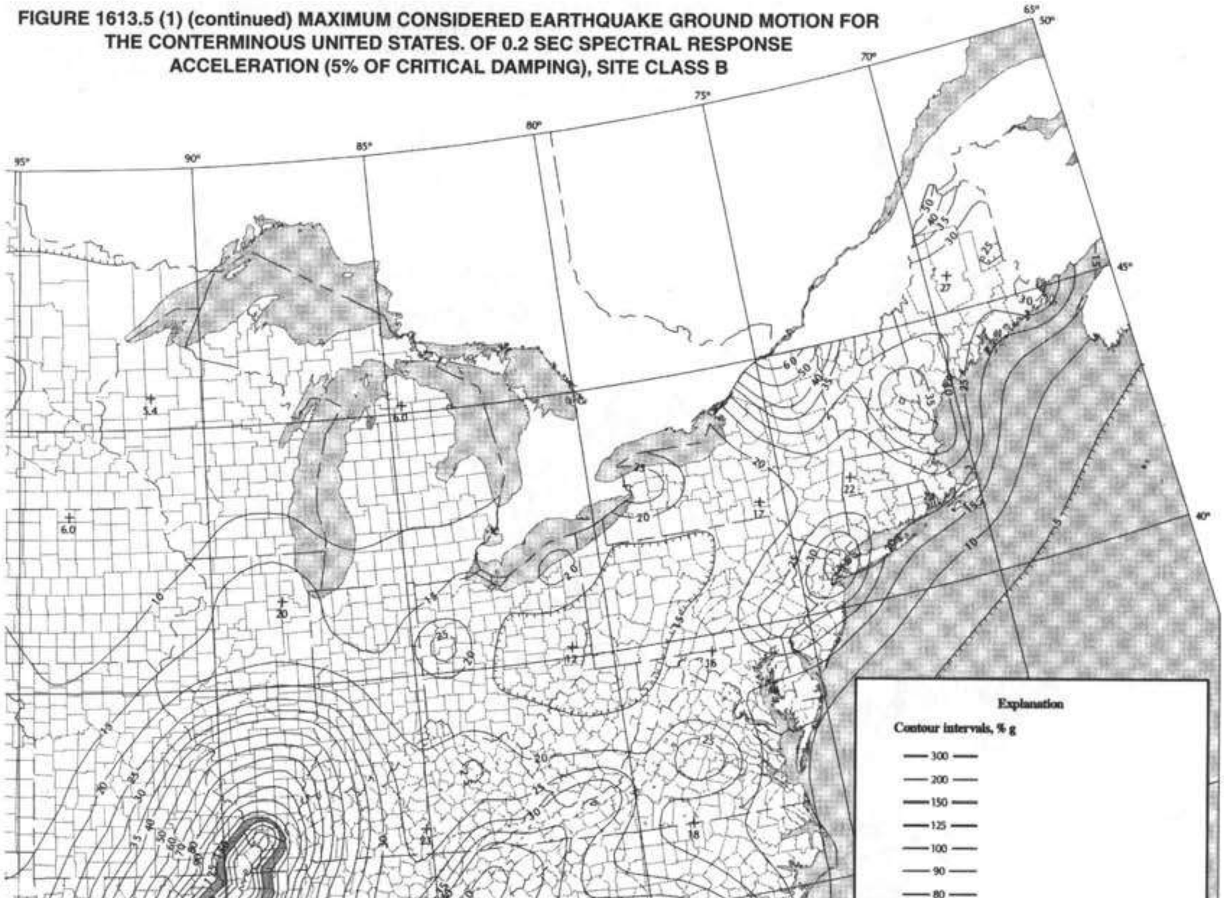


Figure 20.1a *(continued)*

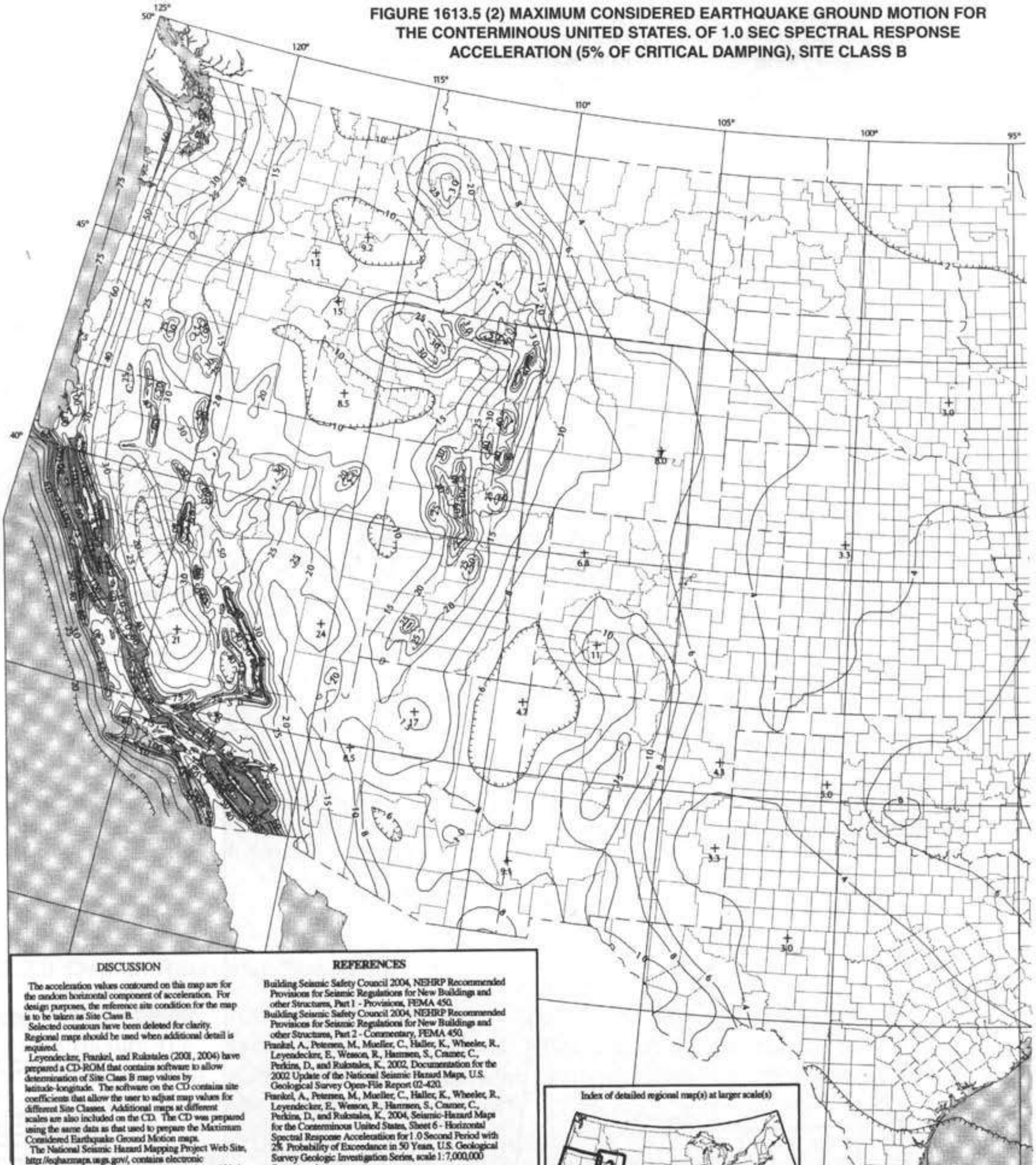


Figure 20.1b Maximum considered earthquake spectral response acceleration at 1-second periods, S_1 (IBC Fig. 1613.5(2)). Courtesy of International Building Code (IBC 2006).

FIGURE 1613.5 (2) (continued) MAXIMUM CONSIDERED EARTHQUAKE GROUND MOTION FOR THE CONTERMINOUS UNITED STATES. OF 1.0 SEC SPECTRAL RESPONSE ACCELERATION (5% OF CRITICAL DAMPING), SITE CLASS B

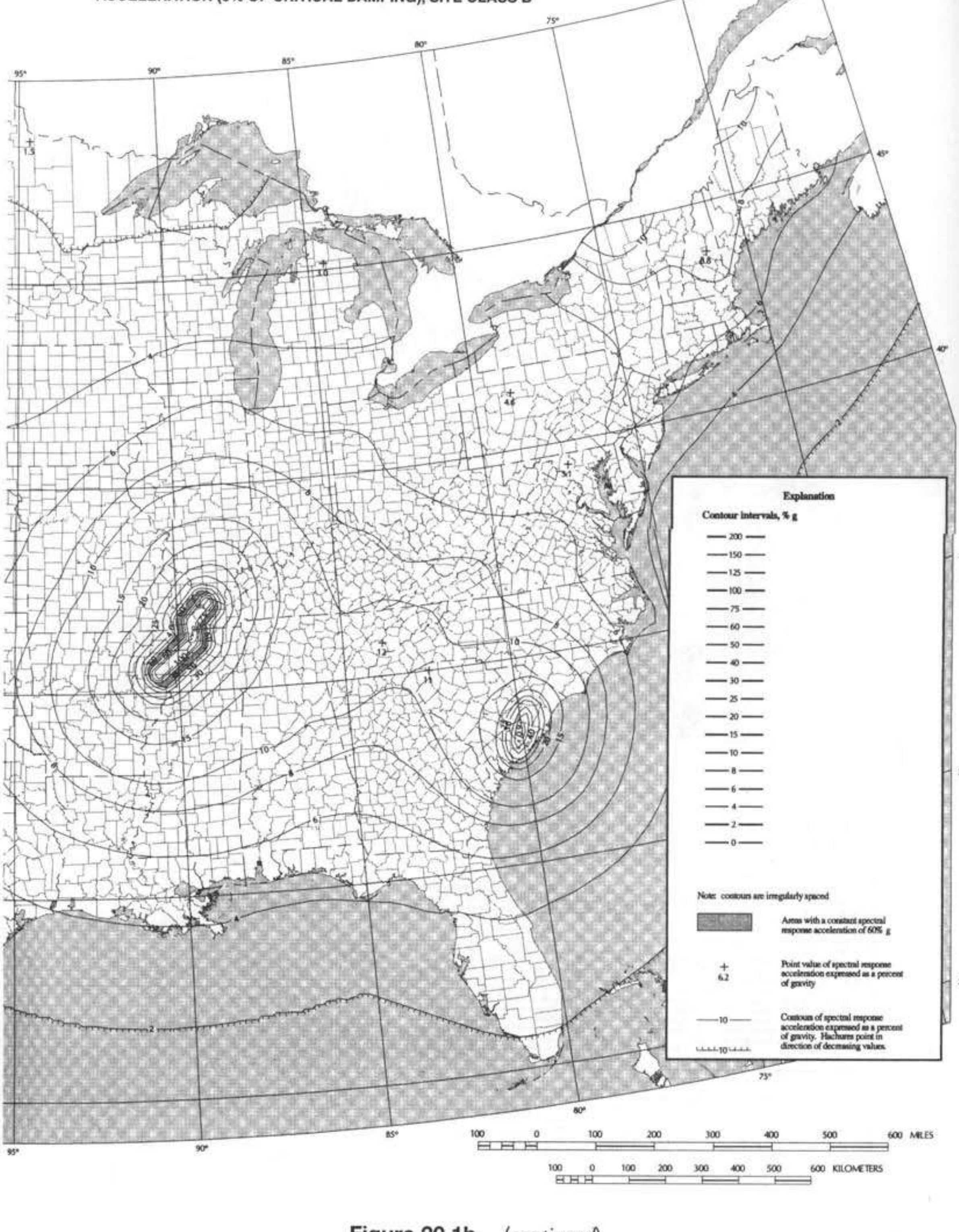


Figure 20.1b *(continued)*

Table 20.2 Values of Site Coefficient, F_a[a] (Table 1613.5.3(1) of IBC 2006)

Site Class	Mapped Spectral Response Acceleration at Short Periods				
	$S_S \le 0.25$	$S_S = 0.50$	$S_S = 0.75$	$S_S = 1.00$	$S_S \ge 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E	2.5	1.7	1.2	0.9	0.9
F	Note b	Note b	Note b	Note b	Note b

[a]Use straight-line interpolation for intermediate values of mapped spectral response acceleration at short period, S_s.
[b]Site-specific geotechnical investigation and dynamic site response analysis shall be performed to determine appropriate values, or in accordance with Section 11.4.7 of ASCE 7.

Table 20.3 Values of Site Coefficient, F_v[a] (Table 1613.5. 3(2) of IBC 2006)

Site Class	Mapped Spectral Response Acceleration at 1-Second Period				
	$S_1 \le 0.1$	$S_1 = 0.2$	$S_1 = 0.3$	$S_1 = 0.4$	$S_1 \ge 0.5$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.7	1.6	1.5	1.4	1.3
D	2.4	2.0	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	2.4
F	Note b	Note b	Note b	Note b	Note b

[a]Use straight-line interpolation for intermediate values of mapped spectral response acceleration at 1-second period, S_1.
[b]Site-specific geotechnical investigation and dynamic site response analysis shall be performed to determine appropriate values, or in accordance with Section 11.4.7 of ASCE7.

20.2.3 Design Response Spectrum

Design response spectrum is used to determine the design spectral response acceleration for a given structure (i.e., given period of vibration). After calculating design response acceleration coefficients S_{DS} and S_{D1} from Section 20.2.2, the design response spectrum curve (ASCE 7-05, Section 11.4.5) should be constructed as follows:

1. For periods $T \le T_o$, the design spectral response acceleration, S_a, shall be determined as

$$S_a = 0.6\,\frac{S_{DS}}{T_o}T + 0.4S_{DS} \tag{20.3}$$

where

$$T_o = 0.2\frac{S_{D1}}{S_{DS}} \tag{20.4}$$

T = Fundamental period of the structure (in seconds) and will be determined later in Section 20.3.1 (Eq. 20.15 or 20.16).

Table 20.4 Site Classification (Table 1613.5.2 of IBC 2006)

Site Class	Soil Profile Name	Average Properties in Top 100, as per Section 1613.5.5		
		Soil Shear Wave Velocity, $\overline{V}_S$, (ft/s)	**Standard Penetration Resistance, $\overline{N}$**	**Soil Undrained Shear Strength, $\overline{S}_u$**
A	Hard rock	$\overline{V}_S > 5{,}000$	N/A	N/A
B	Rock	$2500 < \overline{V}_S \leq 5000$	N/A	N/A
C	Very dense soil and soft rock	$1200 < \overline{V}_S \leq 2500$	$\overline{N}$)50	$S_u \geq 2000$
D	Stiff soil profile	$600 < \overline{V}_S \leq 1200$	$15 \leq \overline{N} \leq 50$	$1000 \leq \overline{S}_u \leq 2000$
E	Stiff soil profile	$\overline{V}_S \leq 600$	$\overline{N} < 15$	$\overline{S}_u < 1000$
E	—	Any profile with more than 10 ft of soil having the following characteristics: 1. Plasticity index $(PI) > 20$ 2. Moisture content $(w) \geq 40\%$ 3. Undrained shear strength $\overline{S}_u) < 500$ psf		
F	—	Any profile containing soils having one or more of the following characteristics: 1. Soils vulnerable to potential failure or collapse under seismic loading such as liquefiable soils, quick and highly sensitive clays, collapsible weakly cemented soils. 2. Peats and/or highly organic clays ($H > 10$ ft of peat and/or highly organic clay where H = thickness of soil) 3. Very high plasticity clays ($H > 25$ ft with plasticity index $(PI) > 75$) 4. Very thick soft/medium stiff clays ($H > 120$ ft)		

2. For periods $T_o \leq T \leq T_s$, the design spectral response acceleration, S_a, shall be determined as

$$S_a = S_{DS} \tag{20.5}$$

where

$$T_s = \frac{S_{D1}}{S_{DS}} \tag{20.6}$$

3. For periods $T_L \geq T > T_s$, the design spectral response acceleration, S_a, shall be determined as

$$S_a = \frac{S_{D1}}{T} \tag{20.7}$$

4. For periods greater than T_L, S_a shall be taken as

$$S_a = \frac{S_{D1}T_L}{T^2} \tag{20.8}$$

where

T_L = long-period transition period(s) showns in Fig. 20.3a (Conterminous United States), Fig. 20.3b (Region 1), Fig. 20.3c (Alaska), Fig. 20.3d (Hawaii). Fig 20.3e (Puerto Rico, Culebra, Vieques, St. Thomas, St. John, and St. Croix), and Fig. 20.3f (Guam and Tutuila).

The shape of the design response spectrum curve is shown in Fig. 20.2.

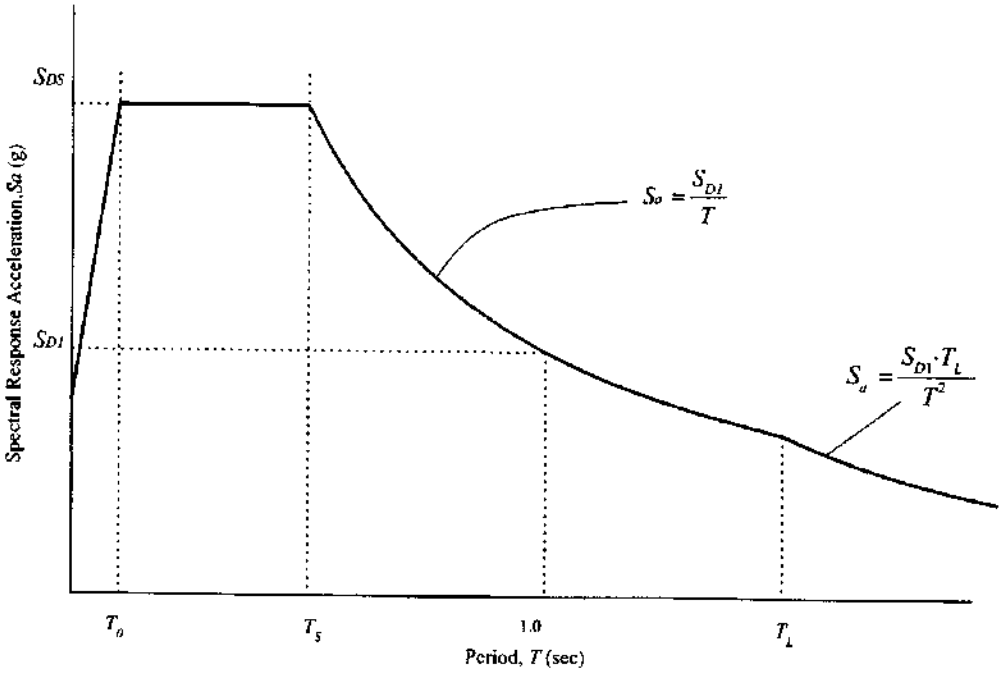


Figure 20.2 Design response spectrum (courtesy of ASCE 7.05, Section 11.4.5).

20.2.4 Determination of Seismic Design Category (SDC)

Structures shall be assigned SDCs, which are classified as A, B, C, D, E, and F are determined from Table 20.5 and Table 20.6. These have no relation to the site class types that, are also named A, B, C, D, E, and F as described in Table 20.4. To determine the SDC, the values of S_{DS} and S_{D1} are utilized and the occupancy category must be defined.

"Occupancy Category I, II, or III structures located where the mapped spectral response acceleration parameter at 1-s period. S_1, is greater than or equal to 0.75 shall be assigned to Seismic Design Category E. Occupancy Category IV structures located where the mapped spectral response acceleration parameter at 1-s period, S_1, is greater than or equal to 0.75 shall be assigned to Seismic Design Category F. All other structures shall be assigned to a Seismic Design Category based on their Occupancy Category and the design spectral response acceleration parameters, S_{DS} and S_{D1}, determined in Section 20.2.2. Each building and structure shall be assigned to the more severe Seismic Design Category in accordance with Table 20.5 or 20.6, irrespective of the fundamental period of vibration of the structure, T.

Where S_1 is less than 0.75, the Seismic Design Category is permitted to be determined from Table 20.5 alone where all of the following apply:

1. In each of the two orthogonal directions, the approximate fundamental period of the structure, T_a, determined in accordance with Section 20.3.1 is less than $0.8T_s$, where T_s is determined in accordance with Section 20.2.3.
2. In each of two orthogonal directions, the fundamental period of the structure used to calculate the story drift is less than T_s.
3. The seismic response coefficient C_s is determined from $C_s = \frac{S_{DS}}{\left(\frac{R}{I_E}\right)}$
4. The diaphragms are rigid or for diaphragms that are flexible, the distance between vertical elements of the seismic force-resisting system does not exceed 40 ft.

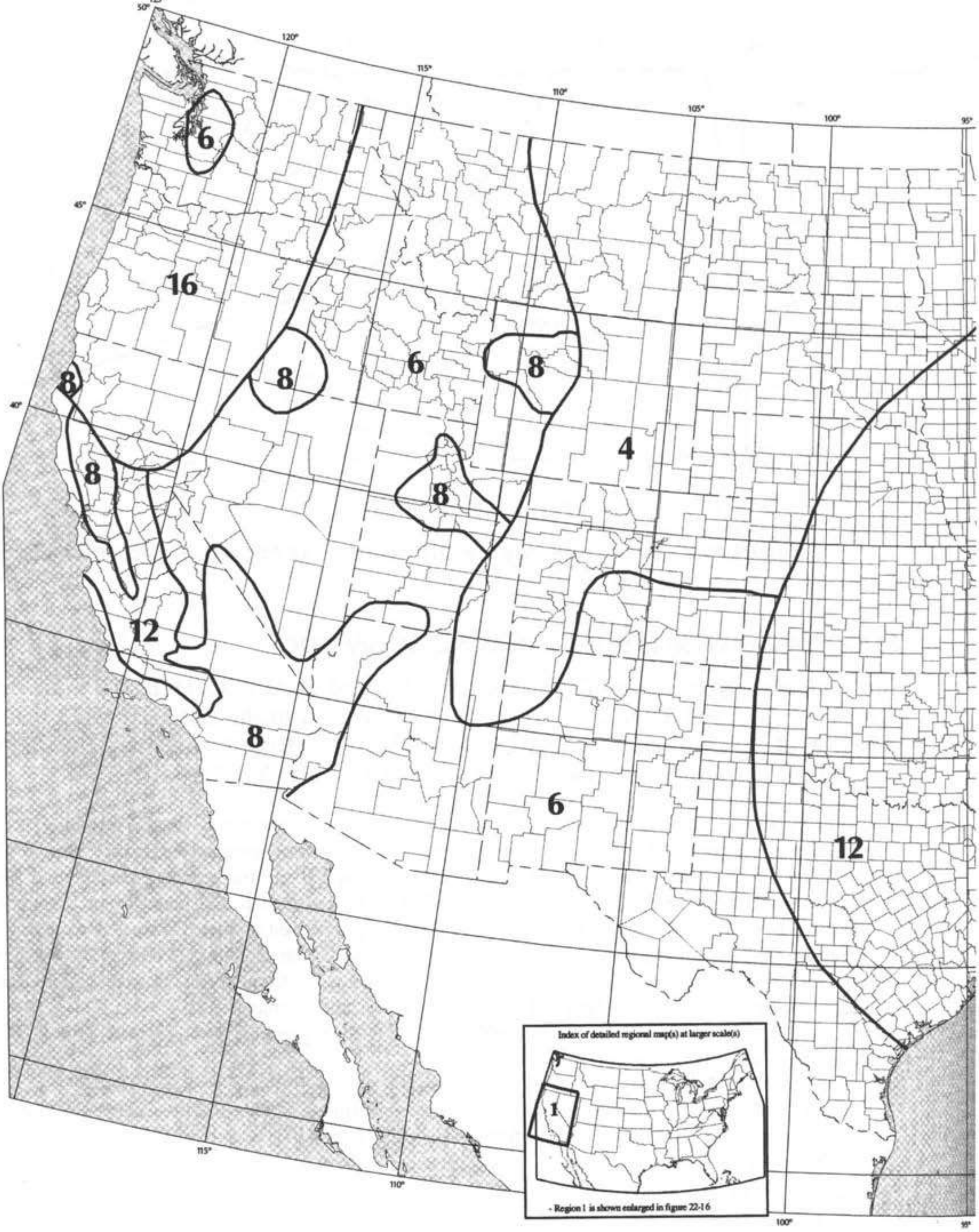


Figure 20.3a Long-period transition period, T_L(SEC), for the conterminous United States (courtesy of ASCE 7-05).

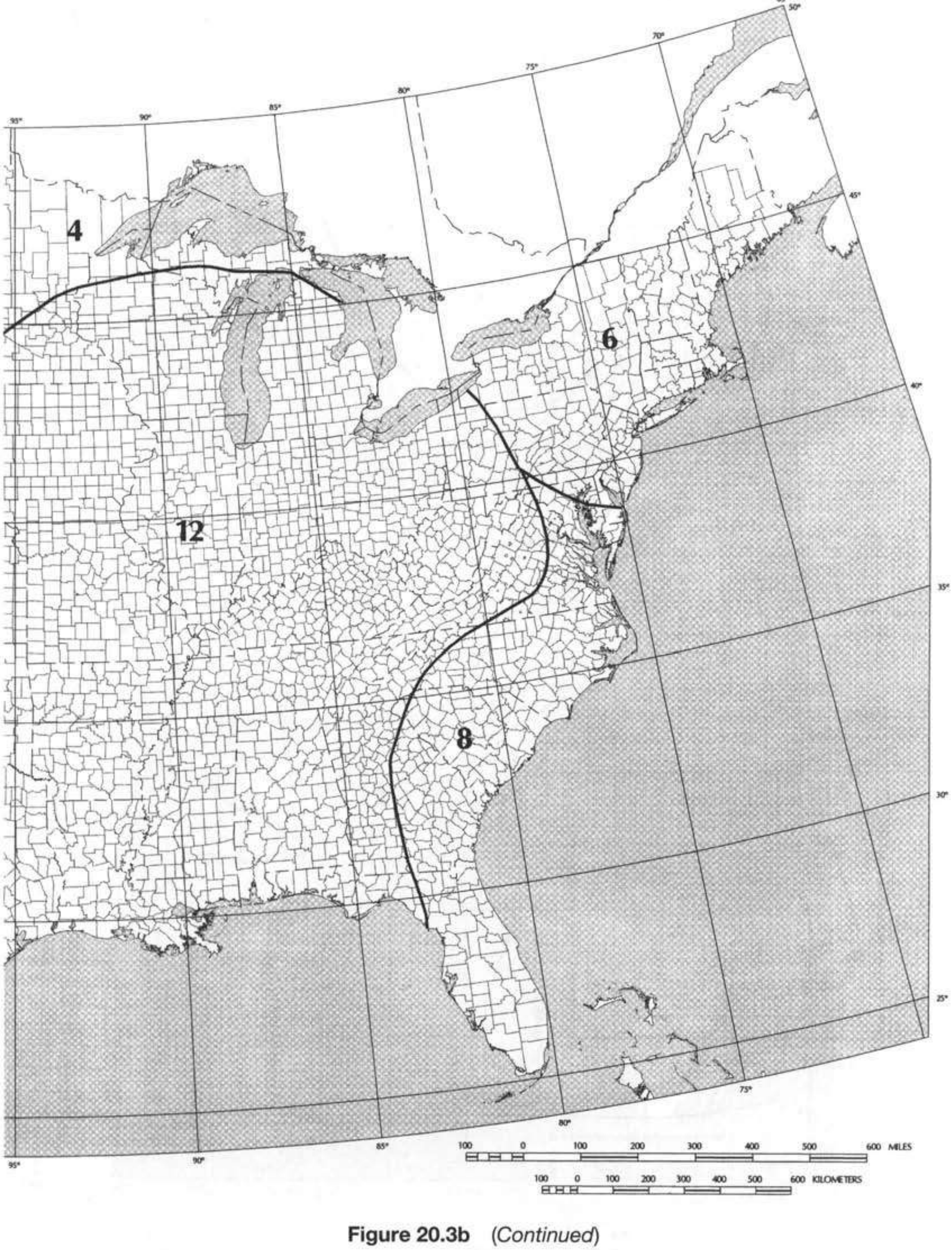


Figure 20.3b *(Continued)*

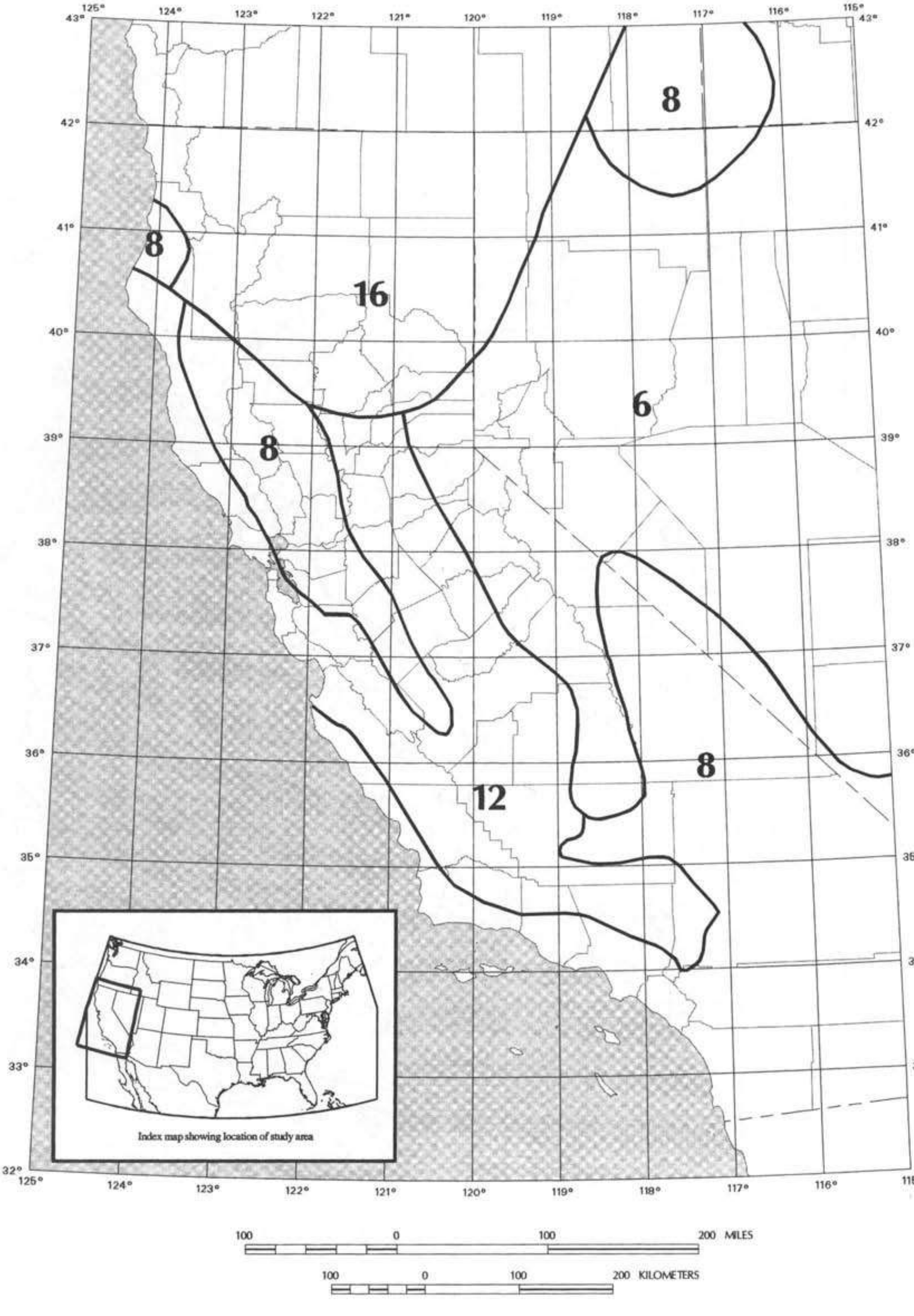


Figure 20.3c Long-period transition period, T_L(SEC), for region 1 (courtesy of ASCE 7-05).

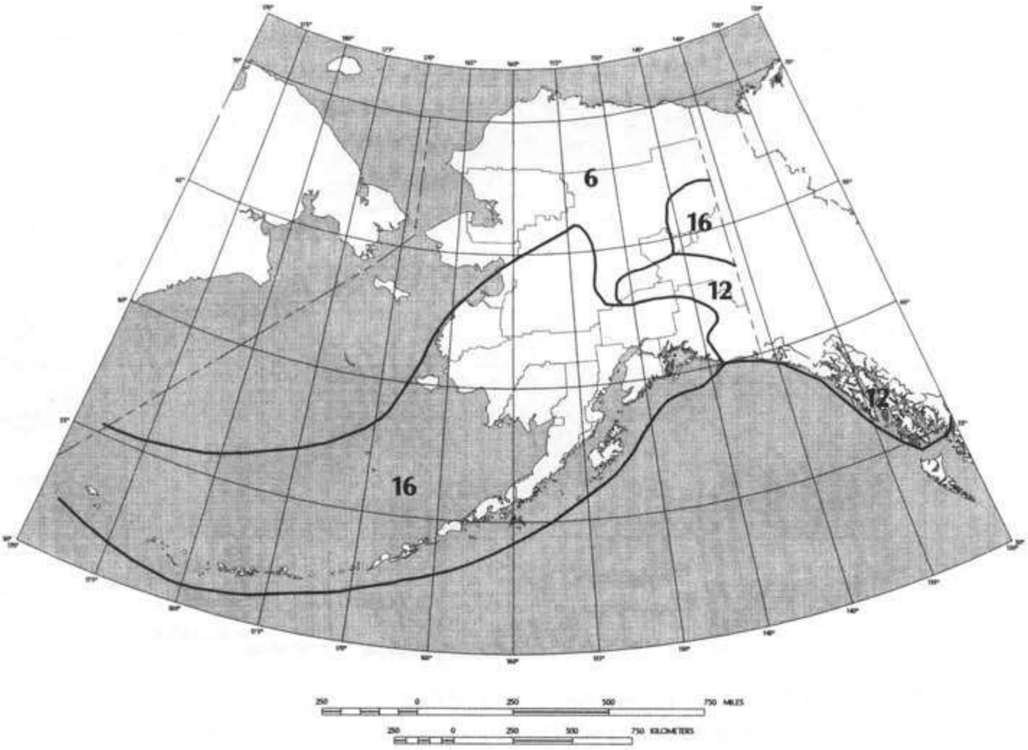


Figure 20.3d Long-period transition period, T_L(SEC), for Alaska (courtesy of ASCE 7-05).

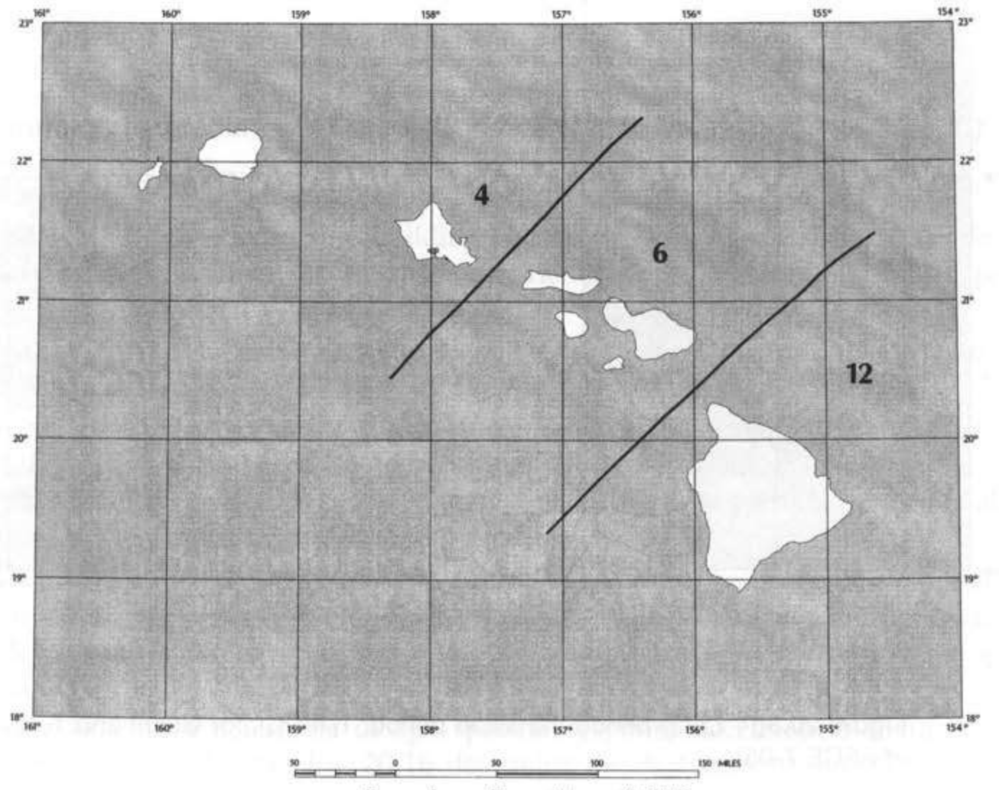


Figure 20.3e Long-period transition period, T_L(SEC), for Hawaii (courtesy of ASCE 7-05).

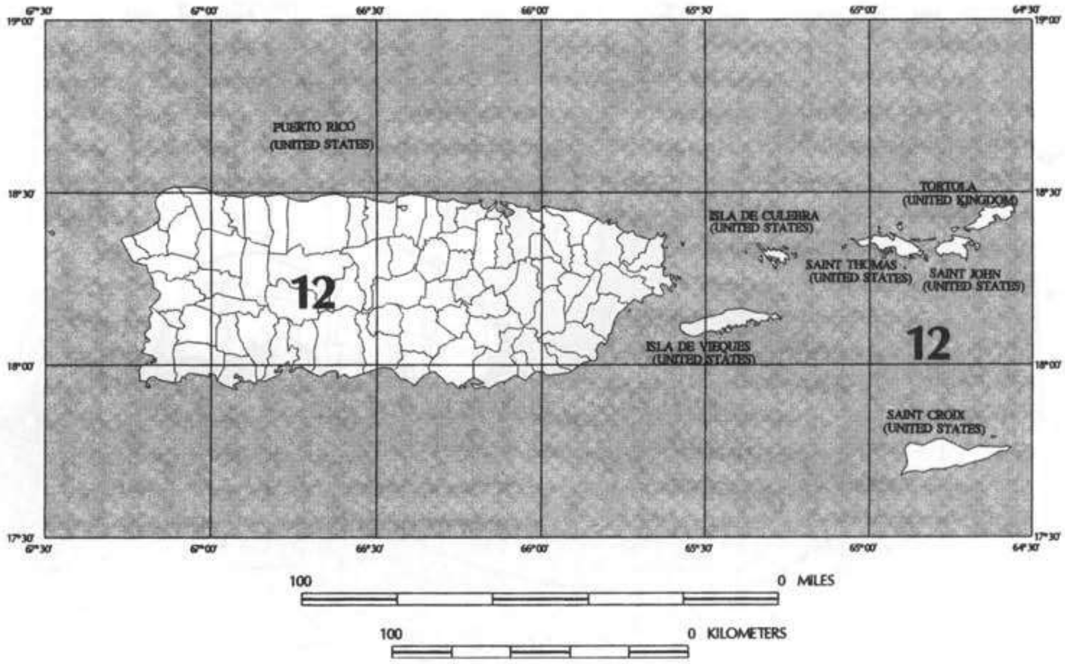


Figure 20.3f Long-period transition period, T_L(SEC), for Puerto Rico, Culebra, Vieques, St. Thomas, St. John, and St. Croix (courtesy of ASCE 7-05).

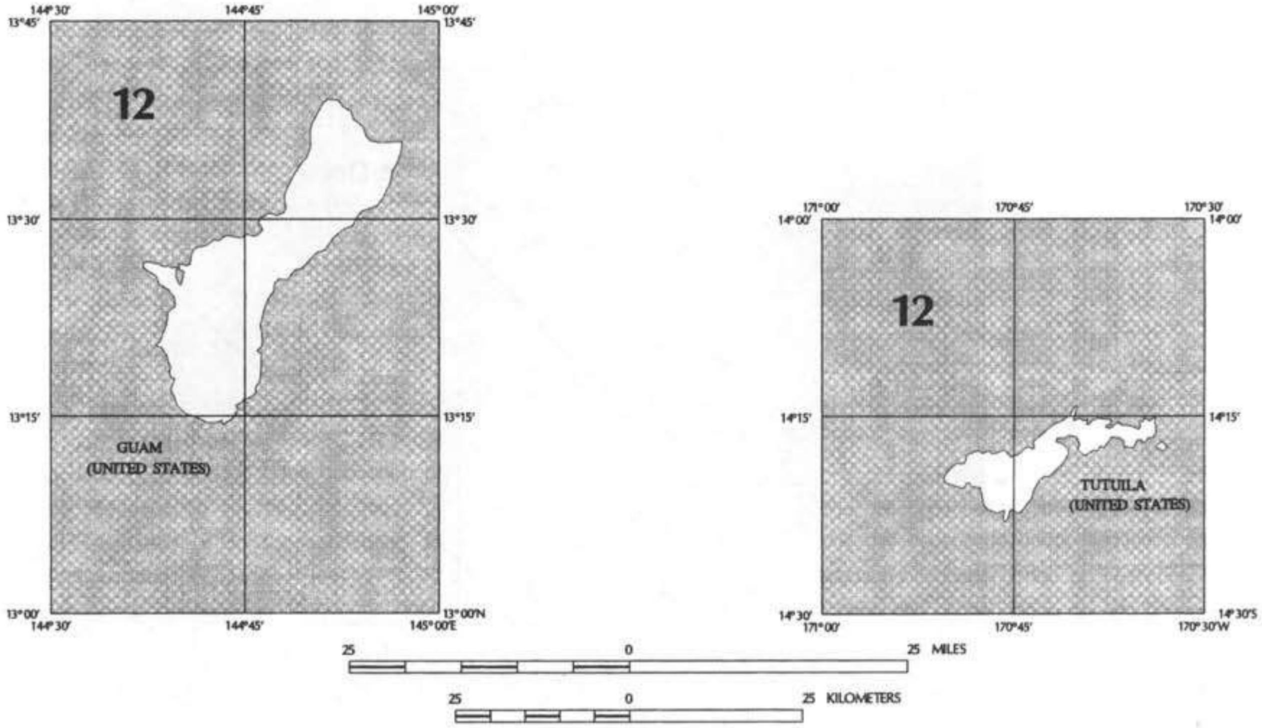


Figure 20.3g Long-period transition period, T_L(SEC), for Guam and Tutuila (courtesy of ASCE 7-05).

Table 20.5 Seismic Design Category Based on Short-Period Response Accelerations (Table 1613.5.6(1) of IBC 2006)

	Occupancy Category		
Value of S_{DS}	**I or II**	**III**	**IV**
$S_{DS} < 0.167\,g$	A	A	A
$0.167\,g \le S_{DS} < 0.33\,g$	B	B	C
$0.33\,g \le S_{DS} < 0.50\,g$	C	C	D
$0.50\,g \le S_{DS}$	D	D	D

Table 20.6 Seismic Design Category Based on 1-Second Period Response Acceleration (Table 1613.5.6(2) of IBC 2006)

	Occupancy Category		
Value of S_{D1}	**I or II**	**III**	**IV**
$S_{D1} < 0.067\,g$	A	A	A
$0.067\,g \le S_{D1} < 0.133\,g$	B	B	C
$0.133\,g \le S_{D1} < 0.20\,g$	C	C	D
$0.20\,g \le S_{D1}$	D	D	D

Where the alternate simplified design procedure of Section 20.3.3 is used, the Seismic Design Category is permitted to be determined from Table 20.5 alone, using the value of S_{DS} determined in Section 20.2.2." (Source: ASCE 7-05, Section 11.6)

20.2.5 Summary: Procedure for Calculation of Seismic Design Category (SDC)

Step 1. Determine seismic use group as described in Section 20.2.1. (Table 20.1)

Step 2. Based on the location of the building determine the mapped spectral accelerations for short periods, S_s, and the mapped spectral accelerations for a 1-second period. Use Fig. 20.1*a* and Fig. 20.1*b* of Section 20.2.2

Step 3. Use Table 20.4 to determine site class based on the soil profile name and properties of soil.

Step 4. Using Table 20.2 determine site coefficient F_a based on mapped maximum considered earthquake spectral response accelerations at short periods, S_S. Also using Table 20.3 determine site coefficient F_v based on mapped maximum considered earthquake spectral response accelerations at 1-second period, S_1.

Step 5. Calculate the maximum considered earthquake spectral response accelerations for short periods for specific soil class, S_{MS}, using Eq. 20.2a. Also calculate the maximum considered earthquake spectral response accelerations for 1-second period for specific soil class, S_{M1}, using Eq. 20.2b.

Step 6. Using Eq. 20.1a determine design spectral response acceleration coefficient for short periods, S_{DS}, and using Eq. 20.1b determine spectral response acceleration coefficient for 1-second period, S_{D1}.

Step 7. Determine SDC according to Section 20.2.4. Utilize Table 20.5 and Table 20.6.

Example 20.1

Determine seismic design category for a minor storage facility building in San Francisco on soft rock.

Solution

1. According to Table 20.1, minor storage facilities buildings are classified in occupancy category I.
2. $S_S = 2.02\,\text{g}$ (Fig. 20.1*a*)
   $S_1 = 0.60\,\text{g}$ (Fig. 20.1*b*)
3. According to the Table 20.4, a site with soft rock is considered to be class C.
4. According to the Table 20.2 for the site class C and $S_S = 2.02 > 1.25$, $F_a = 1.0$.
   According to the Table 20.3 for the site class C and $S_1 = 0.60 > 0.5$, $F_v = 1.3$.
5. $$S_{MS} = F_a S_S = (1.0)(2.02) = 2.02\text{ g} \qquad \text{(Eq. 20.2}a\text{)}$$
   $$S_{M1} = F_v S_1 = (1.3)(0.60) = 0.78\text{ g} \qquad \text{(Eq. 20.2}b\text{)}$$
6. $$S_{DS} = \frac{2}{3}\,S_{MS} = \frac{2}{3}(2.02) = 1.35\text{ g} \qquad \text{(Eq. 20.1}a\text{)}$$
   $$S_{D1} = \frac{2}{3}\,S_{M1} = \frac{2}{3}(0.78) = 0.52\text{ g} \qquad \text{(Eq. 20.1}b\text{)}$$
7. According to Table 20.5 for $S_{DS} = 1.35\,\text{g} > 0.50\,\text{g}$, occupancy category I, and since $S_1 < 0.75\,\text{g}$, therefore SDC is D.
   According to Table 20.6 for $S_{D1} = 0.52\,\text{g} > 0.20\,\text{g}$, occupancy category I, and since $S_1 < 0.75\,\text{g}$, therefore SDC is D.

Therefore, seismic design category D is assigned to the structure.

Example 20.2

Determine seismic design category for a hospital building in Oakland, California, on soft soil.

Solution

1. According to Table 20.1, hospital buildings are classified in the occupancy category IV.
2. $$S_S = 2.08\text{ g} \qquad \text{(Fig. 20.1}a\text{)}$$
   $$S_1 = 0.92\text{ g} \qquad \text{(Fig. 20.1}b\text{)}$$
3. According to Table 20.4, the site class for soft soil is E.
4. According to the Table 20.2, for the site class E and $S_S = 2.08 > 1.25$, $F_a = 0.9$.
   According to the Table 20.3, for the site class E and $S_1 = 0.92 > 0.5$, $F_v = 2.4$.
5. $$S_{MS} = F_a S_S = (0.9)(2.08) = 1.87\text{ g} \qquad \text{(Fig. 20.2}a\text{)}$$
   $$S_{M1} = F_v S_1 = (2.4)(0.92) = 2.21\text{ g} \qquad \text{(Fig. 20.2}b\text{)}$$

6.

$$S_{DS} = \frac{2}{3}S_{MS} = \frac{2}{3}(1.87) = 1.25 \text{ g} \qquad \text{(Fig. 20.1}a\text{)}$$

$$S_{D1} = \frac{2}{3}S_{M1} = \frac{2}{3}(2.21) = 1.47 \text{ g} \qquad \text{(Fig. 20.1}a\text{)}$$

7. According to Table 20.5, for $S_{DS} = 1.25\,\text{g} > 0.50\,\text{g}$, occupancy category IV, and since $S_1 > 0.75\,\text{g}$, SDC is F.
   According to Table 20.6, for $S_{D1} = 1.47\,\text{g} > 0.20\,\text{g}$, occupancy category IV, and since $S_1 > 0.75\,\text{g}$, SDC is F.

Therefore, seismic design category F is assigned to the structure.

20.3 ANALYSIS PROCEDURES

During the earthquake motions, the structure is subjected to the deformation that produces internal forces and stresses. Earthquake engineering philosophy is to relate earthquake dynamic forces to the equivalent static forces, and then using static analysis of the structure, determine deformations, internal forces, and stresses in the structure. IBC describes two analysis procedures to determine the equivalent static forces that will simulate an earthquake action on the structure. These are

1. The equivalent lateral force procedure (used for SDC B, C, D, E, and F)
2. The simplified analysis (used for SDC B, C, D, E, and F, and for constructions limited to two stories in height and three stories in height for light frame constructions)

It should be noted that for the structures in SDC A neither the simplified analysis nor the equivalent lateral force procedure can be utilized. This type of structure should be designed so that the lateral resisting-force system can resist the minimum design lateral force, F_x, applied at each floor level (ASCE 7-05, Section 11.7.2). The design lateral force can be determined for this type of structure using the following equation:

$$F_x = 0.01 w_x \tag{20.9}$$

where

w_x = the portion of the dead load of the structure located or assigned to level x.

20.3.1 Equivalent Lateral Force Procedure

This procedure describes how to calculate the seismic base shear and lateral seismic forces. (ASCE 7-05, Section 12.8)

Seismic Base Shear Calculation. The total seismic force that acts at the base of the structure, called seismic base shear, can be determined according to the following equation:

$$V = C_s W \tag{20.10}$$

where

C_s = seismic response coefficient
W = the effective weight of the structure including the total dead load and other loads listed below:

1. In areas used for storage, a minimum of 25% of the reduced floor live load (floor live load in public garages and open parking structures need not be included)
2. Where an allowance for partition load is included in the floor load design, the actual partition weight or a minimum weight of 10 psf of floor area, whichever is greater (0.48 kN/m^2)
3. Total weight of permanent operating equipment
4. 20 percent of flat roof snow load where flat snow load exceeds 30 psf (1.44 kN/m^2)

Seismic Response Coefficient Calculation. The seismic response coefficient, C_s, shall be determined from:

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_E}\right)} \tag{20.11}$$

where

S_{DS} = design spectral response acceleration parameter
R = response modification factor given in Table 20.7
I_E = occupancy importance factor determined from Table 20.1

The value of C_s should not exceed

$$C_{s\,\max} = \frac{S_{D1}}{T\left(\frac{R}{I_E}\right)} \quad \text{for } T \le T_L \tag{20.12}$$

where

S_{D1} = the design spectral response acceleration parameter at a period of 1.0 s, as determined from Section 11.4.4
T = the fundamental period of the structure (s) determined in Section 20.3.1 (Eq. 20.15 or Eq. 20.16)
T_L = lone-period transition period (s) determined in Section 20.2.3
S_1 = the mapped maximum considered earthquake spectral response accerelation parameter determined in accordance with Figure 20.1b

Also, C_S should not be less than the following:

1. For buildings and structures in seismic design categories A, B, C, and D and in buildings and structures for which 1-second spectral response acceleration, S_1 is less than 0.6 g, the value of the seismic coefficient, C_s, should not be taken less than

$$C_{s\,\min} = 0.01 \tag{20.13}$$

Table 20.7 Design Coefficients and Factors for Basic Seismic-Force-Resisting Systems (ASCE 7-05, Section 12.2.1)

Basic Seismic-Force — Resisting System	R^a	$\Omega_o{}^b$	$C_d{}^c$
1.Bearing wall systems			
Special reinforced concrete shear walls	5	2.5	5
Ordinary reinforced concrete shear walls	4	2.5	4
Detailed plain concrete shear walls	2	2.5	2
Ordinary plain concrete shear wall	1.5	2.5	1.5
2. Building frame systems			
Special reinforced concrete shear walls	6	2.5	5
Ordinary reinforced concrete shear walls	5	2.5	4.5
Detailed plain concrete shear walls	2	2.5	2
Ordinary plain concrete shear walls	1.5	2.5	1.5
3. Moment-resisting frame systems			
Special reinforced concrete moment frames	8	3	5.5
Intermediate reinforced concrete moment frames	5	3	4.5
Ordinary reinforced concrete moment frames	3	3	2.5
4. Dual systems with special moment frames			
Special reinforced concrete shear walls	7	2.5	5.5
Ordinary reinforced concrete shear walls	6	2.5	5
5. Dual systems with intermediate moment frames			
Special reinforced concrete shear wall	6.5	2.5	5
Ordinary reinforced concrete shear wall	3	3	2.5
6. Shear wall-frame intermediate system with ordinary reinforced concrete moment frames and ordinary reinforced concrete shear walls	4.5	2.5	4
7. Inverted pendulum systems			
Special reinforced concrete moment frames	2.5	1.25	2.5

[a] Response modification coefficient
[b] System overstrength factor
[c] Deflection amplification factor

2. For buildings and structures in seismic design categories E and F and in buildings and structures for which the 1-second spectral response acceleration, S_1, is equal to or greater than 0.6 g, the value of the seismic coefficient, C_s, should not be taken less than

$$C_{s\,\min} = \frac{0.5S_1}{\dfrac{R}{I_E}} \tag{20.14}$$

The response modification factor, R, is a function of several factors. Some of them are ductility capacity and inelastic performance of structural materials and systems during past earthquakes. Values of R for concrete structures are given in Table 20.7 and are selected by defining the type of basic seismic force resisting system. (Table 12.2-1 of ASCE 7-05)

Fundamental period. Elastic fundamental period, T, is a function of the mass and the stiffness of the structure. If the building is not designed, the period T cannot be precisely determined. On the other hand, to design a building, the period of vibration should be known and included in equations for design. That is why building codes provide equations for calculation of approximate periods of vibrations, T_a. Calculated approximate periods are shorter than the real periods of structure, which leads to the higher base shear and safe design.

An approximate period of vibration, T, can be determined using the following equation:

$$T_a = C_t\ h_n^x \tag{20.15}$$

where h_n is the height in ft above the base to the highest level of the structure and the coefficients C_t and x are determined from Table 20.8.

For the concrete moment-resisting frame buildings that do not exceed 12 stories in height and have a minimum story height of 10 ft, the approximate period of vibration, T, can be determined using the following equation:

$$T_a = 0.1N \tag{20.16}$$

where

N = number of stories in the building

The lateral seismic force calculation. Vertical distribution of the base shear force produces seismic lateral forces, F_x, at any floor level. Seismic lateral forces act at the floor levels because masses of the structure are concentrated at the floor levels. It is known that the force is a product of mass and acceleration. Earthquake motions produce accelerations of the structure and induce forces at the places of mass concentrations (i.e., floor levels).

The lateral force that will be applied to level x of the structure, F_x, can be determined from the following equation:

$$F_x = C_{vx} V \tag{20.17}$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k} \tag{20.18}$$

where

C_{vx} = vertical distribution factor

k = distribution exponent related to the building period

= 1 for building having a period of $T \leq 0.5$ sec

Table 20.8 Values of Approximate Period Parameters C_t and x

Structure Type	C_t	x
Moment-resisting frame systems in which the frames resist 100% of the required seismic force and are not enclosed or adjoined by components that are more rigid and will prevent the frames from deflecting where subjected to seismic forces:		
Steel moment-resisting frames	0.028	0.8
Concrete moment-resisting frames	0.016	0.9
Eccentrically braced steel frames	0.03	0.75
All other structural systems	0.02	0.75

Source: ASCE 7-05, Section 12.8.2.1

$= 2$ for building having a period of $T \geq 2.5\,\text{sec}$

$= 2$, or linear interpolation between 1 and 2, for building having a period of $0.5\,\text{sec} \leq T \leq 2.5\,\text{sec}$

h_i, h_x = height from the base to level i and x

w_i, w_x = portion of W assigned to level i or x

20.3.2 Summary: Equivalent Lateral Procedure

Step 1. Determine seismic design category according to Section 20.2 and choose an appropriate I_E value from Table 20.1.

Step 2. Choose R value from Table 20.7

Step 3. Determine T using Eq. 20.15 or Eq. 20.16, as applicable.

Step 4. Calculate C_s using Eq. 20.10 and check for $C_{s\,\max}$ (Eq. 20.11) and $C_{s\,\min}$ (Eq. 20.12 or Eq. 20.13, whichever is applicable). Ensure that $C_{s\,\min} \leq C_s \leq C_{s\,\max}$ and

$$\text{if } C_s > C_{s\,\max}, \text{ then choose } C_s = C_{s\,\max}.$$

$$\text{if } C_s < C_{s\,\min}, \text{ then choose } C_s = C_{s\,\min}.$$

Step 5. Calculate total gravity load, W, as described in Section 20.3.1.

Step 6. Calculate seismic base shear using Eq. 20.10.

Step 7. Using Eq. 20.17 calculate seismic lateral load, F_x, for every level of the structure.

20.3.3 The Simplified Analysis

The simplified analysis procedure for seismic design described in Section ASCE 7-05, Section 12.14.8.1 is applicable to any structure that satisfies the following limitations and conditions:

1. Seismic design category B, C, D, E, or F.
2. Light-framed construction not exceeding three stories in height, excluding basement, or any construction.

The seismic base shear and lateral seismic forces are calculated as follows:

"**1.** The seismic base shear, V, in a given direction shall be determined in accordance with

$$V = \frac{F S_{DS}}{R} W \tag{20.19}$$

where

$$S_{\text{DS}} = \frac{2}{3} F_a S_s$$

where F_a is permitted to be taken as 1.0 for rock sites, 1.4 for soil sites, or determined in accordance with Section 20.2.2. For the purpose of this section, sites are permitted to be considered to be rock if there is no more than 10 ft (3 m) of soil between the rock surface and the bottom of spread footing or mat foundation. In calculating S_{DS}, S_s shall be in accordance with Section 20.2.2, but need not be taken larger than 1.5.

F = 1.0 for one-story buildings

F = 1.1 for two-story buildings

F = 1.2 for three-story buildings

R = the response modification factor from Table 20.7 (ASCE 7-05, Table 12.2)

W = effective seismic weight of structure that shall include the total dead and other loads listed in the following text

1. In areas used for storage, a minimum of 25 percent of the floor live load (floor live load in public garages and open parking structures need not be included.)
2. Where provision for partitions is required by Section 4.2.2 (Provision for Partitians) in the floor load design, the actual partition weight, or minimum weight of 10 psf (0.48 kNm^2) of floor area, whichever is greater.
3. Total operating weight of permanent equipment.
4. Where the flat roof snow load, P_f, exceeds 30 psf (1.44 kN/m^2), 20 percent of the uniform design snow load, regardless of actual roof slope." (Source: ASCE 7-05, Section 12.14.8.1)

2. The lateral seismic forces calculation. The lateral seismic forces can be determined from (ASCE 7-05, Section 12.14.8.2)

$$F_x = \frac{FS_{\text{DS}}}{R} w_x \tag{20.20}$$

where

F_x = the seismic force applied at level x

w_x = the portion of the effective seismic weight of the structure, W, at level x.

20.3.4 Summary: Simplified Analysis Procedure

Step 1. Check whether the structure satisfies the three conditions described in Section 20.3.6 for qualification for the simplified analysis procedure.

Step 2. Determine the value of S_{DS} as described in Section 20.2.2.

Step 3. Choose appropriate R factor from Table 20.7.

Step 4. Determine the total gravity load, W, of the structure as described in Section 20.3.1.

Step 5. Utilize Eq. 20.19 to calculate seismic base shear, V.

Step 6. Determine the seismic lateral forces acting on the structure, F_x, using Eq. 20.20.

20.3.5 Design Story Shear

The seismic lateral forces will produce seismic design story shear, V_x, at any story x that can be determined from the following equation:

$$V_x = \sum_{i=1}^{n} F_i \tag{20.21}$$

where

F_i = the portion of seismic base shear, V, assigned to level i

n = number of stories

The seismic story shear in any story x should be collected and transferred to the story below by vertical elements of lateral-force-resisting system (walls). The distribution of story shear on vertical elements depends on flexibility of the diaphragm, which those elements (walls) support.

There are two types of diaphragm:

1. Flexible diaphragm
2. Rigid diaphragm

Diaphragm is flexible when the lateral deformation of diaphragm is more than two times the average story drift of the story that supports diaphragm. Lateral deformation of diaphragm is maximum in-plane deflection of the diaphragm under lateral load, and the story drift is the difference between the deflections of the center of mass at the top and the bottom of the story being considered.

A diaphragm that is not flexible by the above definition is rigid.

For flexible diaphragms, the seismic story shear, V_x, is distributed to vertical elements in the story x based on the area of the diaphragm tributary to each line of resistance. The vertical elements of the seismic-force–resisting system may be considered to be in the same line of resistance if the maximum out-of-plane offset between such elements is less than 5% of the building dimension perpendicular to the direction of the lateral force.

For rigid diaphragms, V_x is distributed to the vertical elements in the story x based on the relative lateral stiffness of the vertical resisting elements and the diaphragm.

20.3.6 Torsional Effect

For rigid diaphragms the eccentricity between center of mass and center of rigidity can occur. The lateral shear force is applied to the center of mass. Distribution of V_x to the vertical elements can be determined when the shear force acts to the center of rigidity. When the shear force moves from center of mass to the center of rigidity it produces torsional moment. Effect of torsion will increase horizontal forces on vertical elements. Forces are not to be decreased due to torsional effects.

$$T = V_x e \tag{20.22}$$

where

V_x = base shear at level x in any direction

e = eccentricity between center of mass and center of rigidity. It can occur in both directions x and y.

20.3.7 Overturning Moment

The lateral seismic force F_x produces overturning moments. Overturning moment M_x should be calculated using the following equation:

$$M_x = \tau \sum_{i=1}^{n} F_i (h_i - h_x) \tag{20.23}$$

where

F_i = portion of the seismic base shear, V, induced at level i

h_i, h_x = height from the base to level i and x

τ = overturning moment reduction factor

= 1.0 for the top 10 stories

= 0.8 for the twentieth story from the top and below

= linear interpolation between 1.0 and 0.8 for stories between the twentieth and tenth stories below the top

τ is permitted to be taken as 1.0 for the full height of the structure.

20.3.8 Lateral Deformation of the Structure

The seismic lateral forces should be used in calculating deformations of the structure. The value that is of interest for engineers is story drift—the difference between the deflections of the center of mass at the top and the bottom of the story being considered. The value of story drift under seismic forces is important from different perspectives: stability of the structure, potential damage to nonstructural elements, and human comfort. The allowable values for story drift are shown in Table 20.9 (Table 12.12-1 of ASCE 7–05).

For structures that can be designed based on the simplified analysis procedure described in Section 20.3.3 the drift can be taken as 1% of the story height unless a more exact analysis is provided.

$$\Delta = 0.01h_x \tag{20.24}$$

The value of the design story drift should be less than or equal to the value of allowable story drift, Δ_a, given in Table 20.9.

For all other structures that cannot be analyzed using the simplified analysis procedure, the drift should be determined as follows:

1. Calculate the deflection δ_x at level x from the following equation:

$$\delta_x = \frac{C_d \delta_{xe}}{I_E} \tag{20.25}$$

where

δ_{xe} = the elastic lateral deflection at floor level x under seismic lateral forces
C_d = deflection amplification factor from Table 20.7
I_E = occupancy importance factor from Table 20.1

2. The design story drift can then be calculated as the difference between the deflections of the centers of mass of any two adjacent stories. Definition of story drift is shown in Fig. 20.3.

$$\Delta = \delta_x - \delta_{x-1} \tag{20.26}$$

3. Check for the P-delta effect and adjust for magnification factor if needed.

Table 20.9 Allowable Story Drift, Δ_a (in.)[a]

Building	**Occupancy Category**		
	I or II	**III**	**IV**
Buildings, other than masonry shear wall or masonry wall frame buildings four stories or less in height with interior walls, partitions, ceilings, and exterior wall system that have been designed to accommodate the story drift	$0.025h_{sx}$[b]	$0.020h_{sx}$	$0.015h_{sx}$
Masonry cantilever shear wall buildings[c]	$0.010h_{sx}$	$0.010h_{sx}$	$0.010h_{sx}$
Other masonry shear wall buildings	$0.007h_{sx}$	$0.007h_{sx}$	$0.007h_{sx}$
All other buildings	$0.020h_{sx}$	$0.015h_{sx}$	$0.010h_{sx}$

[a]There shall be no drift limit for single-story building with interior walls, partitions, ceilings, and exterior wall systems that have been designed to accommodate the story drift.
[b]h_{sx} is the story height below level x.
[c]Building in which the basic structural system consist of masonry shear walls designed as vertical elements cantilevered from their base or foundation support that are so constructed that moment transfer between shear walls (coupling) is negligible.

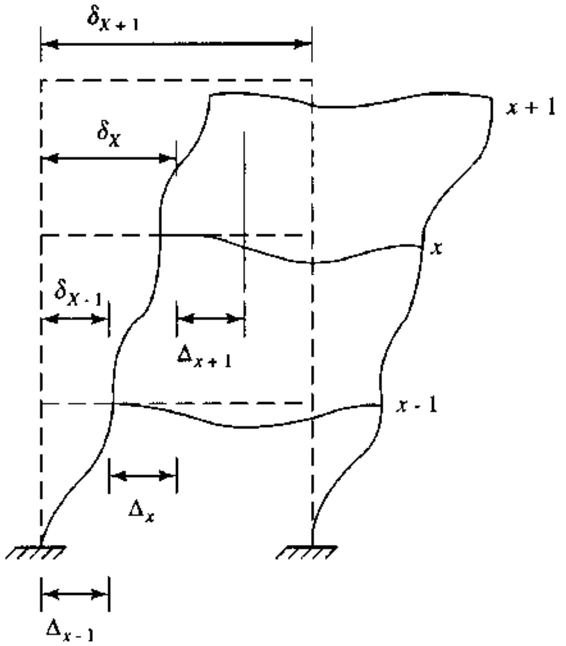


Figure 20.4 Definition of drift.

P-Delta Effect. An accurate estimate of story drift can be obtained by the P-delta analysis. In first order structural analysis the equilibrium equations are formulated for the undeformed shape of structure. When deformations are significant the second-order analysis must be applied and the P-delta effect must be considered in determining the overall stability of the structure. The P-delta effect does not need to be applied when the ratio of secondary to primary moment, θ, does not exceed 0.1. This ratio is given by the following equation:

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d} \tag{20.27}$$

where

θ = stability coefficient
P_x = total unfactored vertical design load at and above level x (dead, floor live and snow load)
Δ = design story drift (in.)
V_x = seismic shear force between level x and x-1
h_{sx} = story height below level x (ft)
C_d = deflection amplification factor in Table 20.7 (Table 12.2-1 of ASCE 7–05)

The stability coefficient, θ, should not exceed

$$\theta_{\max} = \frac{0.5}{C_d \beta} \tag{20.28}$$

where

β = ratio of shear demand to shear capacity for the story between level x and $x - 1$.break A value $\beta = 1$ can be used where the ratio is not calculated.

If $\theta > \theta_{max}$, then the structure is potentially unstable and must be redesigned. For $0.1 < \theta < \theta_{max}$, the interstory drift and element forces need to be computed using the P-delta effect. The design story drift considering P-delta effect, Δ_p, can be calculated from

$$\Delta_p = \Delta \frac{1}{(1-\theta)} \tag{20.29}$$

The computed values of story drift should not exceed the allowable values described in Table 20.9.

20.3.9 Summary: Lateral Deformation of the Structure

Step 1. If the structure satisfies the limitations for the simplified analysis procedure listed in Section 20.3.3, use Eq. 20.24 to determine the story drift.

Step 2. For structures that do not satisfy the limitations for the simplified analysis procedure listed in Section 20.3.3, use Eqs. 20.24, 20.25, 20.26,and 20.27 to calculate δ_x, δ_{x-1}, Δ, θ, and θ_{max}. Check whether the P-delta effect must be considered and adjust Δ to Δ_p using Eq. 20.29.

Step 3. Determine allowable drift from Table 20.9 and compare with the calculated design drift. If calculated drift exceeds the allowable drift, redesign the structure.

Example 20.3: Equivalent Lateral Procedure

Determine the design seismic force and seismic shear for a six-story concrete special moment-resisting frame building located in the area of high seismic risk where $S_s = 1.5\,g$ and $S_1 = 0.6\,g$, on the soil class B. The story heights are all 12 ft, and the story weights are all 1700 k. Check the lateral deformation of the structure. Building elevation is given in Fig. 20.5.

Solution

1.

$$I_E = 1.25 \qquad \text{(Table 20.1)}$$

$$S_s = 1.5\text{ g}, \qquad S_1 = 0.6\text{ g} \qquad \text{(Fig. 20.1}a\text{, 20.1}b\text{)}$$

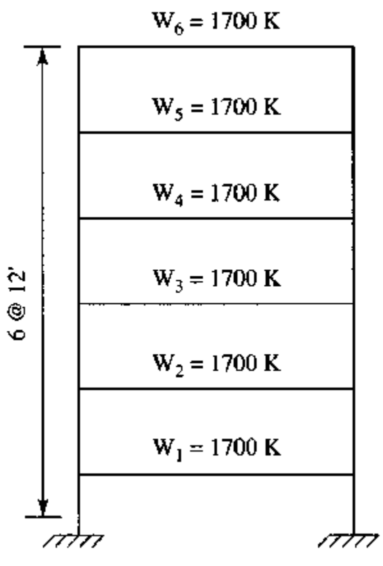


Figure 20.5 Example 20.3 building elevation.

Soil class **B**:

$$Fa = 1.0, \ Fv = 1.0 \qquad \text{(Table 20.3}a\text{, 20.3}b\text{)}$$

$$S_{MS} = 1.5 \text{ g}, S_{M1} = 0.6 \text{ g} \qquad \text{(Eq. 20.2}a\text{, 20.2}b\text{)}$$

$$S_{DS} = 1.0 \text{ g} \qquad \text{(Eq. 20.1}a\text{)}$$

$$S_{D1} = 0.4 \text{ g} \qquad \text{(Eq. 20.1}b\text{)}$$

SDC is D.

2. According to Table 20.7 for special moment-resisting frame, select $R = 8$.
3. Equation 20.18 is not applicable since $h_x > 10$ ft. Period of vibration of structure is calculated according to Eq. 20.14 as follows:

$$T_a = C_T h^{3/4} = 0.030 \times (6 \times 12)^{3/4} = 0.74 \text{ s}$$

4. Calculate seismic response coefficient as follows and check for the limits:

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_E}\right)} = \frac{1}{\left(\frac{8}{1.25}\right)} = 0.156$$

$$C_{s\,\max} = \frac{S_{D1}}{\left(\frac{R}{I_E}\right)T} = \frac{0.4}{\left(\frac{8}{1.25}\right)0.74} = 0.084$$

Since $S_1 = 0.6$ g, Eq. 20.14 should be used to calculate $C_{s\,\min}$:

$$C_{s\,\min} = \frac{0.5S_1}{\frac{R}{I_E}} = \frac{0.5 \times (0.6)}{\frac{8}{1.25}} = 0.047$$

Since $C_s > C_{s\,\max}$, $C_s = 0.084$.

5. The total gravity load is calculated as follows:

$$W = w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 6 \times (1700) = 10{,}200 \text{ kips}$$

6. Calculate the seismic base using Eq. 20.10:

$$V = C_s W = 0.084 \times 10200 = 857 \text{ kips}$$

7. Calculation of F_x, V_x, and M_x (Fig. 20.6). Use Eq. 20.17 to calculate the seismic lateral force, F_x, as shown in the following table. The table also calculates the shear force for each floor level and the overturning moments as described in Eqs. 20.21 and 20.23.

Floor Level	Weight W_i (kips)	Height h_i (ft)	$W_i h_i^{k}$* (kips-ft)	C_{vx}	Lateral Force, F_x (kips)	Shear Force, V_x (kips)	Overturning Moment M_x (kips-ft)
6	1700	72	204,485	0.30	257	257	0
5	1700	60	166,716	0.24	206	463	3084
4	1700	48	129,849	0.19	163	626	8640
3	1700	36	94,082	0.14	120	746	16,152
2	1700	24	59,743	0.09	77	823	25,104
1	1700	12	27,487	0.04	34	857	34,980
0			682,362			857	45,264

*To calculate k, use Section 20.3.1. For $T = 0.74$ s, using linear interpolation, $k = 1.12$

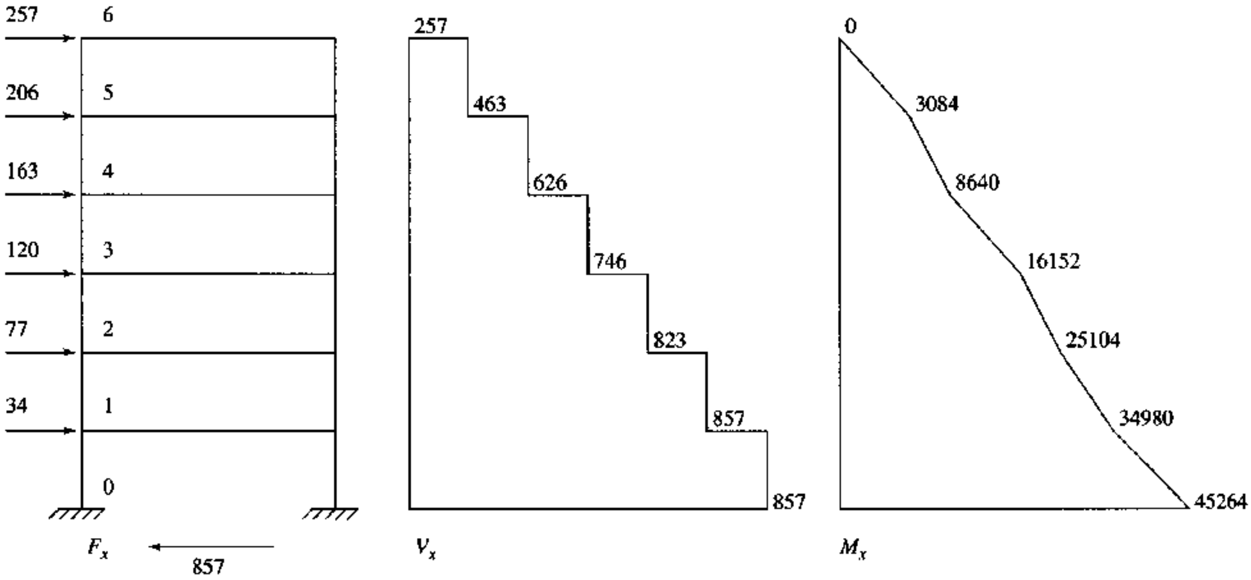


Figure 20.6 Example 20.3: distribution of lateral seismic force, F_x, base shear, V_x, and overturning moment, M_x.

8. Calculation of drift. According to Table 20.7 for special moment resisting frame $C_d = 5.5$,

$$I_E = 1.25 \quad \text{(Table 20.1)}$$

$$h_{sx} = 12 \text{ ft} = 144 \text{ in.}$$

Floor Level	δ_{xe} (in.)	δ_x (in.)	Δ (in.)	P_x (kip)	V_x (kip)	θ
6	1.26	5.54	1.23	1882	257	0.011
5	0.98	4.31	1.19	3943	463	0.013
4	0.71	3.12	1.1	6004	626	0.013
3	0.46	2.02	0.96	8065	746	0.013
2	0.24	1.06	0.8	10,126	823	0.012
1	0.06	0.26	0.26	12,187	857	0.005

$$\theta_{max} = \frac{0.5}{C_d\beta} = \frac{0.5}{(5.5) \times (1.0)} = 0.09 > \theta \text{ in every floor level}$$

Which is o.k. (Eq. 20.28) Also, $\theta < 0.1$ in every floor level, which means that the P-delta effect can be disregarded.

9. Allowable drift, according to the Table 20.9, is $\Delta_a = 0.010\, h_{sx} = 0.010 \times (12 \times 12) = 1.44$ in $> \Delta$ in every floor level, which is o.k.

Example 20.4: Simplified Analysis

Calculate the seismic base shear for a two-story concrete building assuming that the first floor weight is $w_x = 35$ kip and the second floor weight is 40 kip. The height of the first floor is $h_x = 15$ ft, and of the second floor is 12 ft. Seismic-force– resisting system is ordinary reinforced shear wall system. Utilize the value of S_{DS} from in Example 20.1. Check the lateral deformation of the structure.

Solution

1. The building is classified SDC D (from Example 20.1), and is two stories in height. This building satisfies the conditions for simplified analysis.
2. $S_{DS} = 1.35\,g$ (Example 20.1).
3. The R factor is chosen from Table 20.7 based on the seismic-force–resisting system of the structure. For ordinary reinforced concrete shear wall, R is equal to 4.
4. Calculate the total gravity load (Fig. 20.7): $W = w_1 + w_2 = 35 + 40 = 75\,\text{kip}$
5. For two story building F = 1.1 as desribed in 20.3.3

$$V = \frac{1.1S_{DS}}{R}W = \frac{1.1(1.35)}{4} \times 75 = 27.8\ \text{kip} \qquad \text{(Eq. 20.19)}$$

6. Calculate the seismic lateral forces acting at the first and second floors using Eq. 20.20, (Fig. 20.8).

$$F_1 = \frac{1.1S_{DS}}{R}w_1 = \frac{1.1(1.35)}{4} \times 35 = 13\ \text{kip} \qquad \text{(first floor)}$$

$$F_2 = \frac{1.1S_{DS}}{R}w_2 = \frac{1.1(1.35)}{4} \times 40 = 14.9\ \text{kip} \qquad \text{(second floor)}$$

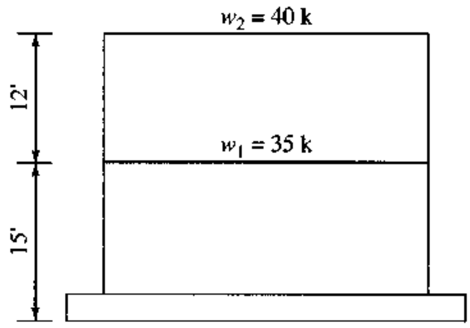


Figure 20.7 Example 20.4 building elevation.

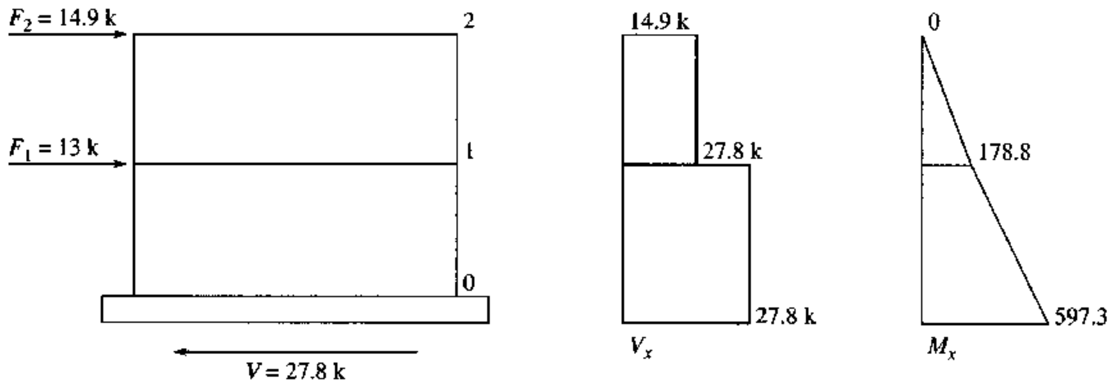


Figure 20.8 Example 20.4: distribution of lateral seismic force, F_x, base shear, V_x, and overturning moment, M_x.

7. Calculate the story shear force using Eq. 20.20:

$$V_2 = 13 \text{ kip} \qquad \text{(second floor)}$$

$$V_1 = 27.8 \text{ kip} \qquad \text{(first floor)}$$

8. Calculate the overturning moment using Eq. 20.23:

$$M_2 = 0 \qquad \text{(second floor)}$$

$$M_1 = 14.9 \times 12 = 178.8 \text{ kip-ft} \qquad \text{(first floor)}$$

$$M_0 = 14.9(12 + 15) + 13 \times 15 = 597.3 \text{ kip-ft} \qquad \text{(at the base of the structure)}$$

9. Determine the seismic lateral story drift using Eq. 20.24:

$$\Delta_1 = 0.01h_1 = 0.01 \times 15 = 0.15 \text{ feet} = 0.0125 \text{ in.} \qquad \text{(first floor)}$$

$$\Delta_2 = 0.01h_2 = 0.01 \times 12 = 0.12 \text{ feet} = 0.01 \text{ in.} \qquad \text{(second floor)}$$

Check for allowable drift using Table 20.9.

$\Delta_a = 0.020h_{sx}$, where h_{sx} is the story height below level x.

$$\Delta_{a_1} = 0.020h_1 = 0.020 \times 15 = 0.3 \text{ ft} = 0.025 \text{ in.} > 0.0125 \text{ in.} \quad \text{(o.k.)} \quad \text{(first floor)}$$

$$\Delta_{a_2} = 0.020h_2 = 0.020 \times 12 = 0.24 \text{ ft} = 0.02 \text{ in.} > 0.01 \text{ in.} \quad \text{(o.k.)} \quad \text{(second floor)}$$

Example 20.5: Torsional Effect

Determine the shear forces V_1 and V_2 acting on the shear wall 1 and 2 of the building with floor plan shown in Fig. 20.9. Assume that the value of story shear, V_x, is 15 kip. Consider torsional effect.

Solution

Center of mass is in the centroid of the rigid diaphragm. The center of rigidity can be determined as follows (Fig. 20.10):

$$x = (25 \times 30 \times 2 + 10 \times 120 \times 2)/(2 \times 25 + 2 \times 10) = 55.7 \text{ ft}$$

$$e_x = 150/2 - 55.7 = 19.3 \text{ ft}$$

For the story shear force $V_x = 15$ kip and excentricity of 19.3 ft, the torsional moment is

$$T = 15 \times 19.3 = 289.5 \text{ kip-ft} \qquad \text{(Eq. 20.22)}$$

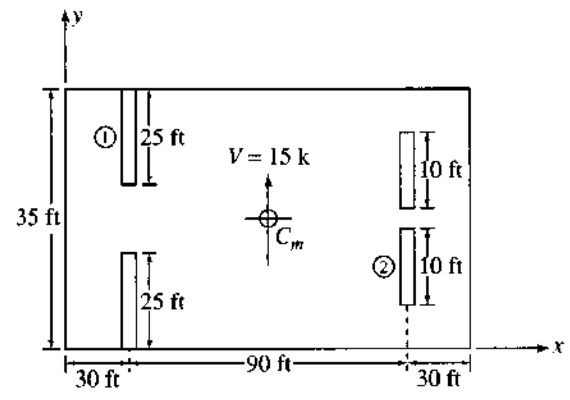


Figure 20.9 Example 20.5: floor plan.

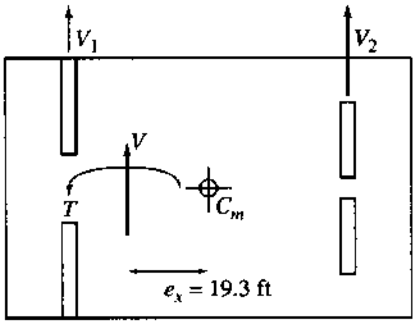


Figure 20.10 Example 20.5: torsional effect.

The shear force acting on the wall is the sum of the shear force due to story shear, V_x, and shear force due to torsional moment, T_x.

For wall 1, shear force V_1 is

$$V_1 = 15 \times 25/(25 + 25 + 10 + 10) = 5.4 \text{ kip} \quad (\text{due to } V_x)$$

$$V_1 = 289.5\ (25 \times 25.7)/(2 \times 25 \times 25.7^2 + 2 \times 10 \times 64.3^2) = 1.6 \text{ kip-ft} \quad (\text{due to } T)$$

Therefore, $V_1 = 5.4 + 1.6 = 7$ kip. For wall 2, shear force V_2 is

$$V_2 = 15 \times 10/(25 + 25 + 10 + 10) = 2.1 \text{ kip} \quad (\text{due to } V_x)$$

$$V_2 = 289.5\ (10 \times 64.3)/(2 \times 25 \times 25.7^2 + 2 \times 10 \times 64.3^2) = 1.6 \text{ kip-ft} \quad (\text{due to } T)$$

Therefore, $V_2 = 2.1 + 1.6 = 3.7$ kip.

20.4 LOAD COMBINATIONS

A structure should be designed to resist the combined effects of the loadings. Basic load combinations for strength design are given (IBC 2006, Section 1605.2.1):

1. $4D$
2. $1.2\ D + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (f_1 L \text{ or } 0.8\ W)$
4. $1.2D + 1.6W + f_1 L + 0.5(L_r \text{ or } S \text{ or } R)$
5. $1.2D + 1.0E + f_1 L + f_2 S$
6. $0.9D + 1.6W + 1.6H$
7. $0.9D + (1.0E \text{ or } 1.6W)$

where

$f_1 = 1.0$ for the floors in places of public assembly, for live loads in excess of 100 pounds per square foot, and for parking garage live load

$f_1 = 0.5$ for other live loads

$f_2 = 0.7$ for roof configurations that do not shed snow off the structure

$f_2 = 0.2$ for the other roof configurations

D = dead load
L = live load excluding roof live load
L_r = roof live load
S = snow load
R = rain load
W = wind load
E = seismic load effect

Special seismic load combinations for strength design should be used in a case of structures having certain plan or vertical irregularities in SDC B or higher. Special seismic load combinations for strength design are (IBC 2006, Section 1605.4):

1. $1.2D + f_1 L + E_m$
2. $0.9D + E_m$

where

f_1 = 1.0 for floors in places of public assembly, for live loads in excess of 100 psf, and for parking garage live load
f_1 = 0.5 for other live loads
E_m = the maximum effect of horizontal and vertical forces

20.4.1 Calculation of Seismic Load Effect, *E*

The seismic load effect (ASCE 7-05, Section 12.4) can be determined from the following two conditions:

1. The seismic load effect E is calculated from

$$E = \rho Q_E + 0.2 S_{DS} D \tag{20.30}$$

where

Q_E = effect of horizontal seismic forces
ρ = redundancy coefficient
S_{DS} = the design spectral response acceleration at short periods determined in Section 20.2.2
D = effect of dead load

2. When the effect of gravity and seismic ground motions are counteractive, the seismic load effect is calculated from

$$E = \rho Q_E - 0.2 S_{DS} D \tag{20.31}$$

20.4.2 Redundancy Coefficient, ρ

Redundancy coefficient can be determined as follows (ASCE 7-05, Section 12.3.4)

1. For structures assigned to seismic design category A, B, or C, the value of the redundancy coefficient, ρ, is 1.
2. For structures assigned to seismic design category D, E, or F, the redundancy coefficient ρ, shall be taken equal to 1.3.

20.4.3 Seismic Force Effect, E_m

When the effects of gravity and seismic forces are additive, the seismic force effect, E_m, should be calculated using the following equation:

$$E_m = \Omega_0 Q_E + 0.2 S_{DS} D \tag{20.32}$$

where

Ω_0 = the system overstrength factor given in Table 20.7

When the effects of gravity and seismic forces counteract the seismic force effect, E_m, should be calculated using the following equation:

$$E_m = \Omega_0 Q_E - 0.2 S_{DS} D \tag{20.33}$$

20.5 SPECIAL REQUIREMENTS IN DESIGN OF STRUCTURES SUBJECTED TO THE EARTHQUAKE LOADS

The ACI Code (2008), Section 20.1.1.9.1 and 21.1, define five seismic design categories (SDCs) for earthquake-resistant structures. These are A, B, C, D, E, and F. The classification of these zones described in ACI Section R21.1.1 can be given in three different categories:

1. SDC D, E, and F indicate high seismic risk zones with strong ground shaking
2. SDC C indicates moderate/intermediate seismic risk zones with moderately strong ground shaking.
3. SDC A and B indicate low seismic risk zones with SDC A corresponding to the lowest seismic hazard zone.

For structures in high seismic risk (SDC D, E, and F) special requirements in flexural design and detailing are required. Special moment frames (Section 20.5.1) and special structural walls (Section 20.5.1.0) should be used as the structural system of a building.

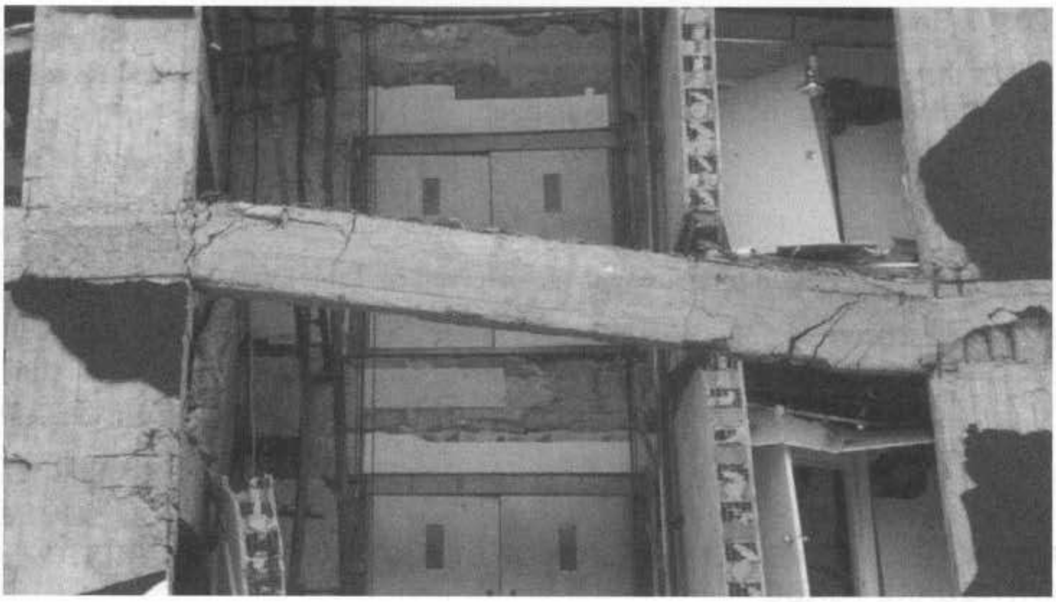
For the structures in moderate seismic risk (SDC C) some special provisions are required for satisfactory intermediate seismic performance (Section 20.5.2). Structure can be designed as intermediate moment-frame or intermediate structural-walls systems. Structures from a higher category can also be utilized.

For the structures in low seismic risk (SDC A and B), no special requirements in flexural design and detailing are required. Ordinary moment frames and ordinary structural walls and systems should be utilized as the structural system of a building.

20.5.1 Structures in the High Seismic Risk: Special Moment Frames (ACI 2008, Section 21.5)

A special moment frame is a structural system that is designed and detailed to sustain strong earthquakes. Special provisions for designing and detailing are given for

1. Flexural members of special moment frames such as members subjected to only bending
2. Special moment frame members subjected to bending and axial load such as columns
3. Joints of special moment frames



Strong column-weak beam connection.

Flexural members of special moment frame (Section 20.5.1.1).

General requirements (Section 20.5.1.1.1). If factored axial compressive force $P_u < A_g f_c'/10$, then the member is considered to be subjected to bending. A_g represents the gross area of the concrete member. Flexural member should satisfy following the conditions (ACI 2008, Section 21.5.1):

1. Clear span $l_n \geq 4 \times$ effective depth (d).
2. The flexural member width-to-depth ratio, $b_w/d \geq 0.3$.
3. Flexural member width (b_w) $\geq$ 10 in.
4. Flexural member width (b_w) < width of supporting member (column), $b_s + (1.5 \times$ depth of the flexural member, h).

Longitudinal Reinforcement Requirements (Section 20.5.1.1.2). According to the ACI Code (2008), Section 21.5.2, the longitudinal reinforcement at any section should satisfy the following (Fig. 20.11):

1. Longitudinal reinforcement for both top and bottom steel (A_s) should be in the range defined as follows:

$$\left.\begin{matrix} \dfrac{3\sqrt{f_c'}bd}{f_y} \\ \dfrac{200bd}{f_y} \end{matrix}\right\} \leq (A_s) \leq 0.025bd \tag{20.34}$$

At least two bars should be provided continuously at both top and bottom. For the statically determined T-sections with flanges in tension the value of b in the expression $3\sqrt{f_c'}bd/f_y$ should be replaced by either $2b$ (width of web) or the width of the flange, whichever is smaller (ACI 2008, Section 10.5.2).

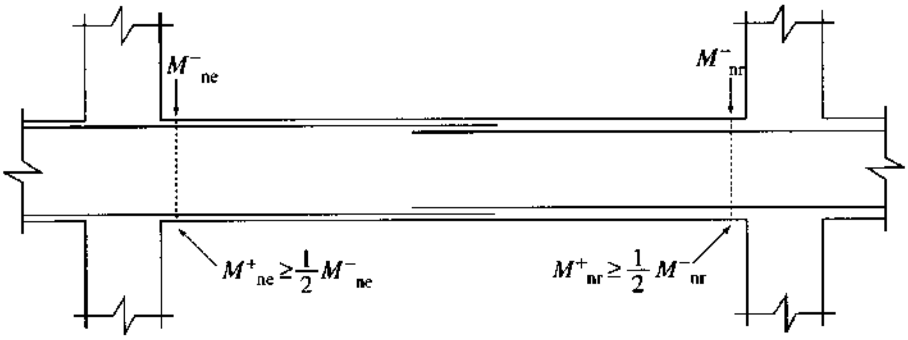


Figure 20.11 Longitudinal reinforcement requirements.

2. The positive moment strength at joint face should be greater or equal $\frac{1}{2}$ negative moment strength at that face of the joint (ACI Section 21.5.2.2):

$$\phi M^{+}_{n_l} \geq \frac{1}{2}\phi M^{-}_{n_l} \qquad \text{(left joint)} \tag{20.35a}$$

$$\phi M^{+}_{n_r} \geq \frac{1}{2}\phi M^{-}_{n_r} \qquad \text{(right joint)} \tag{20.35b}$$

where

M_{n_l} = moment strength at left joint of flexural member
M_{n_r} = moment strength at right joint of flexural member

3. Neither the negative nor positive moment strength at any section along the member should be less than $\frac{1}{4}$ the maximum moment strength provided at the face of either joint.

$$(\phi M^{+}_{n} \text{ or } \phi M^{-}_{n}) \geq \frac{1}{4}(\max \phi M_n \text{ at either joint}) \tag{20.36}$$

4. Anchorage of flexural reinforcement in support can be calculated using the following equation:

$$l_{dh} \geq \begin{cases} \dfrac{f_y d_b}{65\sqrt{f'_c}} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{20.37}$$

where d_b is the diameter of longitudinal reinforcement.

5. Lap splices of flexural reinforcement are permitted only if hoop or spiral reinforcement is provided over the lap length. Hoop or spiral reinforcement spacing should not exceed $d/4$ or 4 in., whichever is smaller. Lap splices should not be used within a joint, within a distance of twice the member depth from the face of the joint, or at locations of plastic hinges.

Transverse Reinforcement Requirements (Section 20.5.1.1.3) For the special moment-resisting frame, plastic hinges will form at the ends of flexural members. Those locations should be specially detailed to ensure sufficient ductility of the frame members. Transverse reinforcement

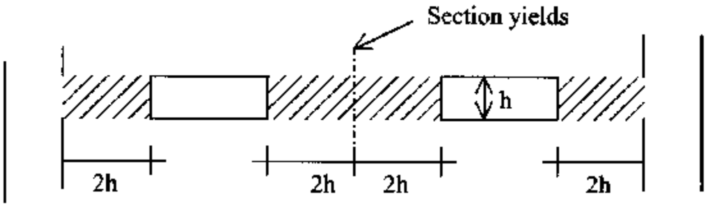


Figure 20.12 Areas of the flexural member where hoops are required. (Note: These areas do not necessarily occur at midspan.)

gives lateral support for the longitudinal reinforcement and assists concrete to resist shear. It should satisfy the following: (ACI 2008, Section 2.1.5.3)

1. Hoops are required over a length equal to twice the member depth from the face of the support at both ends of flexural member. Also, hoops are required over lengths equal to twice the member depth on both sides of section where flexural yielding may occur, as shown in Fig. 20.12.
2. The spacing of the hoops, s, should not exceed the smallest of the following values:
   a. $d/4$
   b. Eight times the diameter of the smallest longitudinal bar
   c. 24 times the diameter of the hoop bars
   d. 12 in.

   The first hoop should be located not more than 2 in. from the face of the support.
3. Where hoops are not required, stirrups with seismic hooks at both ends should be used. Spacing between stirrups should be less than or equal to $d/2$.
4. Transverse reinforcement should be designed to resist the design shear force (Figs. 20.13 and 20.14). Design shear force for flexural members of special moment frames can be determined using the following equation (Fig. 20.15):

$$V_l = \frac{M_{pr}^{-} + M_{pr}^{+}}{l_n} + \frac{w_u l_n}{2} \tag{20.38a}$$

$$V_r = \frac{M_{pr}^{+} + M_{pr}^{-}}{l_n} - \frac{w_u l_n}{2} \tag{20.38b}$$

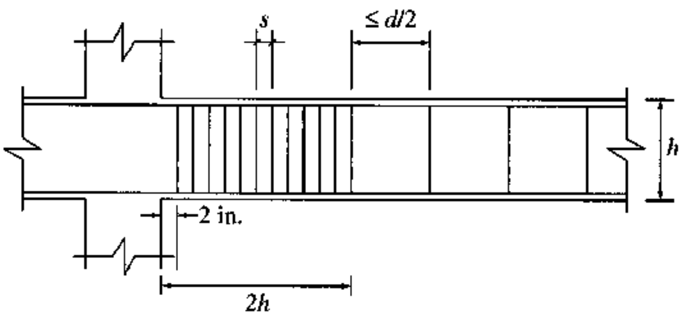


Figure 20.13 Transverse reinforcement requirements.

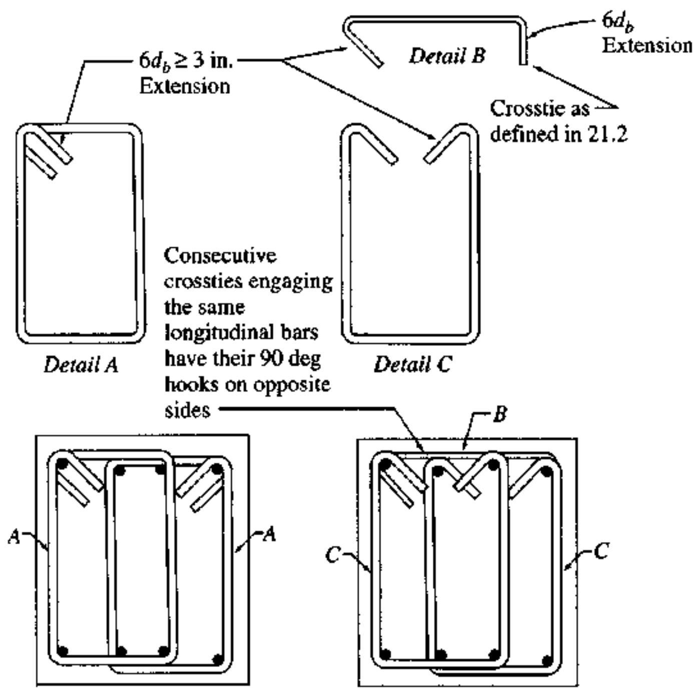


Figure 20.14 Transverse reinforcement requirements. Courtesy of American Concrete Institute (ACI 2008).

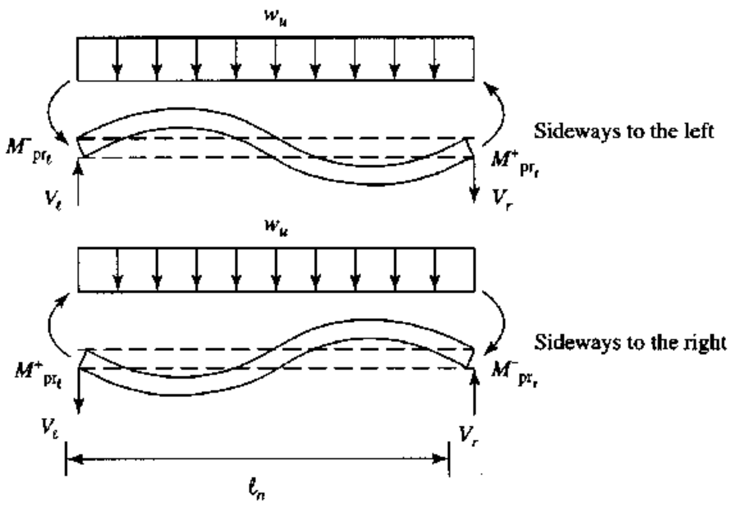


Figure 20.15 Design shear force.

where

V_l = design shear force at left joint of flexural member

V_r = design shear force at right joint of flexural member

M_{pr} = probable moment strength at the end of the beam determined as strength of the beam with the stress in the reinforcing steel equal to 1.25 f_y and a strength reduction factor of $\phi = 1.0$.

l_n = clear span of flexural member

w_u = factored distributed load determined by Eq. 20.47

$$w_u = 1.2D + 1.0L + 0.2S \tag{20.39}$$

where

D = dead load

L = live load

S = snow load

Probable moment strength at the end of the beam, M_{pr}, can be calculated from the following equation:

$$M_{\text{pr}} = A_s(1.25f_y)\left(d - \frac{a}{2}\right) \tag{20.40}$$

where

$$a = \frac{A_s(1.25f_y)}{0.85f_c'b} \tag{20.41}$$

The shear strength of concrete can be taken to be 0 when the earthquake-induced shear force is greater than or equal to 50% of the total shear force and the factored axial compressive force is less than $A_g f_c'/20$, where A_g is the gross area of the beam.

Summary: Design of the Special Moment-Resisting Frame Members Subjected to Bending (Section 20.5.1.1.4)

Step 1. Determine the seismic design category, base shear, lateral seismic force, and seismic shear according to Sections 20.2 and 20.3.

Step 2. Calculate the member forces, and use the different load combinations to determine the values of member forces that govern the design (Section 20.4). Design for flexural reinforcement.

Step 3. Check whether the frame member is a flexural member and check the general requirements for the special moment frame member according to Section 20.5.1.1.1.

Step 4. Check the special requirements for the longitudinal reinforcement according to Section 20.5.1.1.2.

Step 5. Design the transverse reinforcement for confinement and shear resistant using Section 20.5.1.1.3.

Example 20.6

Design a beam AB on the second floor of a building, as shown on Fig. 20.16. The building is constructed in the region of high seismic risk on soil class B. Additional information:

Material properties: Concrete: $f_c' = 4000$ psi, $w_c = 150$ pcf
Steel $f_y = 60{,}000$ psi

Loads: Live loads = 40 psf
Superimposed dead load = 35 psf

Member dimensions: Beams = 20 × 24 in.
Columns = 24 × 24 in.
Slab thickness = 7 in.

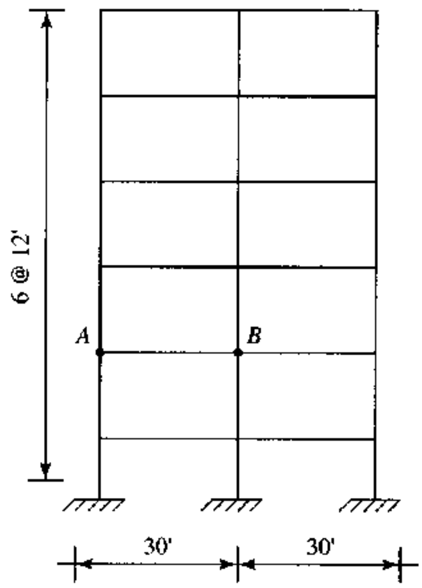


Figure 20.16 Example 20.6:1 building elevation.

Solution

1. Seismic design category, base shear, lateral seismic force, and seismic shear are determined in Example 20.3.
2. Load combinations are given as follows:

$$1.4D \quad \text{(I)}$$

$$1.2D + 1.6L \quad \text{(II)}$$

$$1.2D + 1.0E + f_1 L \quad f_1 = 0.5 \text{ according to Section 20.4} \quad \text{(III)}$$

$$0.9D + 1.0E \quad \text{(IV)}$$

Redundancy coefficient, ρ, can not be taken less than 1. For seismic design catagory D to F use $f = 1.3$.

Seismic load effect, E, can be determined using Eqs. 20.30 and 20.31:

$$E = \rho Q_E + 0.2S_{\mathrm{DS}}D = Q_E + 0.2(1.0)D = Q_E + 0.2D$$

$$E = \rho Q_E - 0.2S_{\mathrm{DS}}D = Q_E - 0.2(1.0)D = Q_E - 0.2D$$

Replacing the E in Eq. III gives

$$1.4D + 0.5L + Q_E$$

$$D + 0.5L + Q_E$$

Replacing the E in Eq. IV gives

$$1.1D + Q_E$$

$$0.7D + Q_E$$

The member forces for the beam AB on the second floor (Fig. 20.16) are calculated using the software for load analysis, and the values of required flexural strengths are determined using different load combinations, as shown in Table 20.10.

Table 20.10 Calculated Member Forces

Load Cases	Location	Bending Moment (kip-ft)	Shear (kip)
D	Support	−95	24
	Midspan	−465	
L	Support	−22	11
	Midspan	15	
Q_E	Support	±290	
	Midspan	0	±25
Load Combinations			
1.4 D	Support	−133	33.6
	Midspan	91	
$1.2D + 1.6L$	Support	−149	46.4
	Midspan	102	
$1.4D + 0.5L + Q_E$	Support	−434/146*	64.1
	Midspan	98.5	
$D + 0.5L + Q_E$	Support	−396/184*	54.5
	Midspan	−473	
$1.1D + Q_E$	Support	−395/186*	51.4
	Midspan	−471.5	
$0.7D + Q_E$	Support	−357/224*	41.8
	Midspan	−446	

*Q_E has negative and positive value.

Table 20.11 Calculation of Longitudinal Reinforcement

Location	M_u (kip-ft)	A_s (in.^{2})	Reinforcement Used	ϕM_n (kip-ft)
Support	−434	5.20	7 no. 8 ($A_s = 5.53\,\text{in.}^2$)	−474
	224	3.01	6 no. 7 ($A_s = 3.6\,\text{in.}^2$)	358
Midspan	102	1.21	2 no. 7 ($A_s = 1.2\,\text{in.}^2$)	113

From the previous table the most critical loads are chosen and summarized in Table 20.11. Longitudinal reinforcement for the beam is also determined in Table 20.11.
Table 20.12 summarizes the reinforcement used for the beam.

3. General requirements for flexural members of special moment frame are checked as follows:

a. Clear span ≥ 4 effective depth

$$28\text{ ft} \geq 4\frac{21.5}{12} = 7.2\text{ ft} \quad \text{(o.k.)}$$

b. Width-to-depth ratio ≥ 0.3

$$\frac{20}{24} = 0.83 > 0.3 \quad \text{(o.k.)}$$

c. Width = 20 in. ≥ 10 in. (o.k.)

d. Width ≤ width of supporting member + distance on each side of the supporting member not exceeding three-fourths of the depth of the flexural member

$$20\text{ in.} \leq 24\text{ in. (o.k.)}$$

$$20\text{ in.} \leq 24 + (1.5 \times 26) = 63\text{ in.} \quad \text{(o.k.)}$$

Table 20.12 Summary of Reinforcement

Location	Reinforcement Provided: Top	Reinforcement Provided: Bottom
Support	7 no. 8 (5.53 in.^{2})	6 no. 7 (3.6 in.^{2})
Midspan	2 no. 8*(1.58 in.^{2})	2 no. 7 (1.2 in.^{2})

*Two no. 8 bars are extended from seven no. 8 support bars into the negative moment zone at midspan.

4. Special requirements for longitudinal reinforcement are

a.

$$(A_s^- \text{ or } A_s^+) \geq \begin{cases} \dfrac{3\sqrt{f_c'}b_w d}{f_y} = \dfrac{3\sqrt{4000} \times 20 \times 21.5}{60000} = 1.36 \text{ in.}^2 \\ \dfrac{200 b_w d}{f_y} = \dfrac{200 \times 20 \times 21.5}{60000} = 1.43 \text{ in.}^2 \end{cases}$$

$$\max A_S = 0.025 b_w d = 0.025 \times 20 \times 21.5 = 10.75 \text{ in.}^2$$

Check the reinforcement limits against the required reinforcement, as shown in Table 20.13.

b. Positive moment strength at joint face $\geq \frac{1}{2}$ negative moment strength at that face of the joint:

$$M_n^+ = 358 \text{ kip-ft} \geq \frac{1}{2} Mn^- = \frac{1}{2} 474 = 237 \text{ kip-ft} \qquad \text{(o.k.)}$$

c. (M_n^- or M_n^+) at any section $\geq \frac{1}{4}$ (max M_n at either joint) (ACI 20.3.2.2)

$$M_n = 148 \text{ kip-ft} > \frac{1}{4} 474 = 119 \text{ kip-ft} \quad \text{(o.k.)}$$

Anchorage of flexural reinforcement in exterior column is determined as follows:

For no. 8 bars,

$$l_{dh} = \begin{cases} \dfrac{60000 \times 1.0}{65\sqrt{4000}} = 14.6 \text{ in.} \\ 8 \times 1.0 = 8 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

Therefore, $l_{dh} = 14.6$ in.

Table 20.13 Longitudinal Reinforcement Requirements According to the Limits of the Reinforcement

	Reinforcement Used A_s (in.^{2})	Limits: Min A_s (in.^{2})	Limits: Max A_s (in.^{2})	Required A_s (in.^{2})	ϕM_n (kip-ft)
Support (joint face)	5.53 (7 no. 8 at the top)			5.53	−474
	3.6 (6 no. 7 at the bottom)	1.43	10.75	3.6	358
Midspan	1.58 (2 no. 8 at the top)			1.58	−148
	1.2* (2 no. 7 at the bottom)			1.8* (3 no. 7)	168

*Since 1.2 in.^{2} < min A_s = 1.43 in.^{2}, use three no. 7 bars at the bottom (A_s = 1.8 in.^{2}).

For no. 7 bars,

$$l_{dh} = \begin{cases} \dfrac{60000 \times 0.875}{65\sqrt{4000}} = 12.8 \text{ in.} \\ 8 \times 0.875 = 7 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

Therefore, $l_{dh} = 12.8$ in.

5. Transverse reinforcement is determined as follows:

$$V_e = \frac{(M_{pr}^{\pm})_l + (M_{pr}^{\pm})_r}{l_n} + \frac{w_u l_n}{2}$$

$$M_{pr} = A_s(1.25 f_y)\left(d - \frac{a}{2}\right)$$

For six no. 7 bottom bars,

$$a = \frac{A_s(1.25 f_y)}{0.85 f_c' b} = \frac{3.6(1.25 \times 60)}{0.85 \times 4 \times 20} = 3.97 \text{ in.}$$

$$M_{pr} = A_s(1.25 f_y)\left(d - \frac{a}{2}\right) = 3.6(1.25 \times 60)\left(21.5 - \frac{3.97}{2}\right) = 5269 \text{ kip-in.} = 439 \text{ kip-ft}$$

For seven no. 8 bars,

$$a = \frac{A_s(1.25 f_y)}{0.85 f_c' b} = \frac{5.53(1.25 \times 60)}{0.85 \times 4 \times 20} = 6.1 \text{ in.}$$

$$M_{pr} = A_s(1.25 f_y)\left(d - \frac{a}{2}\right) = 5.53(1.25 \times 60)\left(21.5 - \frac{6.1}{2}\right)$$

$$= 7652 \text{ kip-in.} = 638 \text{ kip-ft}$$

$$w_u = 1.2 w_D + 0.5 w_L = 2.78 \text{ kip/ft}$$

$$V_l = \frac{M_{pr}^- + M_{pr}^+}{l_n} + \frac{w_u l_n}{2} = \frac{638 + 439}{26} + \frac{2.78 \times 26}{2} = 77.6 \text{ kip}$$

$$V_r = \frac{M_{pr_l}^+ + M_{pr_r}^-}{l_n} - \frac{w_u l_n}{2} = \frac{638 + 439}{26} - \frac{2.78 \times 26}{2} = 5.3 \text{ kip}$$

Maximum earthquake induced shear force $= \dfrac{439 + 638}{26} = 41.4 \text{ kip} > \dfrac{77.6}{2} = 38.8$ kip

$$\Rightarrow V_c = 0$$

$$\phi V_s = V_u - V_c$$

$$V_s = \frac{77.6}{0.75} - 0 = 104 \text{ kip}$$

$$V_s = 104 \text{ k} < (V_{s\,max} = 8\sqrt{f_c'} b_w d = 8\sqrt{4000} \times 20 \times 21.5 = 217.6 \text{ kip} \qquad \text{(o.k.)}$$

$$V_s = 104 \text{ k} < 4\sqrt{f_c'} b_w d = 4\sqrt{4000} \times 20 \times 21.5 = 109 \text{ kip} \qquad \text{(o.k.)}$$

Required spacing for no. 3 stirrups is determined as

$$s = \frac{A_s f_y d}{V_s} = \frac{(4 \times 0.11) \times 60 \times 21.5}{104} = 5.5 \text{ in.}$$

Maximum spacing of the hoops within a distance of $2h = 2 \times 24 = 48$ in. shall not exceed the smallest of

$$\frac{d}{4} = \frac{21.5}{4} = 5.4 \text{ in.}$$

Eight times the diameter of the smallest longitudinal bar $= 8 \times 0.875 = 7$ in.
24 times the diameter of the hoop bars $= 24 \times 0.375 = 9$ in.
12 in.

Therefore, use 10 no. 3 hoops at each end of the beam at 5 in. center-to-center with the first hoop located at 2 in. from the face of the support.

At the distance 48 in. from the face of the support shear strength is

$$V_u = 77.66 - 2.78 \times \frac{48}{12} = 66.48 \text{ kip}$$

The shear strength contributed by concrete is

$$V_c = 2 \times 1 \times \sqrt{4000} \times 20 \times 21.5 = 54.4 \text{ kip}$$

$$V_s = \frac{66.48}{0.75} - 54.4 = 34.2 \text{ kip}$$

Spacing of the stirrups should not be taken greater than

$$s = \frac{d}{2} = \frac{24}{2} = 12 \text{ in.}$$

or

$$s = \frac{A_s f_y d}{50b} = \frac{(2 \times 0.11) \times 60 \times 21.5}{50 \times 20} = 13.2 \text{ in.}$$

or

$$s = \frac{d}{2} = \frac{24}{2} = 12 \text{ in.}$$

Therefore, use stirrups with seismic hoops spaced 8 in. center-to-center starting at 48 in. from the face of the support. Figure 20.17 shows reinforcement detailing.

Special moment frame members subjected to bending and axial load (Section 20.5.1.2).

General requirements (Section 20.5.1.2.1) The requirements of this section apply to columns and other flexural members that carry a factored axial load $> A_g f_c'/10$. These members should satisfy both of the following conditions (ACI 2008, Section 21.6):

1. Shortest cross-section dimension $\geq$ 12 in.
2. The ratio of shortest cross-sectional dimension to the perpendicular dimension ≥ 0.4

Longitudinal reinforcement requirements (Section 20.5.1.2.2) According to the ACI Code 2008, Section 21.5.2, the flexural strengths of columns should satisfy the following:

$$\sum M_{nc} \geq \frac{6}{5} \sum M_{nb} \tag{20.42}$$

where

$\sum M_{nc}$ = sum of nominal flexural strengths of the columns framing into the joint, evaluated at the faces of the joint.

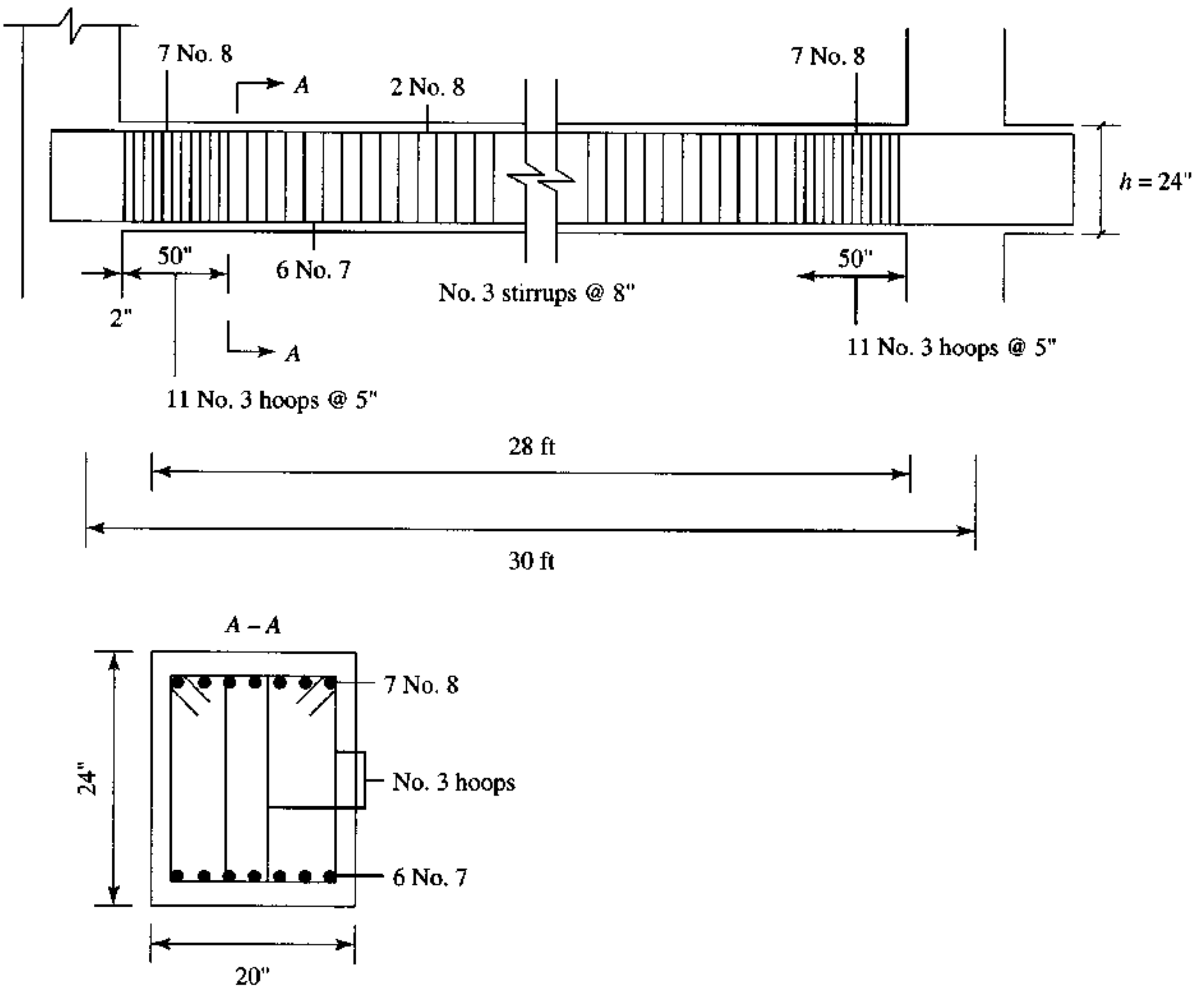


Figure 20.17 Example 20.6 reinforcement detailing.

$\sum M_{nb}$ = sum of nominal flexural strengths of the beams framing into the joint, evaluated at the faces of the joint.

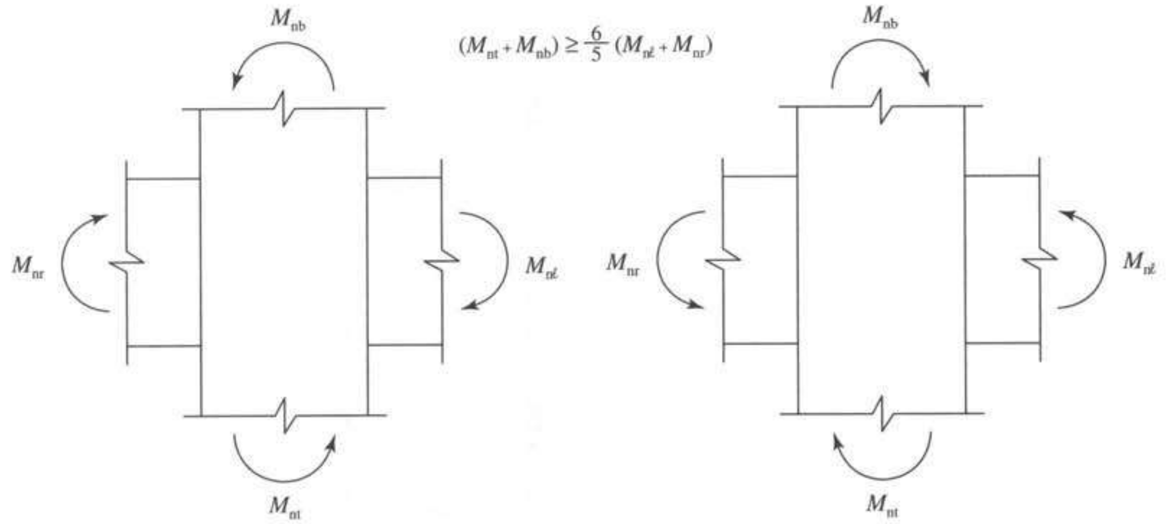
This approach, called strong column—weak beam concept (Fig. 20.18), ensures that columns will not yield before the beams. The main steel reinforcement should be chosen to satisfy Eq. 20.61.

The reinforcement ratio should satisfy the following:

$$0.01 \le \rho_g \le 0.06 \tag{20.43}$$

Transverse reinforcement requirements (Section 20.5.1.2.3) Columns should be properly detailed to ensure column ductility in the case of plastic hinge formation, and should also have the adequate shear strength to prevent shear failure.

The following transverse reinforcement requirements need to be provided only over the length l_o greater or equal to depth of the member, $\frac{1}{6}$ clear span, 18 in., from the each joint face and on both sides of any section where yielding is likely to occur (ACI code 2008, Section 21.6.4.1). The requirements are



Subscripts ℓ, r, t, and b stand for left support, right support, top of column, and bottom of column, respectively.

Figure 20.18 Strong column – weak beam concept. Courtesy of American Concrete Institute (ACI 2008).

Lack of transverse reinforcement.

1. Ratio of spiral reinforcement, ρ_s, should satisfy the following (Fig. 20.19):

$$\rho_s \geq \begin{cases} 0.12 \dfrac{f_c'}{f_{yt}} \\ 0.45 \left(\dfrac{A_g}{A_c} - 1 \right) \dfrac{f_c}{f_{yt}} \end{cases} \tag{20.44}$$

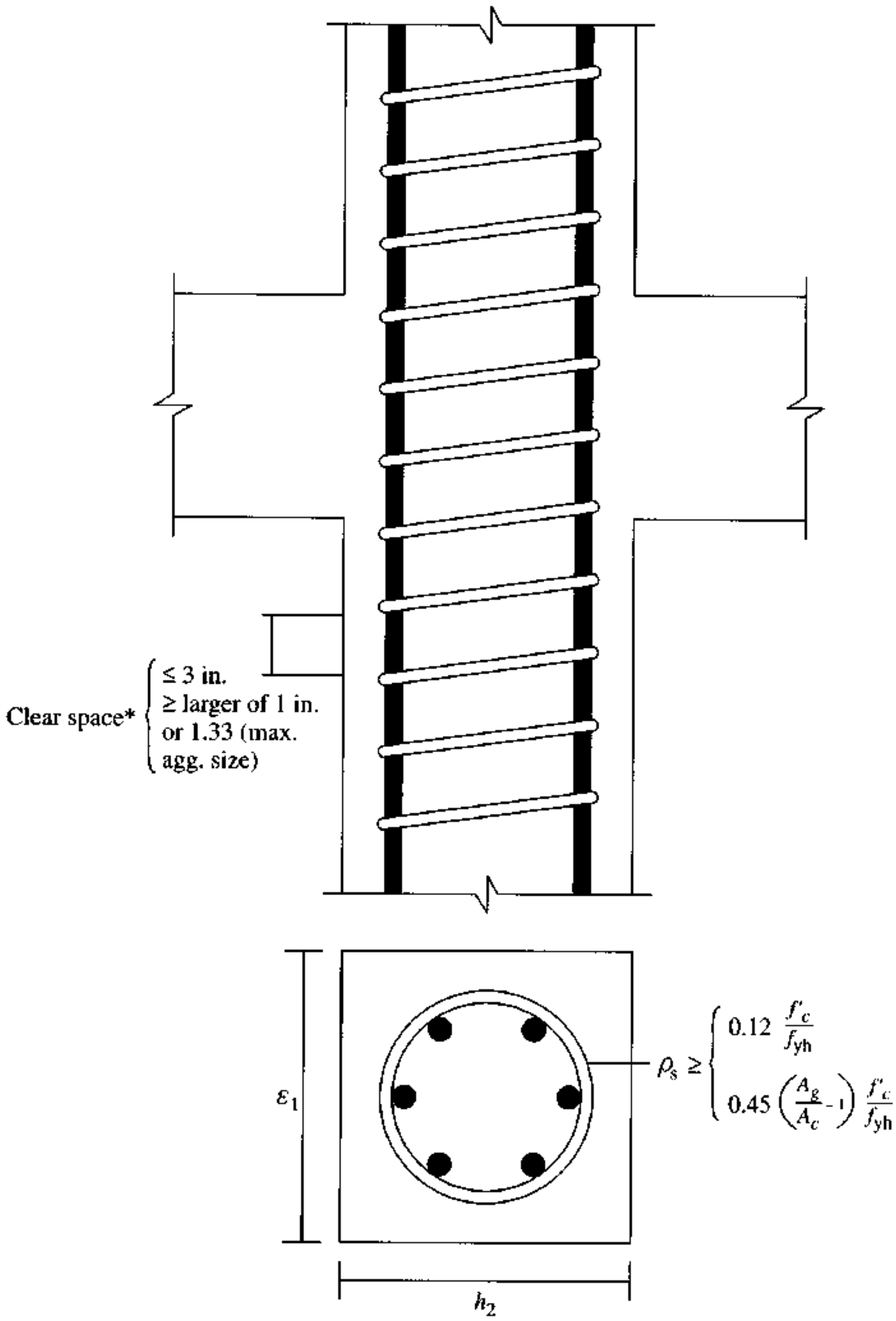


Figure 20.19 Transverse reinforcement requirements for spiral reinforcement. Courtesy of Portland Cement Association (notes on ACI 318).

where

f_{yt} = yield stress of transverse reinforcement

A_c = area of core of spirally reinforced compression member measured to outside diameter of spiral

A_g = gross area of section

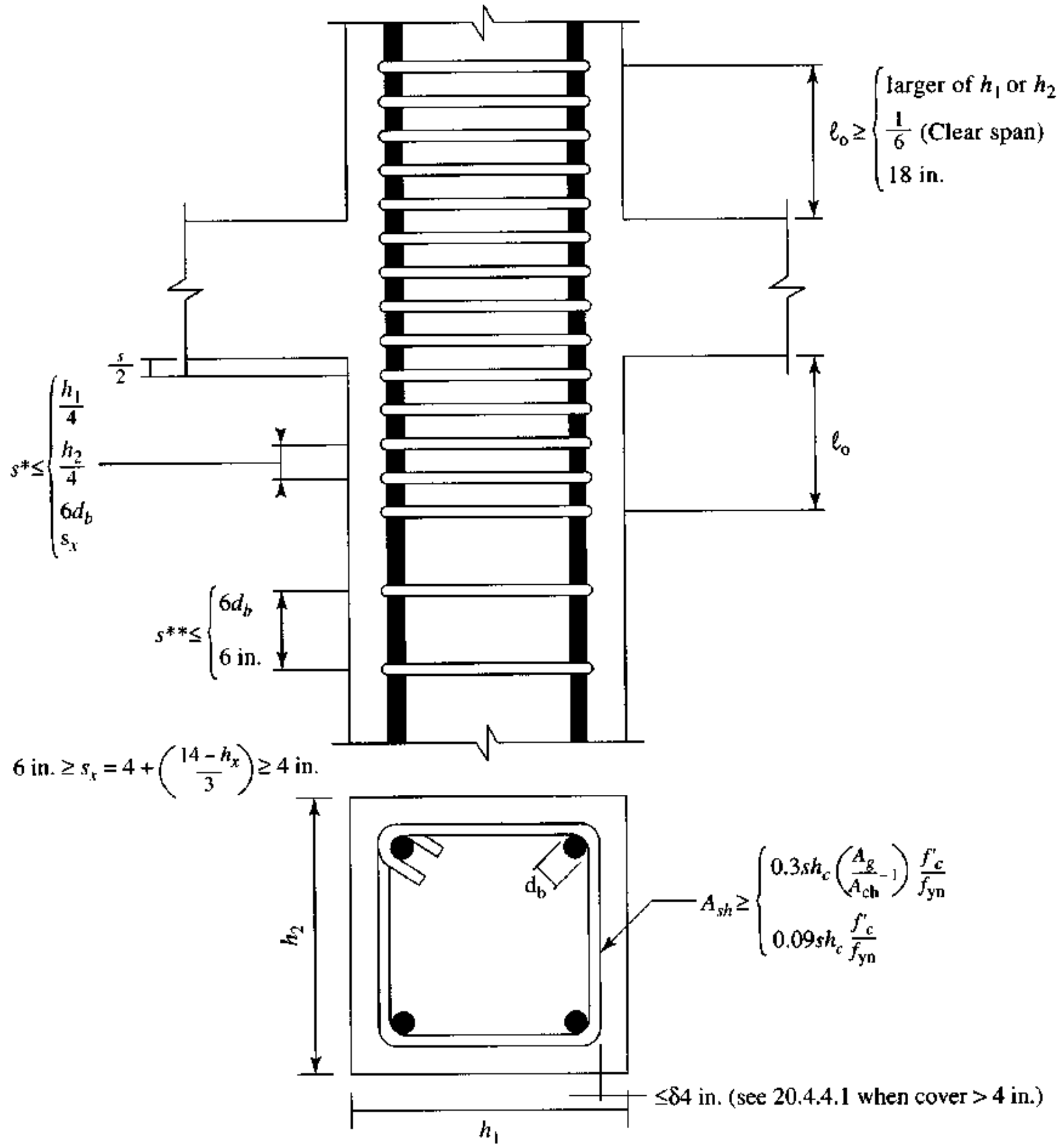


Figure 20.20 Transverse reinforcement requirements for rectangular hoop reinforcement. Courtesy of Portland Cement Association (notes on ACI 318).

2. Total cross-section area of rectangular hoop reinforcement, A_{sh}, should satisfy the following (Fig. 20.20):

$$A_{sh} \geq \begin{cases} 0.3\left(\dfrac{sh_c f'_c}{f_{yt}}\right)\left(\dfrac{A_g}{A_{ch}} - 1\right) \\ 0.09\,\dfrac{sh_c f'_c}{f_{yt}} \end{cases} \tag{20.45}$$

where

s = spacing of transverse reinforcement
h_c = cross-section dimension of column core measured center-to-center of the confining reinforcement.

3. If the thickness of the concrete outside the confining transverse reinforcement exceeds 4 in., additional transverse reinforcement should be provided at a spacing $\leq$ 12 in. Concrete cover on additional reinforcement should not exceed 4 in.
4. Spacing of the transverse reinforcements should satisfy the following:

$$s \leq \begin{cases} \dfrac{h}{4} \\ 6 \times \text{longitudinal diameter bar} \\ s_o \end{cases} \tag{20.46}$$

$$\text{Also, } 4 \text{ in.} \leq s_o = 4 + \frac{14 - h_x}{3} \leq 6 \text{ in.} \tag{20.47}$$

where

s_o = longitudinal spacing of transverse reinforcement within the length l_o.
h_x = maximum horizontal spacing of hoop or crosstie legs on all faces of the column.

The remaining member length should be reinforced with the spiral or hoop transverse reinforcement spaced as follows:

$$s \leq \begin{cases} 6 \times \text{longitudinal bar diameter} \\ 6 \text{ in.} \end{cases} \tag{20.48}$$

Transverse reinforcement should be designed to resist the design shear force. Design shear force for flexural members of special moment frames can be determined using the following equation:

$$V_u = \frac{M_{\mathrm{Pr}_t} + M_{\mathrm{Pr}_b}}{l_c} \tag{20.49}$$

index t is for top and index b is for bottom of the column) where,
l_c = length of the column

Summary: Design of the Special Moment-Resisting Frame Members Subjected to Bending and Axial Force (Section 20.5.1.2.4)

Step 1. Determine seismic design category, base shear, lateral seismic force and seismic shear according to Sections 20.2 and 20.3.

Step 2. Calculate the member forces and using the different load combinations determine the values of member forces that govern the design. Design the reinforcement.

Step 3. Check whether the frame member is a flexural member or whether the member is subjected to the bending and axial force, and check general requirements for the special moment frame member according to Section 20.5.1.2.1.

Step 4. Check the special requirements for the longitudinal reinforcement according to Section 20.5.1.2.2.

Step 5. Design the transverse reinforcement for confinement and shear resistant using Section 20.5.1.2.3.

Example 20.7

Design the edge column on the second floor of a building from Example 20.6.

Solution

1. The load combinations gave the following results:

$$P_u = 1022 \text{ kip} \quad \text{(maximum force at the first floor)}$$

$$P_u = 935 \text{ kip} \quad \text{(maximum force at the second floor)}$$

2.
$$P_u = 1022k > \frac{A_g f_c'}{10} = \frac{(24 \times 24) \times 4}{10} = 230 \text{ kip}$$

Member is subjected to bending and axial load. General requirements should be checked as follows:

a. Shortest cross-section dimension = 24 in. ≥ 12 in., which is o.k.

b. The ratio of shortest cross-sectional dimension to the perpendicular dimension, $\frac{24}{24} = 1 \geq 0.4$, which is o.k.

3. Longitudinal reinforcement for the column with $P_u = 1022$ kip is eight no. 8 bars.
The reinforcement ratio is $\rho_g = 0.011 < 0.06$, which is o.k. and > 0.01, which is also o.k.

$$\sum M_{nc} \geq \frac{6}{5} \sum M_{nb}$$

For $P_u = 1022$ kip, $M_n = 580$ kip-ft. For $P_u = 935$ kip, $M_n = 528$ kip-ft. A minimum nominal flexural strength of the beam at the joint including the slab reinforcement is $M_n = 723$ kip-ft.

$$\sum M_{nc} = 580 + 528 = 1108 \text{ kip-ft}$$

$$\sum M_{nb} = 723 \text{ kip-ft}$$

$$\sum M_{nc} = 1108 \text{ kip-ft} \geq \frac{6}{5} \sum M_{nb} = \frac{6}{5} 723 = 868 \text{ kip-ft} \qquad \text{(o.k.)}$$

4. Length l_o is determined as follows:

$$l_o \geq \begin{cases} \text{depth of the member} = 24\text{in.} \\ \frac{1}{6}\text{clear height} = (12 \times 12)/6 = 24 \text{ in.} \\ 18 \text{ in.} \end{cases}$$

Choose $l_o = 24$ in.

$$\text{Spacing is} s \leq \begin{cases} \frac{h}{4} = \frac{24}{4} = 6 \text{ in.} \\ 6 \times \text{longitudinal diameter bar} = 6 \times 1.0 = 6 \text{ in.} \\ s_o = 4 + \left(\frac{14 - 11}{3}\right) = 5 \text{ in.} \end{cases}$$

Therefore, $s = 5$ in.

Required cross-section area of reinforcement is

$$A_{sh} \geq \begin{cases} 0.3\left(\dfrac{sh_c f'_c}{f_{yt}}\right)\left(\dfrac{A_g}{A_{ch}} - 1\right) = 0.3\left(\dfrac{5 \times 20.5 \times 4}{60}\right)\left(\dfrac{576}{441} - 1\right) = 0.63 \text{ in.}^2 \\ 0.09\dfrac{sh_c f'_c}{f_{yt}} = 0.09\dfrac{5 \times 20.5 \times 4}{60} = 0.62 \text{ in.}^2 \end{cases}$$

Choose no. 4 hoops and no. 5 crossties:

$$A_{sh} = 2 \times 0.2 + 0.31 = 0.71 \text{ in.}^2 > 0.63 \text{ in.}^2$$

Detailing of the reinforcement can be found in Fig. 20.21.

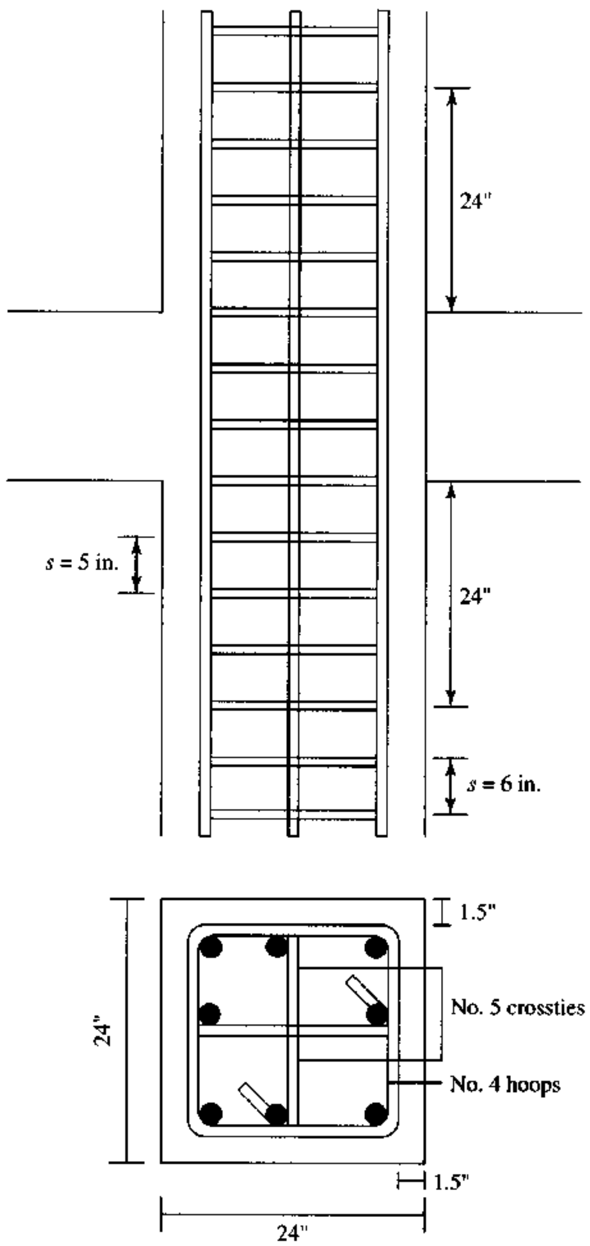


Figure 20.21 Example 20.7 reinforcement detailing.

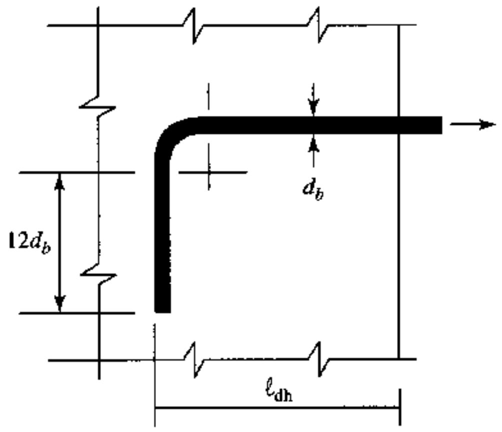


Figure 20.22 Standard 90° hooks. Courtesy of American Concrete Institute (ACI 2008).

Joints of the special moment-resisting frame (Section 20.5.1.3). Joint of special moment-resisting frame should be detailed according to the ACI Code 2008, Section 20.5, as follows:

Longitudinal Reinforcement Requirements (Section 20.5.1.3.1) The development length l_{dh} for a bar with a standard 90° hook using normal-weight concrete, for bar size no. 3 through no. 11, should be determined according to the following (Fig. 20.22):

$$l_{\mathrm{dh}} \geq \begin{cases} \dfrac{f_y d_b}{65\sqrt{f_c'}} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{20.50}$$

where d_b is the diameter of longitudinal reinforcement.

The development length, l_d, for a strait bar for bar size no. 3 through no. 11 should not be less than

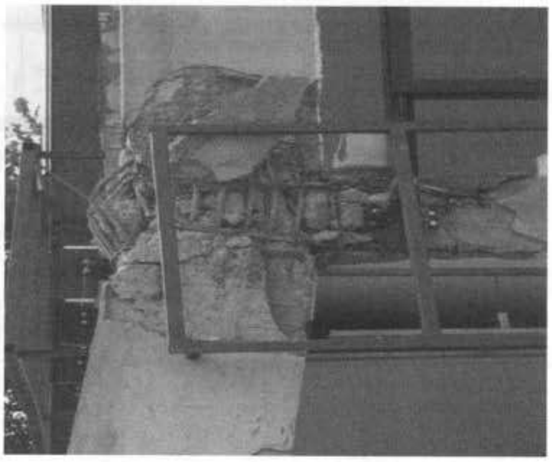
1. 2.5 l_{dh} if the depth of the concrete cast in one lift beneath the bar does not exceed 12 in.
2. 3.5 l_{dh} if the depth of the concrete cast in one lift beneath the bar exceeds 12 in.

When the longitudinal reinforcement passes through the joint, the column dimension parallel to the beam reinforcement should not be less than 20 times the diameter of the largest longitudinal bar for normal-weight concrete. For lightweight concrete, this dimension should not be less than 26 times the bar diameter.

Shear strength requirements (Section 20.5.1.3.2) The nominal shear strength of the joint for normal-weight concrete should not exceed the following:

1. $20\sqrt{f_c'}A_j$ for joints confined on all four faces
2. $15\sqrt{f_c'}A_j$ for joints confined on three faces or on two opposite faces
3. $12\sqrt{f_c'}A_j$ for all other joints

where A_j is the effective area, as shown in Fig. 20.23.



Beam-column connection (joint).

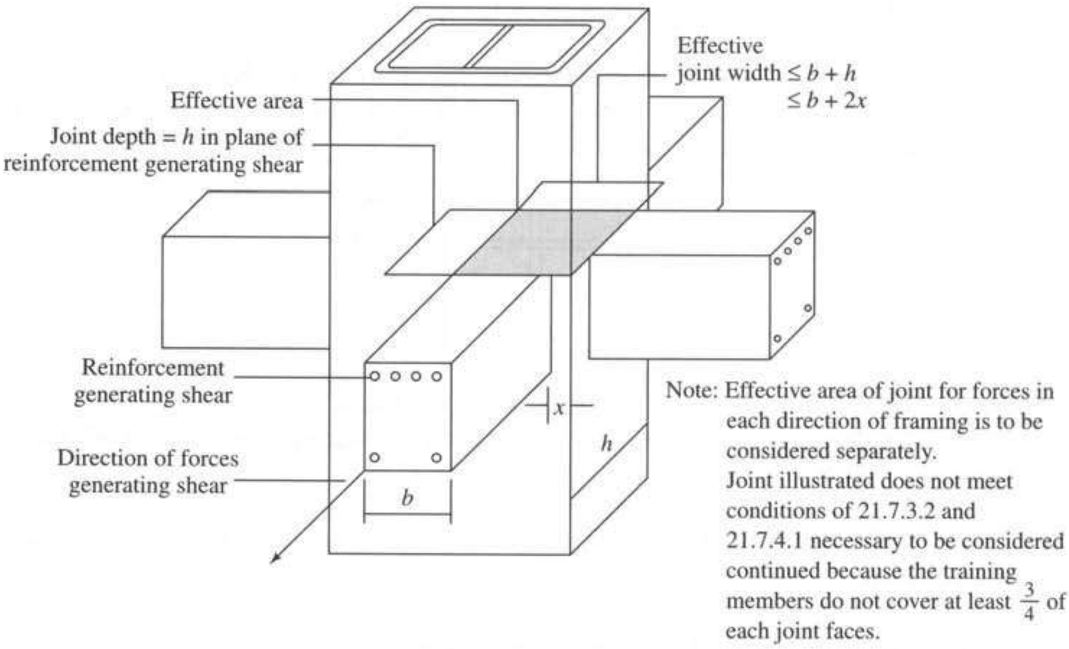


Figure 20.23 Effective *Aj* of joint. Courtesy of American Concrete Institute (ACI 2008).

20.5.2 Structures in the High Seismic Risk: Special Reinforced Concrete Structural Walls and Coupling Beams (ACI 2008 Section 21.9)

Wall system is a structural system that provides support for all gravity loads and all lateral loads applied to the structure. A structural wall system is much stiffer than a frame system and its performance during an earthquake is better than the performance of the frame system.

A structural wall should be properly designed to sustain all loads acting on it. Boundary elements of structural walls are the areas around the structural wall edges, as shown in Fig. 20.24, that strengthen by the longitudinal and transverse reinforcement. Boundary elements increase

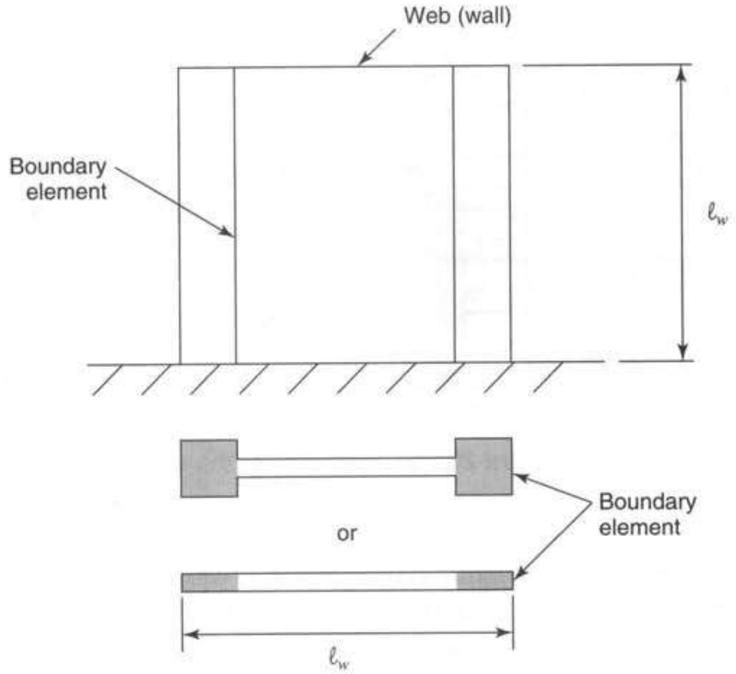


Figure 20.24 Boundary elements of structural wall.

Shear wall after an earthquake.

the rigidity and strength of wall panels. The web reinforcement is anchoraged into the boundary elements.

Figure 20.25 shows the elements of the wall with openings. The vertical wall segment bounded by two openings is called pier. A horizontal wall section between the openings is called a horizontal wall segment. When the openings are aligned vertically over the building height, the horizontal wall segments between the openings are called coupling beams.

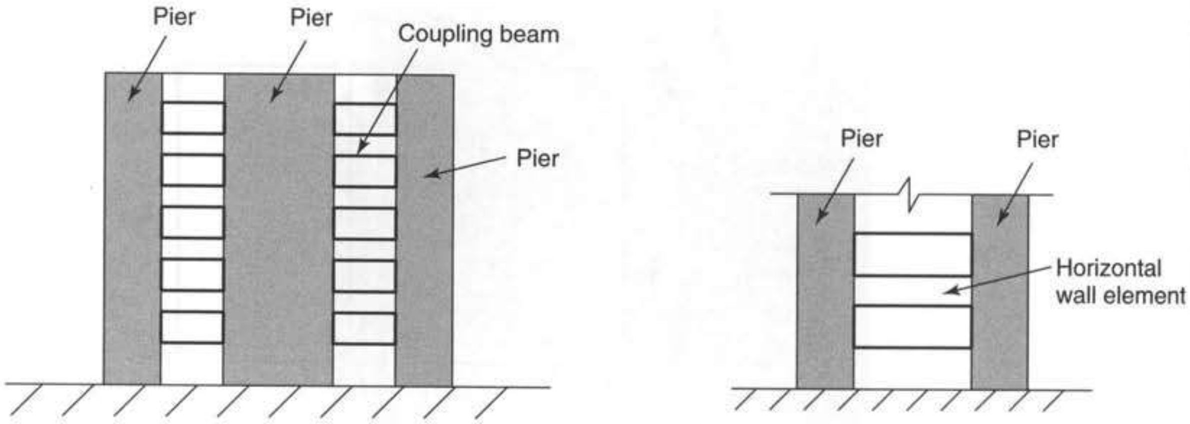


Figure 20.25 Elements of the wall with openings.

In the regions of high seismic risk, structural walls with special reinforcement requirements should be used. The ACI Code (2008) Section 21.9, gives provisions for the design and detailing of structural walls. These are described in the following sections.

Reinforcement Requirements (Section 20.5.2.1). Shear reinforcement should be provided in two orthogonal directions in the plane of the wall. (ACI 2008, Section 21.9.2.1) The minimum reinforcement ratio for both longitudinal and transverse directions can be determined as follows:

1. If the design shear $V_u > A_{cv}\lambda\sqrt{f_c'}$, the distributed web reinforcement ratios, ρ_v and ρ_n, should not be less than 0.0025.

$$\rho_l = \frac{A_{sv}}{A_{cv}} = \rho_n \geq 0.0025 \tag{20.51}$$

where

ρ_t = ratio of area of distributed reinforcement parallel to the plane of A_{cv} to gross concrete area perpendicular to that reinforcement (Fig. 20.26)

ρ_l = ratio of area of distributed reinforcement perpendicular to the plane of A_{cv} to gross concrete area A_{cv}. (Fig. 20.26)

A_{cv} = gross area of concrete section (product of thickness and length of the section in the direction of shear force)

A_{sv} = Projection on A_{cv} of area of shear reinforcement crossing the plane of A_{cv}

λ = factor for lightweight aggregate concrete

2. If the design shear $(V_u) < A_{cv}\lambda\sqrt{f_c'}$, the minimum reinforcement for ordinary structural walls can be utilized:

Minimum vertical reinforcement ratio, ρ_l = 0.0012 for no. 5 bars and smaller
= 0.0015 for no. 6 bars and larger

Minimum horizontal reinforcement ratio, ρ_t = 0.0020 for no. 5 bars and smaller
= 0.0025 for no. 6 bars and larger

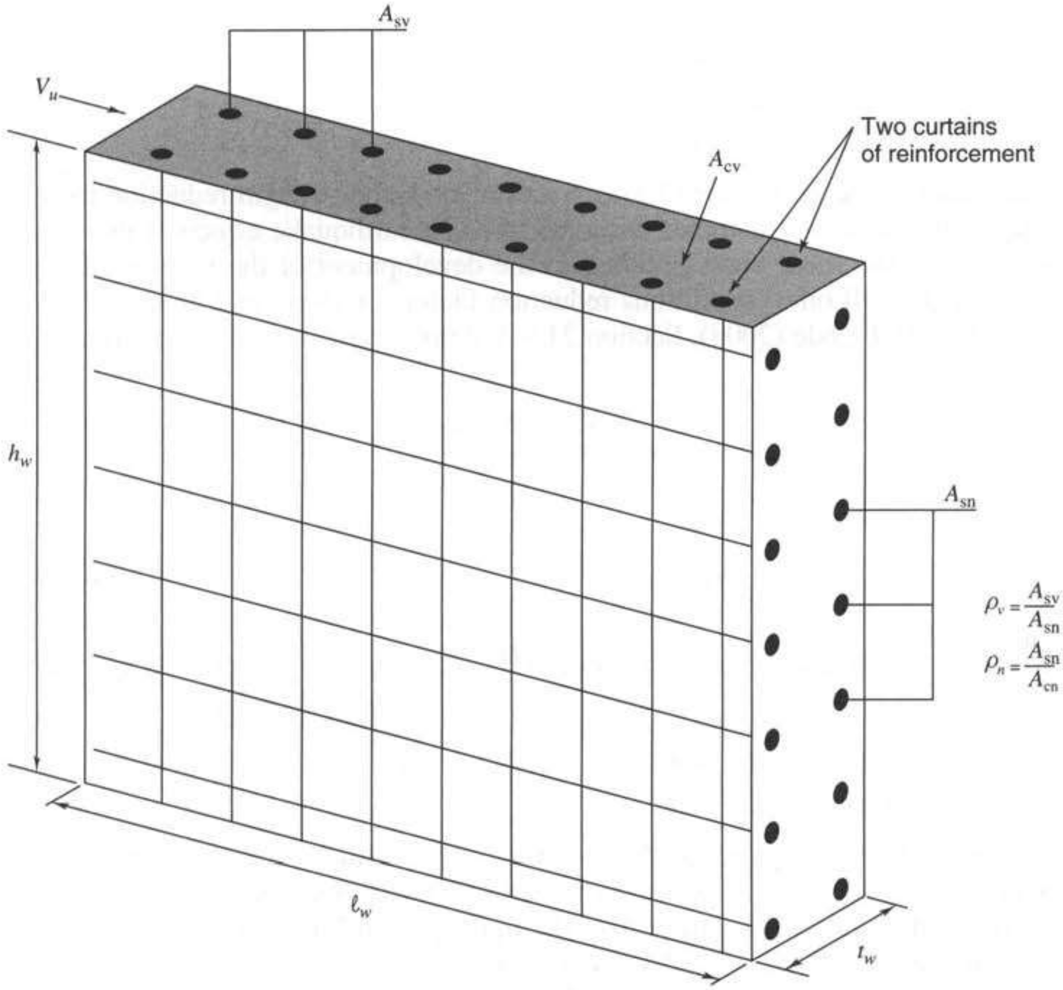


Figure 20.26 Reinforcement requirements.

The spacing of the reinforcement can be calculated as follows:

$$s = 2A_s^1 / A_s \text{required (per foot of wall)}$$

where

A_s^1 = area of one bar (Fig. 20.26)

Maximum spacing of reinforcement is 18 in. each way according to ACI Section 21.9.2.1.

If the in-plane factored shear force assigned to the wall exceeds $2A_{cv}\lambda\sqrt{f_c'}$, at least two curtains of reinforcement should be provided, as shown in Figure 20.26.

All continuous reinforcement in structural walls should be anchored and spliced as reinforcement in tension for special moment frame (Section 21.9.2.3).

Shear Strength Requirements (Section 21.9.2.2). The shear strength of structural wall is adequate if the following condition is satisfied:

$$V_u \leq \phi V_n \quad (20.52)$$

where

V_u = factored axial force
V_n = nominal shear strength
ϕ = strength reduction factor

According to the ACI Code (2008), Section 9.3.4, the strength reduction factor for shear will be 0.6 for any structural member designed to resist earthquake effects if its nominal shear strength is less than the shear corresponding to the development of the nominal flexural strength of the member. For all other conditions reduction factor for shear will be 0.75.

The ACI Code (2008), Section 21.9.4, defines the nominal shear strength of structural walls as follows:

$$V_n = A_{cv}(\alpha_c \lambda \sqrt{f_c'} + \rho_t f_y) \tag{20.53}$$

where

$$\alpha_c = 3.0 \text{ for} \frac{h_w}{l_w} \leq 1.5$$
$$= 2.0 \text{ for} \frac{h_w}{l_w} \geq 2.0$$
$$= \text{linear interpolation between 3.0 and 2.0 for } \frac{h_w}{l_w} \text{ between 1.5 and 2.0}$$

where

h_w = height of the wall
l_w = length of the wall

For the walls with openings, the value of h_w/l_w shall be the larger of the ratios for the entire wall and the segment of wall considered. This ensures that the assigned unit strength of any segment of a wall is not larger than the unit strength for the whole wall.

If the ratio $h_w/l_w \leq 2$, reinforcement ratio ρ_v should not be less than ρ_n.

For the walls with openings, the nominal shear strength, V_n, for vertical and horizontal walls segments should satisfy the following:

1. If the factored shear force is resisted several pier, the nominal shear strength, V_n, for all wall segments should be $\leq 8A_{cv}\sqrt{f_c'}$, where A_{cv} is the total cross-section area of the walls (piers) and $V_n \leq 10A_{cp}\sqrt{f_c'}$, where A_{cp} is the cross-section area of the pier considered.
2. Nominal shear strength of horizontal wall segment and coupling beams should be $\leq 10A_{cp}\sqrt{f_c'}$, where A_{cp} is the cross-section area of the horizontal wall segment or coupling beam.

Design for flexure and axial loads (Section 20.5.2.3). Flexural strength of walls should be determined according to the procedure used for columns subjected to flexure and axial loads (ACI 2008, Section 21.9.5). The reinforcement in the whole cross-section of the wall, including boundary elements and web, should be included in calculations of the capacity of the wall. Openings in walls should also be considered.

Where the wall sections intersect, they from L-sections, T-sections, or other cross-section shapes of the flanges (as shown in Fig. 20.27), which need to be considered in design. Flange width should be determined as follows:

Effective flange width from the face of the web should extend a distance equal to or smaller than $\frac{1}{2}$ the distance to an adjacent wall web or 25% of the total wall height (Fig. 20.28), (ACI Section 21.9.5.2).

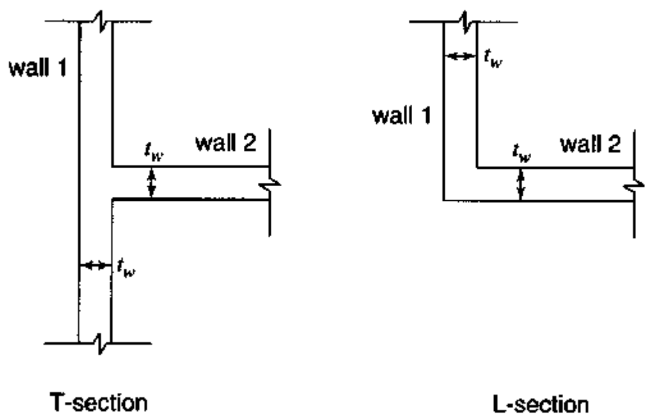


Figure 20.27 Shapes of the wall flanges.

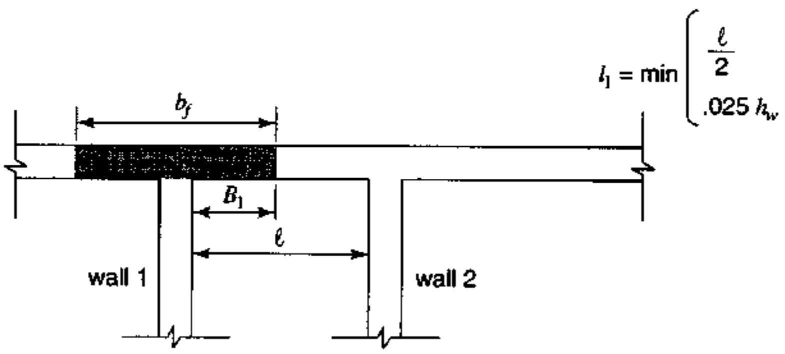


Figure 20.28 Effective flange width, b_f.

Special Boundary Elements of Special Reinforced Structural Walls (Section 20.5.2.4). During an earthquake, a structural wall behaves as cantilever beam (Fig. 20.29). Boundary elements can be very heavily loaded due to earthquake loads. A plastic hinge can form at the base of the wall, which requires special reinforcement detailing to provide necessary strength and ductility of the structural wall. According to the ACI Code (2008), Section 21.9.6.1, there are two design approaches for evaluating the detailing requirements of wall boundary element. These are defined as follows:

1. Displacement based design (ACI Section 21.9.6.2). For the walls or walls pier that are effectively continuous from the base of the structure to the top of the wall, design to have a single critical section for flexure and axial load compression zones should be reinforced with special boundary elements if

$$c \geq \frac{l_w}{600\left(\frac{\delta_u}{h_w}\right)} \tag{20.54}$$

where

$$\frac{\delta_u}{h_w} \geq 0.007$$

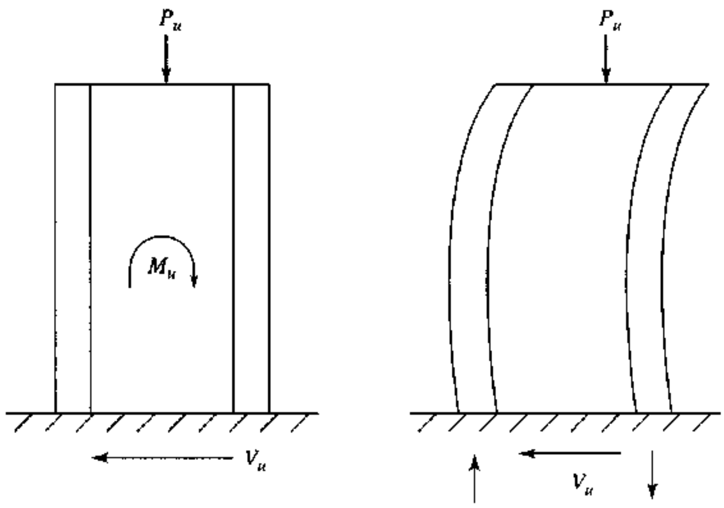


Figure 20.29 Deformation of wall due to earthquake loads.

c = the distance from the extreme compression fiber to the neutral axis, calculated for the factored axial force and nominal moment strength
l_w = the length of the wall in the direction of shear force
δ_u = design displacement

The special boundary reinforcement should extend vertically from a critical section a distance (Fig. 20.30).

$$\geq \begin{cases} l_w \\ \dfrac{M_u}{4V_u} \end{cases} \tag{20.55}$$

2. Shear based design (ACI Section 21.9.6.3). Structural walls not designed to the displacement based approach shall have special boundary elements at boundaries and edges around openings of the structural wall. A special boundary element should be provided where the maximum extreme fiber compressive stress due to factored forces including earthquake effects exceeds $0.2f_c'$. The boundary elements may be discontinued when the compressive stress becomes less than $0.15f_c'$.

Detailing of the special boundary elements should satisfy the following:

1. Extend horizontally from the extreme compression fiber a distance (Fig. 20.30).

$$\geq \begin{cases} c - 0.1l_w \\ \dfrac{c}{2} \end{cases}$$

where

c = the largest neutral axis depth calculated for the factored axial force and nominal moment strength consistent with δ_u.

2. Transverse reinforcement should be designed by the provisions given for the special moment frame members subjected to bending and axial forces (Fig. 20.31).

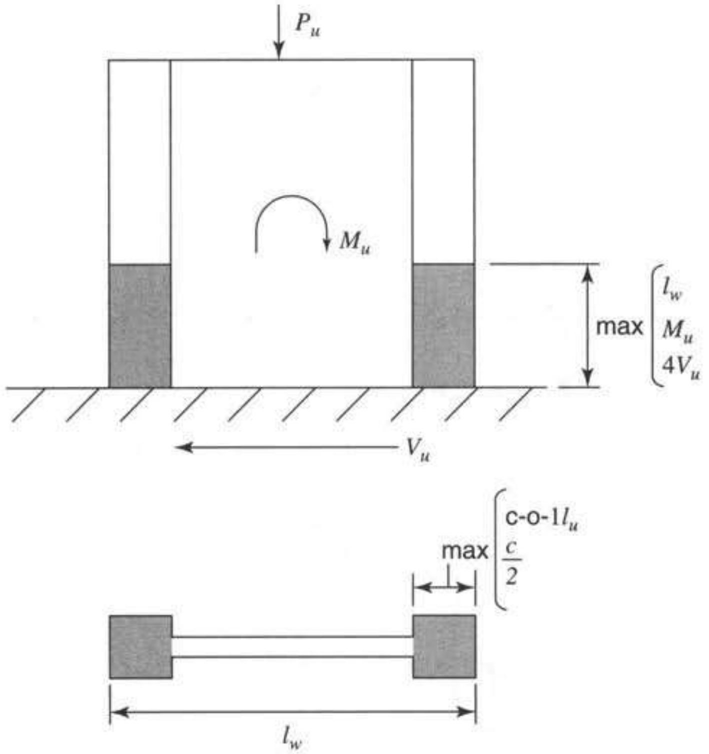


Figure 20.30 Area where special reinforcement requirements should be provided.

Coupling Beams (Section 20.5.2.5). The coupling beam is the structural element that rigidly connects two walls. In a properly designed earthquake-resistant coupled wall system, the coupled beams should yield first, before the base of the wall where the bending moment has the highest value. Also, the beam should have significant ductility and dissipate the energy through the inelastic deformation.

According to the ACI Code, Section 21.9.7, the coupled beams should be designed as follows:

1. If $l_n/h \geq 4$, where l_n is the length and h is the height of the coupled beam, design the coupled beam to satisfy requirements given for flexural members of special moment frame.
2. If $l_n/h < 4$, the beam should be reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan. The diagonal bars are also required for coupling beam with aspect ratio $l_n/h < 2$ and $V_u > 4\lambda\sqrt{f_c'}A_{\mathrm{cw}}$, where A_{cw} is the area of concrete section, resisting shear, of individual pier or horizontal wall segment.

   Two confinement options are described in ACI 318-08 as shown in Figure 20.32. According to ACI Section 21.9.7.4(c) each diagonal element consists of a cage of longitudinal and transverse reinforcement as shown in Figure 20.32(a). Each cage contains at least four diagonal bars and confines a concrete core. The requirement on side dimensions of the cage and its core is to provide adequate toughness and stability to the core section when the bars are loaded beyond yielding.

   ACI Section 21.9.7.4(d) describes a second option for confinement of the diagonals as shown in Figure 20.32(b). This second option is to confine the entire beam cross section instead of confining the individual diagonals. This option can considerably simplify field

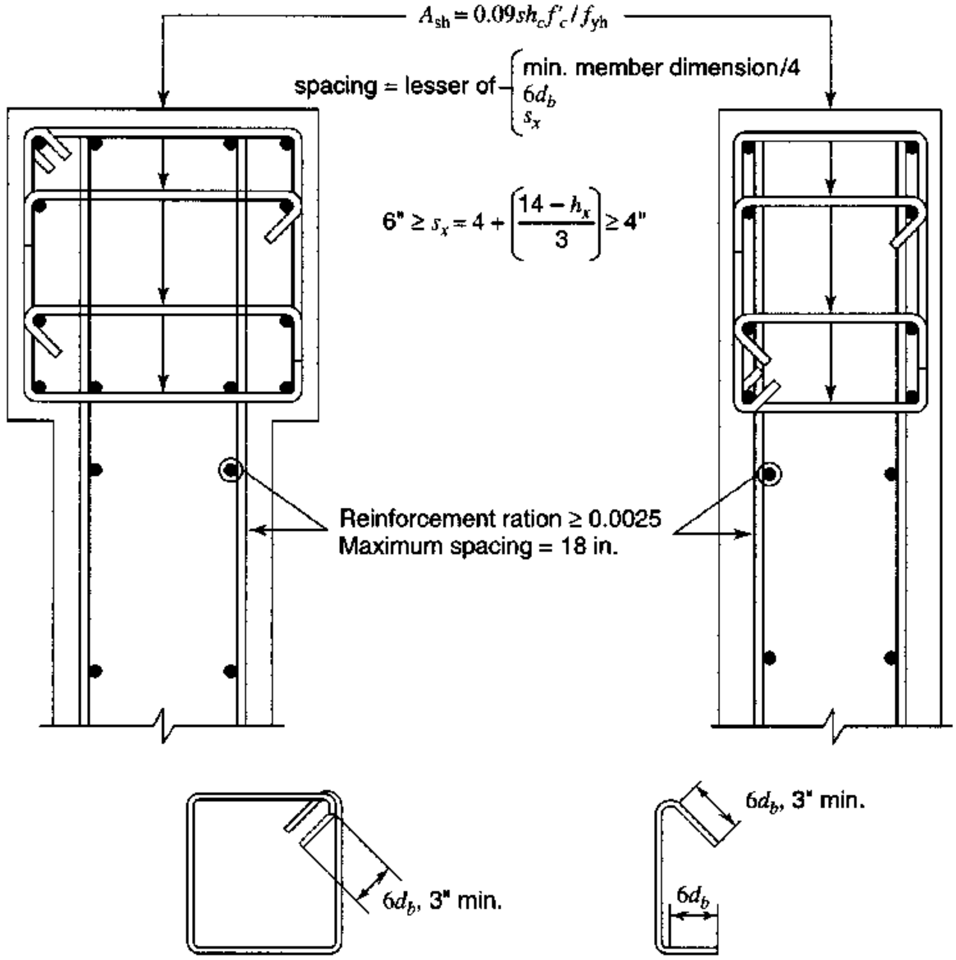


Figure 20.31 Reinforcement details for special boundary elements. Courtesy of Portland Cement Association (notes on ACI 318).

placement for hoops, which can be challenging where diagonal bars intersect each other or entire wall boundary.

Nominal shear strength can be determined using the following equation:

$$V_u = 2A_{vd}f_y \sin\alpha \leq 10\sqrt{f'_c}A_{cw} \tag{20.56}$$

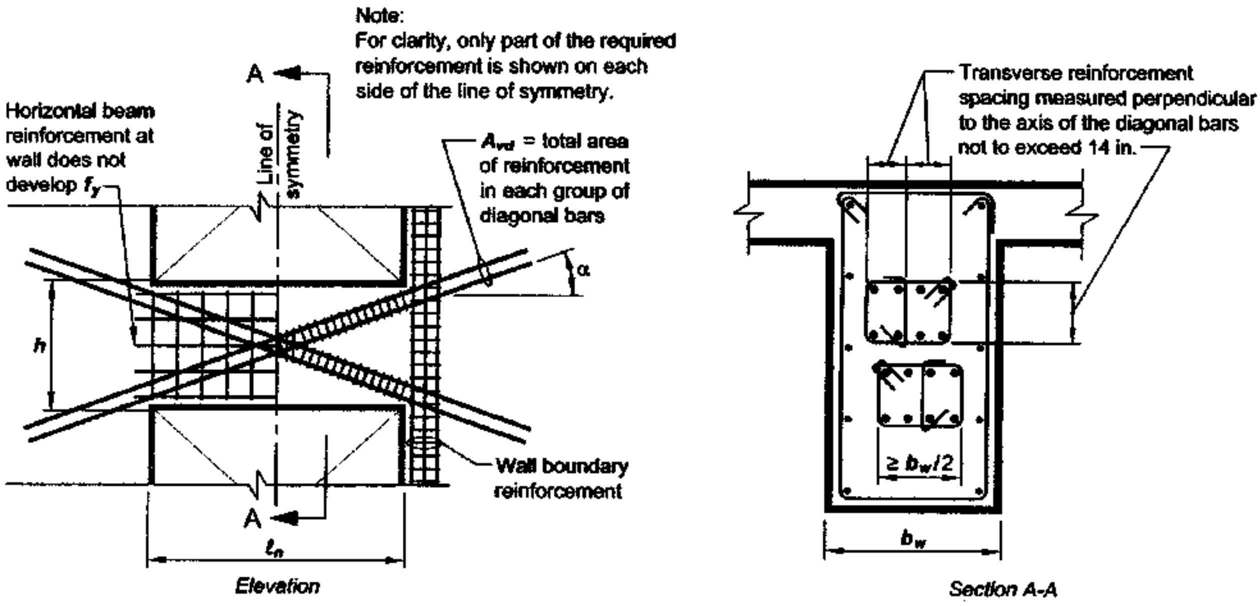
where

A_{vd} = total area of reinforcement in each group of diagonal bars in a diagonally reinforced coupling beam

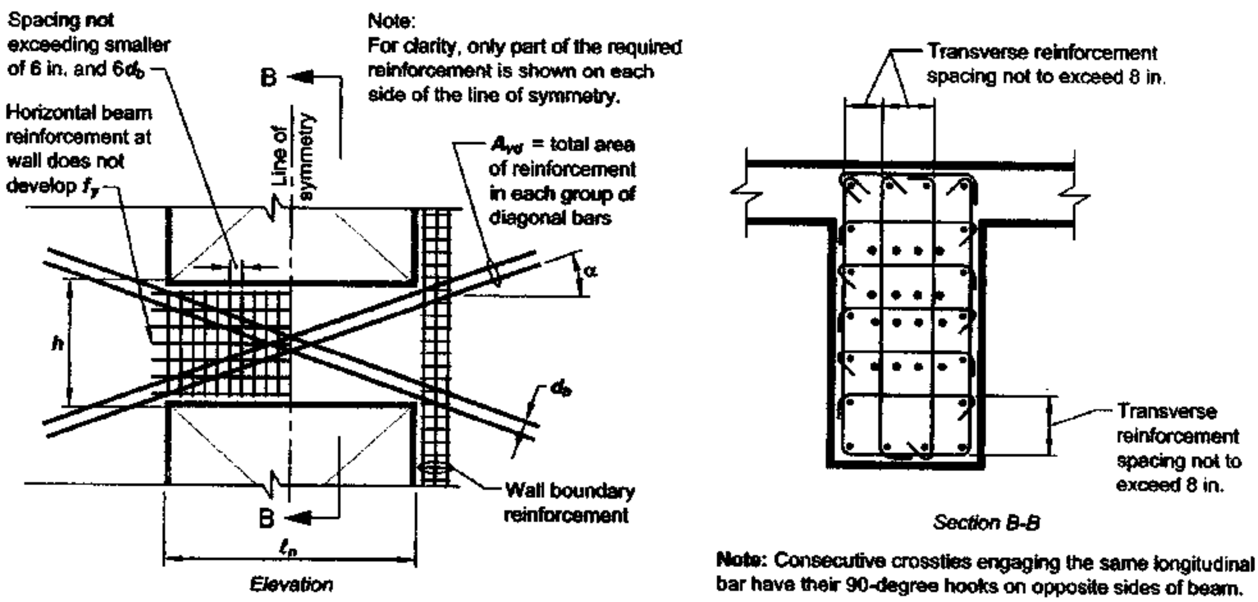
α = angle between the diagonal reinforcement and the longitudinal axis of a diagonally reinforced coupling beam

Transverse reinforcement for each group of diagonally placed bars should be designed as transverse reinforcement for the members of a special moment frame subjected to bending and axial force.

Detailing of coupling beam reinforcement should be in accordance with Fig. 20.32.



(a) Confinement of individual diagonals.
Note: For clarity in the elevation view, only part of the total required reinforcement is shown on each side of the line of symmetry.



(b) Full confinement of diagonally reinforced concrete beam section.

Figure 20.32 Reinforced detailing for coupling beams with diagonally oriented reinforcement. Wall boundary reinforcement shown on one side only for clarity. Courtesy of American Concrete Institute (ACI 2008).

Summary: Design of Special Structural Wall (Section 20.5.2.6).

Step 1. Determine minimum reinforcement ratio according to Section 20.5.2.1 and design horizontal and vertical reinforcement for wall web.

Step 2. Check the shear strength of the wall according to Section 20.5.2.2.

Step 3. Design the wall for flexure and axial force assuming that the wall behaves as a column and include all reinforcement in cross-section of the wall and reinforcement in boundary elements and web in calculations (Section 20.5.2.3).

Step 4. Check whether the boundary elements need to be specially detailed according to Section 20.5.2.4. If conditions are satisfied, design the transverse reinforcement of boundary elements by the provisions given for the special moment frame members subjected to bending and axial forces.

Step 5. Design the coupling beams as shown in Section 20.5.2.5

Example 20.8

Design the wall section given in Fig. 20.33 as a special structural wall.

Given: Forces are $Pu = 4000$ kip, $Mu = 45{,}000$ kip-ft, $Vu = 900$ kip; boundary elements are 24×24 in. columns; wall web thickness is 16 in.; wall length is 28 ft; wall height is 12 ft; normal-weight concrete with $f_c' = 4000$ psi; normal-weight concrete, and $f_y = 60{,}000$ psi. Boundary elements are reinforced with 16 no. 11 bars.

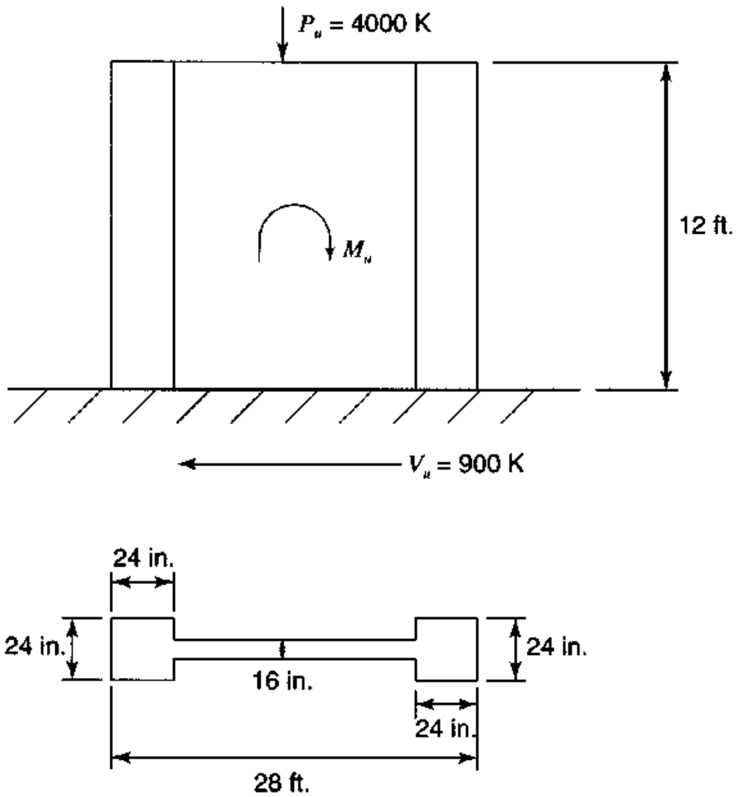


Figure 20.33 Example 20.8: structural wall.

Solution

1. Reinforcement requirements. To determine minimum reinforcement ratio check whether $V_u > A_{cv}\lambda\sqrt{f'_c}$

$$A_{cv} = 16 \times (28 \times 12) = 5376 \text{ in.}^2$$

$$A_{cv}\lambda\sqrt{f'_c} = 5376 \times 1 \times \sqrt{4000}/1000 = 340 \text{ kip} < V_u = 900 \text{ kip}$$

$$\Rightarrow \min \rho_l = \frac{A_{sv}}{A_{cv}} = \rho_n = 0.0025$$

Minimum reinforcement in both directions, longitudinal and transverse, per foot of wall can be determined as follows:

$$A_{cv} = 16 \times 12 = 192 \text{ in.}^2 \text{ per foot of wall}$$

$$A_s = 0.0025 \times 192 = 0.48 \text{ in.}^2/\text{ft}$$

Check whether two curtains of reinforcement are needed:

$$2A_{cv}\lambda\sqrt{f'_c} = 2 \times 1 \times 340 = 680 \text{ kip} < Vu = 900 \text{ kip}$$

Two curtains of reinforcement are required.

Choose no. 5 bars:

$$A_s = 2 \times (0.31) = 0.62 \text{ in.}^2$$

$$\text{Spacing}(s) = \frac{0.62}{0.48} \times 12 = 15.5 \text{ in} < 18 \text{ in.}$$

Choose $s = 15$ in. (See Fig. 20.34.)

2. Shear strength requirements. Check whether the two curtains of no. 5 bars spaced 15 in. on center can sustain applied shear force at the base. For $h_w/l_w = 12/28 = 0.43 < 1.5$,

$$\alpha_c = 3.0$$

$$\rho_t = \frac{0.62}{16 \times 15} = 0.0026$$

$$\phi V_n = \phi A_{cv}(\alpha_c\lambda\sqrt{f'_c} + \rho_t f_y)$$

$$= 0.75 \times 5376(3 \times 1 \times \sqrt{4000} + 0.0026 \times 60000)/1000 = 1394 \text{ kip} > 900 \text{ kip}$$

Two curtains of no. 5 bars spaced 15 in. center-to-center can sustain applied shear force at the base.

3. Design for flexure and axial force. Wall is designed as column subjected to axial load and bending.

$$Pu = 4000 \text{ kip}$$

$$Mu = 45{,}000 \text{ kip}$$

$$e = \frac{Mu}{Pu} = \frac{45,000}{4000} \times 12 = 135 \text{ in.}$$

$$M_n = \frac{M_u}{\phi} = \frac{45,000}{0.65} = 69{,}230 \text{ kip-ft}$$

$$Pn = \frac{Pu}{\phi} = \frac{4000}{0.65} = 6153 \text{ kip}$$

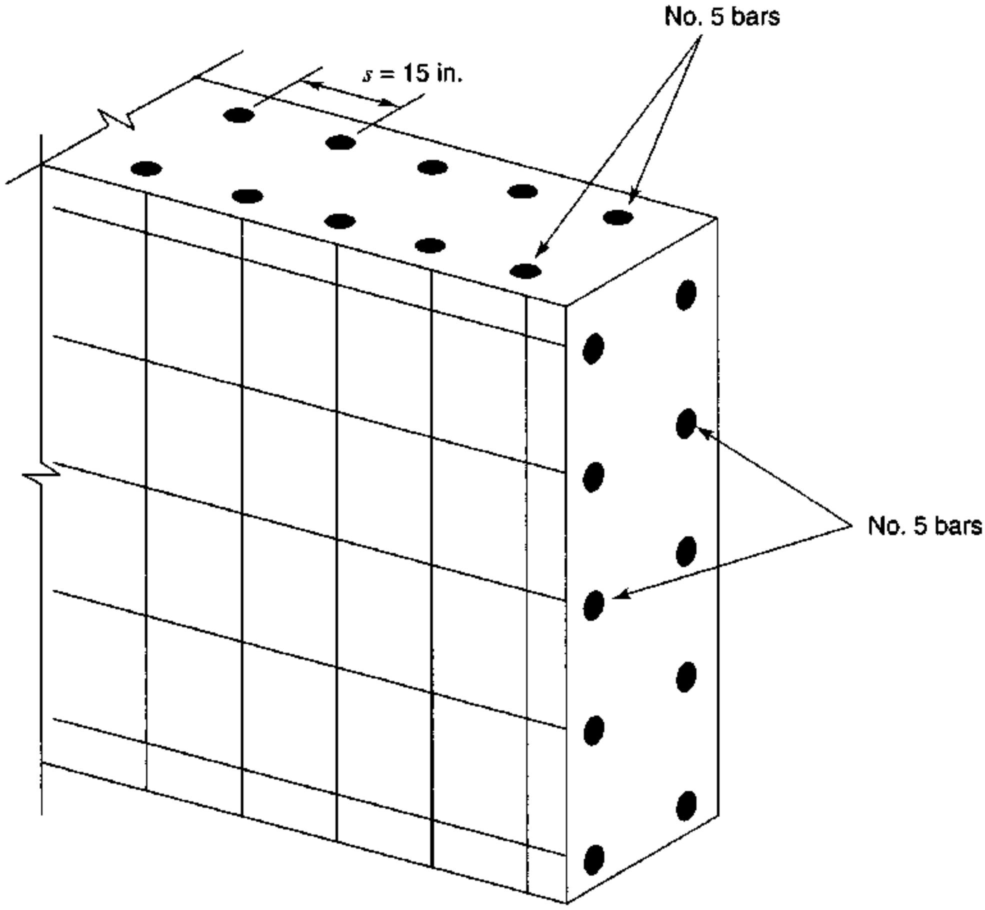


Figure 20.34 Example 20.8: reinforcement detailing of wall web.

Total area of reinforcement consists of 32 no. 11 bars in boundary elements and 40 no. 5 bars in the web.

$$A_s = 32 \times 1.56 + 40 \times 0.31 = 62.3 \text{ in.}^2$$

$$A_g = 5760 \text{ in.}^2$$

$$\rho = \frac{62.3}{5760} = 0.0109 > 0.01 \quad \text{and} \quad < 0.06 \text{ (o.k.)}$$

$$\frac{P_n}{f_c' A_g} = \frac{6{,}153{,}000}{4000 \times 5760} = 0.267$$

From the interaction diagram,

$$\frac{M_n}{f_c' A_g h} = 0.162$$

$$M_n = 0.162 \times 4000 \times 5760 \times 28 \times 12 = 104{,}509 \text{ kip-ft} > 69{,}230 \text{ kip-ft (o.k.)}$$

4. Special boundary elements requirements. The shear-based approach is used to determine whether the special boundary elements are required.

$$A_g = 5376 \text{ in.}^2$$

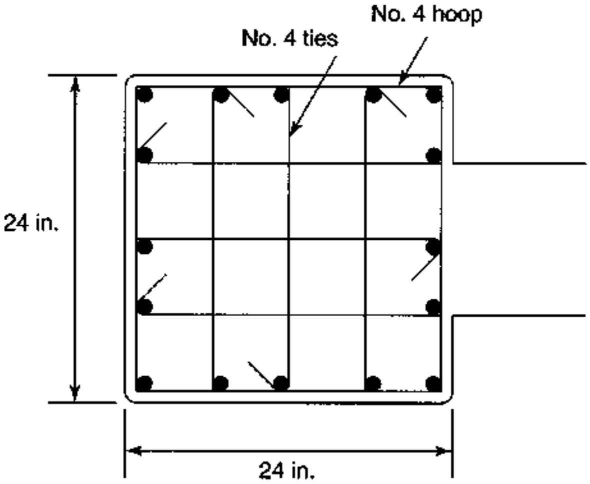


Figure 20.35 Example 20.8: boundary element reinforcement.

$$I_g = \frac{16 \times (28 \times 12)^3}{12} = 50{,}577{,}408 \text{ in.}^4$$

$$c = \frac{28 \times 12}{2} = 168 \text{ in.}$$

Maximum compressive stress in the wall

$$= \frac{P_u}{A_g} + \frac{M_u\, c}{I_g} = \frac{4{,}000{,}000}{5376} + \frac{45{,}000{,}000 \times 12}{50{,}577{,}408}168 = 2538 \text{ psi}$$

$$0.2 f_c' = 0.2 \times 4000 = 800 \text{ psi} < 2538 \text{ psi}$$

A special boundary element is needed. Transverse reinforcement of boundary element should be designed as for members of special moment frame subjected to axial load and bending (Fig. 20.35).

Use no. 4 hoops and crossties around longitudinal bars in both directions. Maximum spacing of transverse reinforcement should be determined as follows:

$$s_{\max} = \begin{cases} 0.25 \times (\text{smallest member dimension}) = 0.25 \times 24 = 6 \text{ in.} \\ 6 \times (\text{diameter of longitudinal bar}) = 6 \times 1.41 = 8.5 \text{ in.} \\ s_x = 4 + \left(\dfrac{14 - h_x}{3}\right) = 4 + \left(\dfrac{14 - 6}{3}\right) = 6.7 > 6 \end{cases}$$

Use $s = 6$ in. (governs).

Required cross-section area:

$$A_{sh} = \frac{0.09 s h_c f_c'}{f_y} = \frac{0.09 \times 6 \times [24 - (2 \times 1.5) - 0.5] \times 4}{60} = 0.738 \text{ in.}^2$$

$$A_{sh} = 0.3 s h_c \left[\frac{A_g}{A_{ch}} - 1\right] \frac{f_c'}{f_{yh}} = 0.3 \times 6 \times 20.5 \left(\frac{576}{420.25} - 1\right) \frac{4}{60} = 0.911 \text{ in.}^2 \quad \text{(governs)}$$

No. 4 hoops with crossties around every longitudinal bar provide

$$A_{sh} = 5 \times 0.2 = 1.0 \text{ in.}^2 > 0.911 \text{ in.}^2 \quad \text{(o.k.)}$$

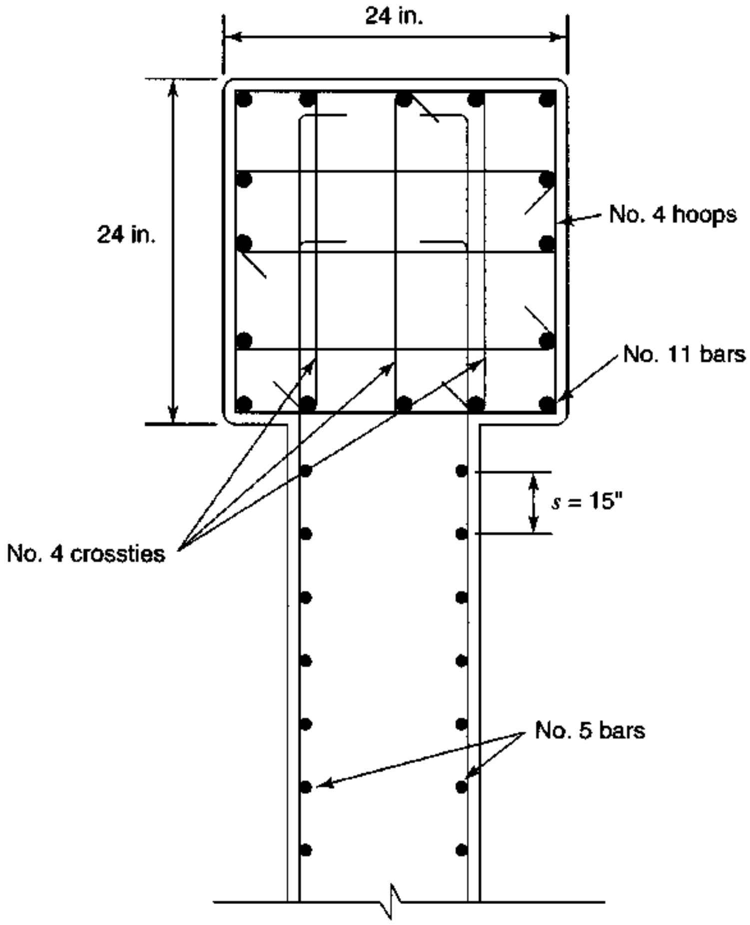


Figure 20.36 Example 20.8: reinforcement detailing.

Development length of no. 5 bars assuming that the hooks are used (ACI Section 21.7.5.1) is

$$l_{\text{dh}} \geq \begin{cases} \dfrac{f_y d_b}{65\sqrt{f_c'}} = \dfrac{60000 \times 0.625}{65\sqrt{4000}} = 9 \text{ in.} \\ 8d_b = 8 \times 0.625 = 5 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

Therefore, $l_{dh} = 9\,\text{in}$. $l_d = 3.5 l_{dh} = 3.5 \times 9 = 32\,\text{in}. >$ the dimension of boundary element $= 24\,\text{in}$. Use the hooks to anchor reinforcement (Fig. 20.36).

20.5.3 Structures in the Areas of Moderate Seismic Risk: Intermediate Moment Frames (ACI 2008, Section 21.3)

In regions of moderate seismic risk (SDC C), the moment frames should be designed as intermediate moment frames. The ACI Code, ACI Section 21.1.1.5, gives provisions for the design and detailing of intermediate moment frames as follows:

Longitudinal Reinforcement Requirements (Section 20.5.3.1). If the compressive axial load for the member $< A_g f_c'/10$, the member is considered to be subjected only to bending and the following is applicable (ACI 2008, Section 21.3.2):

Positive moment strength at joint face ≥ $\frac{1}{3}$ negative moment strength at that face of the joint.

$$M_{\text{nl}}^{+} \geq \frac{1}{3} M_{\text{nl}}^{-} \qquad \text{(left joint)} \tag{20.57a}$$

$$M_{\text{nr}}^{+} \geq \frac{1}{3} M_{\text{nr}}^{-} \qquad \text{(right joint)} \tag{20.57b}$$

Neither the positive nor the negative moment strength at any section along the length of the member should be less than $\frac{1}{5}$ the maximum moment strength provided at the face of either joint.

$$(\phi M_n^{+} \text{ or } \phi M_n^{-}) \geq \frac{1}{5} \max(\phi M_n \text{ at either joint}) \tag{20.58}$$

Transverse Reinforcement Requirements (Section 20.5.3.2).

Beams. It is assumed that the plastic hinges will form at the end of the beams. According to this, the beam ends should be specially detailed to provide the beam with necessary ductility.

Hoops should also be provided over a length equal to $2d$ (d is the effective depth of the beam) measured from the face of support towards midspan. The first hoop should be located at distance ≤ 2 in. from the face of support.

Maximum spacing of transverse reinforcement should not exceed the smallest of:

$$S_{\max} \leq \begin{cases} \dfrac{d}{4} \\ 8 \times \text{(diameters of the smallest longitudinal bar enclosed)} \\ 24 \times \text{(diameter of the hoop bar)} \\ 12 \text{ in.} \end{cases} \tag{20.59}$$

When hoops are not required, stirrups should be used. Spacing of stirrups should be ≤ $d/2$ through the length of the member (ACI Section 21.3.4.3).

Columns. Transverse reinforcement of columns of intermediate moment frame should be designed with spiral reinforcement or with hoops and stirrups as follows: Spiral reinforcement should satisfy requirements for ordinary compression member (ACI 7.10.4): Hoops should be provided at both ends of the member over a length l_o measured from the face of the joint, spaced a distance s_o. (ACI Section 21.3.5.2). Spacing S_o shall not exceed the smallest of the four items listed below or

$$s_o \leq \begin{cases} 8 \times \text{(diameter of the smallest longitudinal bar)} \\ 24 \times \text{(diameter of the hoop bar)} \\ \dfrac{1}{2} \text{of the smallest cross-section dimension of the member 12} \end{cases} \tag{20.60}$$

Length l_o shall not be less than the largest of the three items listed below or

$$l_o \geq \begin{cases} \dfrac{1}{6} \text{ of the clear length of the member} \\ \text{Maximum cross-section dimension of the member} \\ 18 \text{ in.} \end{cases} \tag{20.61}$$

The first hoop should be located at distance $\leq s_o/2$ from the joint face. Outside the length l_o, spacing s_o should confirm to ACI Section 7.10 and ACI Section 11.4.5.1 or

$$s_o \leq \begin{cases} \dfrac{d}{2} \\ 24 \text{ in.} \end{cases} \tag{20.62}$$

CODE AND DESIGN REFERENCES

1. American Concrete Institute. "Building Code Requirements for Structural Concrete." ACI 318-08 and Commentary ACI 318R-08. Farmington Hills, Michigan, 2008.
2. International Code Council. "International Building Code 2006." Country Club Hills, Illinois.
3. S. K. Ghosh, and A. David, Fanella "Seismic and Wind Design of Concrete Buildings" 2000 IBC, ASCE 7-98, ACI 318-99. Country Club Hills, Illiois, 2003 International Code Council.
4. S. K., Ghosh August W. Domel Jr., and A. David Fanella. *Design of Concrete Buildings for Earthquake and Wind Forces*, 2d ed., Skokie, Illinois, Portland Cement Association, 1995.
5. Portland Cement Association. "Notes on ACI 318-02 Building Code Requirements for Structural Concrete with Design Applications." Skokie, Illinois, 2002.
6. ASCE 7-05, "Minimum Design Loads for Buildings and other Structures." American Society of Civil Engineering, 2005.

PROBLEMS

20.1 Determine seismic design category for a five-story building in the area of northern California if the soil is hard rock.

20.2 Determine base shear for a two-story building located in the area of high seismic risk where $S_S = 1.3\,g$ and $S_1 = 0.6\,g$, on soil class B. Assume that the idealized weight of the first floor is 50 kip and of the second floor is 60 kip.

20.3 Determine lateral seismic forces for the five-story building assuming that the idealized mass of each floor is 1000 kip. Consider the structure a building occupancy category III, site class C.

20.4 Design the longitudinal reinforcement for the beam on the second floor of a special-moment frame four-story building assuming the clear span of a beam is 24 feet. Each story height is 12 feet. Beam dimensions are 20 × 24 in., and the column is 24 × 24 in. Bending moments acting on the beam are given in the following table.

Load	Location	Bending Moment (kip-ft)
Dead	Support	−70
	Midspan	45
Live	Support	25
	Midspan	18
Earthquake	Support	±180
	Midspan	0

20.5 Design the transverse reinforcement for the beam of special moment-resisting frame. The beam is reinforced with five no. 8 bars and is 24 × 30 in. Load acting on the beam is $W_D = 3.0$ kip/ft, $W_L = 1.5$ kip/ft, and clear span is 24 ft.

20.6 Design the reinforcement for a column on the first floor of four-story building following the provisions for special moment-resisting frame reinforcement. The column is 30 × 30 in. and 12 ft high. Nominal flexural strength of the beam framing into the column $M_n = 650\,\text{k} \cdot \text{ft}$. Axial load acting on the on the second-floor column is $P_u = 1920$ kip, axial load acting on the first-floor is $P_u = 2000$ kip, and minimum axial load in load combination is 1010 kip. The shear force is $V_u = 120$ kip. Draw the detail of reinforcement.

20.7 Design the reinforcement for a wall having a total height of 28 ft and span of 35 ft. Total gravity load acting on the wall is 5200 kip, factored moment $(M_u) = 50{,}000$ kip-ft, and base shear is $V = 1000$ kip. Wall thickness is 20 in. and boundary elements are 25 × 25 in.

CHAPTER 21

BEAMS CURVED IN PLAN

Curved beams in an office building.

21.1 INTRODUCTION

Beams curved in plan are used to support curved floors in buildings, balconies, curved ramps and halls, circular reservoirs, and similar structures. In a curved beam, the center of gravity of the loads acting normal to the plane of curvature lies outside the line joining its supports. This situation develops torsional moments in the beam, in addition to bending moments and shearing forces. To maintain the stability of the beam against overturning, the supports must be fixed or continuous. In this chapter, the design of curved beams subjected to loads normal to the plane of curvature is presented. Analysis of curved beams subjected to loads in the plane of curvature is usually discussed in books dealing with mechanics of solids.

Analysis of beams curved in plan was discussed by Wilson [1]. He introduced formulas and coefficients to compute stresses in curved flexural members. Timoshinko [2], [3] also introduced several expressions for calculating bending stresses in square and rectangular sections. Tables and formulas for the calculation of bending and torsional moments, shear, and deflections for different cases of loadings on curved beams and rings are presented by Roark and Young [4].

21.2 UNIFORMLY LOADED CIRCULAR BEAMS

The first case to be considered here is that of a circular beam supported on columns placed at equal distances along the circumference of the beam and subjected to normal loads. Due to

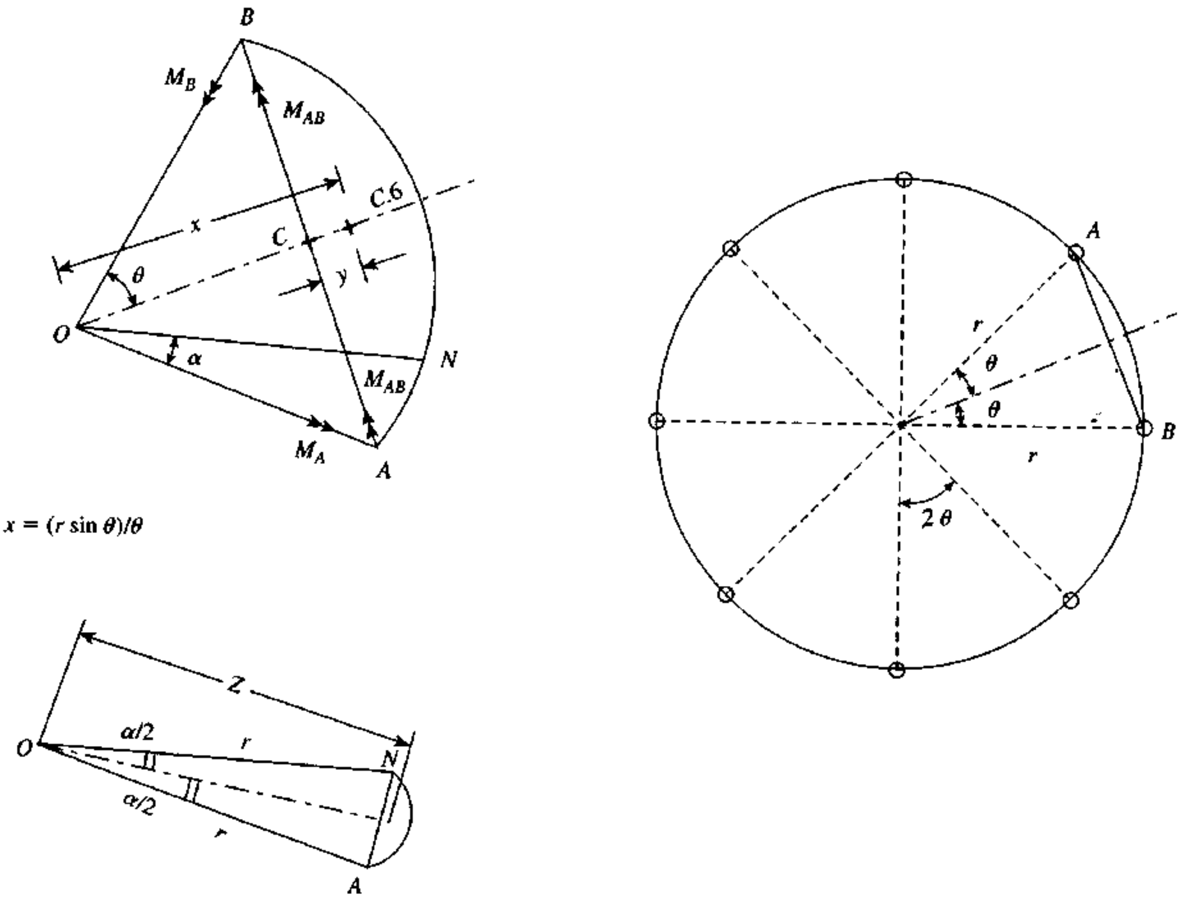


Figure 21.1 Circular Beam.

symmetry, the column reactions will be equal, and each reaction will be equal to the total load on the beam divided by the number of columns. Referring to Fig. 21.1, consider the part AB between two consecutive columns of the ring beam. The length of the curve AB is $r(2\theta)$, and the total load on each column is $P_u = w_u r(2\theta)$, where r is the radius of the ring beam and w_u is the factored load on the beam per unit length. The center of gravity of the load on AB lies at a distance

$$x = \left(\frac{r\ \sin\theta}{\theta}\right)$$

from the center O. The moment of the load P_u about AB is

$$M_{AB} = P_u \times y = P_u(x - r\ \cos\theta) = w_u r(2\theta)\left(\frac{r\ \sin\theta}{\theta} - r\ \cos\theta\right)$$

Consequently, the two reaction moments, M_A and M_B, are developed at supports A and B, respectively. The component of the moment at support A about AB is $M_A\ \sin\theta = M_B\ \sin\theta$. Equating the applied moment, M_{AB}, to the reaction moments components at A and B,

$$2M_A\ \sin\theta = M_{AB} = w_u r(2\theta)\left(\frac{r\ \sin\theta}{\theta} - r\ \cos\theta\right) \tag{21.1}$$

$$M_A = M_B = w_u r^2(1 - \theta\ \cot\theta)$$

The shearing force at support A is

$$V_A = \frac{P_u}{2} = w_u r\theta \tag{21.2}$$

The shearing force at any point N, V_N, is $V_A - w_u\ (r\alpha)$, or

$$V_N = w_u r(\theta - a) \tag{21.3}$$

The load on AN is $w_u\ (r\alpha)$ and acts at a distance equal to

$$Z = \frac{r\ \sin\alpha/2}{\alpha/2}$$

from the center 0. The bending moment at point N on curve AB is equal to the moment of all forces on one side of o about the radial axis ON.

$$M_N = V_A(r\ \sin\alpha) - M_A\ \cos\alpha - (\text{load onthe curve } AN)\left(Z\ \sin\frac{\alpha}{2}\right)$$

$$M_N = w_u r\theta(r\ \sin\alpha) - w_u r^2(1-\theta\ \cot\theta)\cos\alpha$$

$$- (w_u r\alpha)\left(\frac{r\ \sin\alpha/2}{\alpha/2} \times \sin\frac{\alpha}{2}\right)$$

$$= w_u r^2\left[\theta\ \sin\alpha - \cos\alpha + (\theta\ \cot\theta\ \cos\alpha) - 2\ \sin^2\frac{\alpha}{2}\right]$$

$$M_N = w_u r^2[\theta\ \sin\alpha + (\theta\ \cot\theta\ \sin\alpha) - 1] \tag{21.4}$$

(Note that $\cos\alpha = 1 - 2\sin^2\alpha/2$.) The torsional moment at any point N on curve AB is equal to the moment of all forces on one side of N about the tangential axis at N.

$$T_N = M_A\ \sin\alpha - V_A \times r(1-\cos\alpha) + w_r r\alpha\left(r - \frac{r\ \sin\alpha/2}{\alpha/2} \times \cos\alpha/2\right)$$

$$= w_u r^2(1-\theta\ \cot\theta)\sin\alpha - w_u r^2\theta(1-\cos\alpha) + w_u r^2(\alpha - \sin\alpha)$$

$$T_n = w_u r^2(\alpha - \theta + \theta\ \cos\alpha - \theta\ \cot\theta\sin\alpha) \tag{21.5}$$

To obtain the maximum value of the torsional moment T_N, differentiate Eq. 21.5 with respect to α and equate it to 0. This step will give the value of α for maximum T_N.

$$\sin\alpha = \frac{1}{\theta}\left[\sin^2\theta \pm \cos\theta\sqrt{\theta^2 - \sin^2\theta}\right] \tag{21.6}$$

The values of the support moment, midspan moment, the torsional moment, and its angle α from the support can be calculated from Eqs. 21.1 through 21.6. Once the number of supports n is chosen, the angle θ is known,

$$2\theta = \frac{2\pi}{n} \quad \text{and} \quad \theta = \frac{\pi}{n}$$

and the moment coefficients can be calculated as shown in Table 21.1. Note that the angle α is half the central angle between two consecutive columns.

$$\text{Load on each column is } P_u = w_u r(2\theta) = w_u r\left(\frac{2\pi}{n}\right)$$

$$\text{Maximum shearing force is } V_u = \frac{P_u}{2}$$

Table 21.1 Force Coefficients of Circular Beams

Number of Supports, n	$\theta = \frac{\pi}{n}$	K_1	K_2	K_3	$\alpha°$ for T_u (max)
4	90	0.215	0.110	0.0330	19.25
5	72	0.136	0.068	0.0176	15.25
6	60	0.093	0.047	0.0094	12.75
8	45	0.052	0.026	0.0040	9.50
9	40	0.042	0.021	0.0029	8.50
10	36	0.034	0.017	0.0019	7.50
12	30	0.024	0.012	0.0012	6.25

$$\text{Negative moment at any support} = K_1 w_u r^2 \quad (21.7)$$

$$\text{Positive moment at midspan} = K_2 w_u r^2 \quad (21.8)$$

$$\text{Maximum torsional moment} = K_3 w_u r^2 \quad (21.9)$$

The variation of the shearing force and bending and torsional moments along a typical curved beam AB are shown in Fig. 21.2.

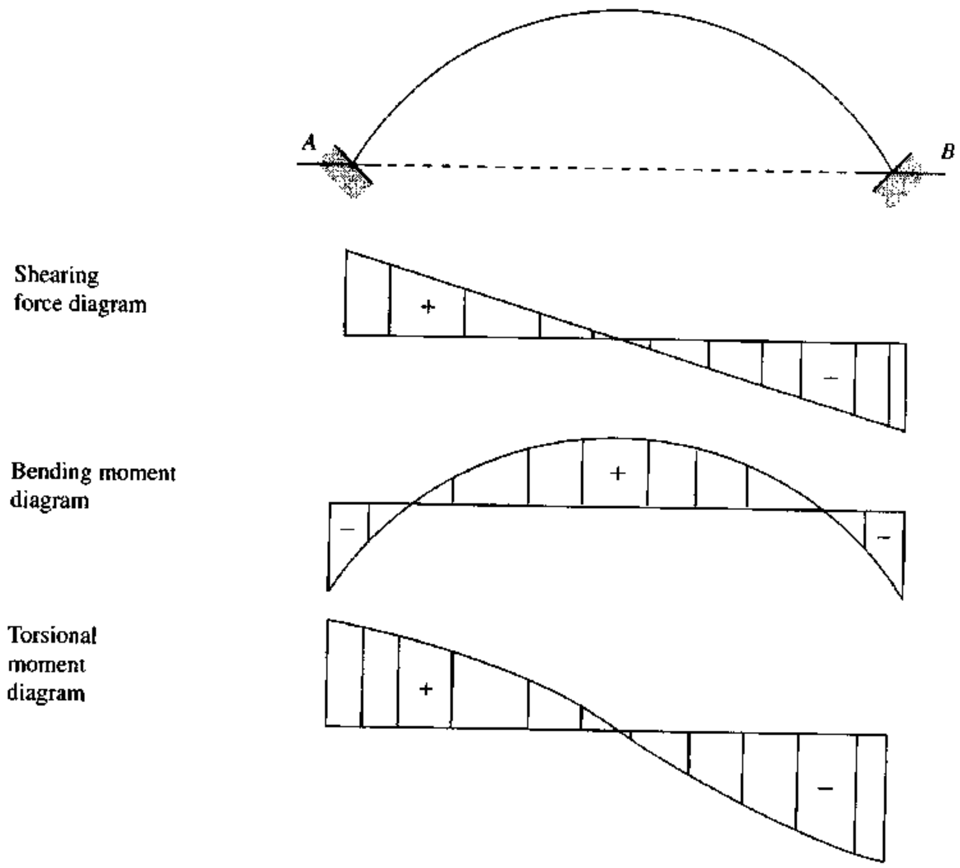


Figure 21.2 Forces in a circular beam.

Example 21.1

Design a circular beam supported on eight equally spaced columns. The centerline of the columns lies on a 40-ft-diameter circle. The beam carries a uniform dead load of 6 K/ft and a live load of 4 K/ft. Use normal-weight concrete with $f'_c = 5$ Ksi, $f_y = 60$ Ksi, and b = 14 in.

Solution

1. Assume a beam size of 14 × 24 in. The weight of the beam is

$$\frac{14 \times 24}{12 \times 12}(0.150) = 0.35 \text{ K/ft}$$

The factored uniform load is $w_u = 1.2(6 + 0.35) + 1.6(5) = 15.7$ K/ft.

2. Because the beam is symmetrically supported on eight columns, the moments can be calculated by using Eqs. 21.7 through 21.9 and Table 21.1 Negative moment at any support is $K_1\, w_u\, r^2 = 0.052(15.7)(20)^2 = 326.6$ K·ft. The positive moment at midspan is $K_2\, w_u\, r^2 = 0.216(15.7)(20)^2 = 163.3$ K·ft. The maximum torsional moment is $K_3\, w_u\, r^2 = 0.004(15.7)(20)^2 = 25.12$ K·ft. Maximum shear is

$$V_u = \frac{P_u}{2} = \frac{w_u r}{2}\left(\frac{2\pi}{n}\right) = (15.7)(20)\left(\frac{\pi}{8}\right) = 123.3 \text{ K}$$

3. For the section at support, $M_u = 326.6$ K ft. Let $d = 21.5$ in.; then

$$R_u = \frac{M_u}{bd^2} = \frac{326.6 \times 12{,}000}{14(21.5)^2} = 605 \text{ psi}$$

For $f'_c = 4$ Ksi and $f_y = 60$ Ksi, $\rho = 0.0126 < \rho_{max} = 0.018$, $\phi = 0.9$

$$A_s = 0.0126 \times 14 \times 21.5 = 3.8 \text{ in.}^2$$

4. For the section at midspan, $M_u = 163.3$ K·ft.

$$R_u = \frac{163.3 \times 12{,}000}{14(21.5)^2} = 303 \text{ psi}$$

$$\rho = 0.006 \quad \text{and} \quad A_s = 0.006 \times 14 \times 21.5 = 1.81 \text{ in.}^2$$

Use two no. 9 bars.

5. Maximum torsional moment is $T_u = 25.12$ K·ft, and it occurs at an angle $\alpha = 9.5°$ from the support (Table 21.1). Shear at the point of maximum torsional moment is equal to the shear at the support minus $w_u\, r\alpha$.

$$V_u = 123.3 - 15.7(20)\left(\frac{9.5}{180} \times \pi\right) = 71.24 \text{ K}$$

The procedure for calculation of the shear and torsional reinforcement for $T_u = 25.12$ K·ft and $V_u = 71.24$ K is similar to Example 15.2

a. Shear reinforcement is required when $V_u > \phi\, V_c/2$.

$$\phi V_c = 2\phi\lambda\sqrt{f_c}bd = 2(0.75)(1.0)\sqrt{4000}(14 \times 21.5) = 28.6 \text{ K}$$

$$\text{since } \phi V_c/2 = 14.3 \text{ K} < V_u = 71.24 \text{ K}$$

Shear reinforcement is required.

b. Torsional reinforcement is required when

$$T_u > T_a = \phi\lambda\sqrt{f'_c}\left(\frac{A_{cP}^2}{P_{cp}}\right)$$

$$A_{cp} = x_0 y_0 = 14 \times 24 = 336 \text{ in.}^2$$

$$P_{cp} = 2(x_0 + y_0) = 2(14 + 24) = 76 \text{ in.}$$

$$T_a = 0.75 \times 1 \times \sqrt{4000}\left(\frac{336^2}{76}\right) = 70.5 \text{ K·in.}$$

$$\text{since } T_u = 25.12 \text{ K·ft} = 301.4 \text{ K·in.} > T_a$$

Therefore torsional reinforcement is required.

c. Design for shear:

i. $V_u = \phi V_c + \phi V_s$ and $\phi V_c = 28.6\,\text{K}.71.24 = 28.6 + 0.75V_s$, so $V_s = 56.8\,\text{K}$

ii. Maximum $V_s = 8\sqrt{f'_c}bd = 8\sqrt{4000}(14 \times 21.5) = 152.3 \text{ K} > V_u$

iii. $A_v/S = V_s/f_y d = 56.8/(60 \times 21.5) = 0.044 \text{ in.}^2/\text{in.}$ (2 legs)
$A_v/2S = 0.022 \text{ in.}^2/\text{in.}$(one leg)

d. Design for torsion:

i. Choose no. 4 stirrups and a 1.5-in. concrete cover:

$$x_1 = 14 - 3.5 = 10.5 \text{ in.}, \quad y_1 = 24 - 3.5 = 20.5 \text{ in.}$$

$$A_{oh} = x_1 y_1 = 10.5(20.5) = 215.25 \text{ in.}^2$$

$$A_o = 0.85 A_{oh} = 183 \text{ in.}^2$$

$$p_h = 2(x_1 + y_1) = 2(10.5 + 20.5) = 62 \text{ in.}$$

For $\theta = 45^\circ$, $\cot\theta = 1.0$.

ii. Check the adequacy of the size of the section using Eq. 15.21:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right)$$

$$\phi V_c = 28.6\text{K}, \qquad V_c = 38.12 \text{ K}$$

$$\text{Left-hand side} = \sqrt{\left(\frac{71{,}240}{14 \times 21.5}\right)^2 + \left[\frac{301{,}400 \times 62}{1.7(215.25)^2}\right]^2} = 335 \text{ psi}$$

$$\text{Right-hand side} = 0.75\left(\frac{38{,}120}{14 \times 21.5} + 8\sqrt{4000}\right) = 558 \text{ psi} > 335 \text{ psi}$$

The section is adequate.

iii. Determine the required closed stirrups due to T_u from:

$$\frac{A_t}{S} = \frac{T_n}{2A_o f_y \cot\theta}, \; T_n = T_n/\phi, \; \phi = 0.75, \; \cot\theta = 1.0$$

$$= \frac{301.4}{0.75 \times 2 \times 183 \times 60} = 0.0183 \text{ in.}^2/\text{in.} \quad \text{(one leg)}$$

iv. The total area of one leg stirrup is $0.022 + 0.0183 = 0.04\ \text{in.}^2/\text{in.}$ For no. 4 stirrups, area of one leg $= 0.2\ \text{in.}^2$ Spacing of closed stirrups is $0.2/0.04 = 5.0$ in., say, 5.5 in.

$$\text{Minimum } S = \frac{p_h}{8} = \frac{62}{8} = 7.75 \text{ in.} > 5.0 \text{ in.}$$

$$\text{Minimum } \frac{A_{vt}}{S} = \frac{50b_w}{f_y} = \frac{50(14)}{60{,}000} = 0.0117 \text{ in.}^2/\text{in.}$$

This is less than the A_t/s provided. Use no. 4 closed stirrups spaced at 5.5 in.

e. Longitudinal bars A_l equal $(A_t/s)\ p_h\ (f_{yv}/f_{yl}) \cot^2\theta$ (Eq. 15.27).

$$A_l = 0.018(62)\left(\frac{60}{60}\right) = 1.13 \text{ in.}^2$$

$$\text{Min. } A_l = \frac{5\sqrt{f'_c}A_{cp}}{f_{yl}} - \left(\frac{A_t}{S}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right)$$

$$= \frac{(5\sqrt{4000})(336)}{60{,}000} - 0.018(62)\left(\frac{60}{60}\right) = 0.64 \text{ in.}^2 < 1.0$$

Use $A_l = 1.13\ \text{in.}^2$, with one-third at the top, one-third at middepth, and one-third at the bottom, or $0.33\ \text{in.}^2$ in each location. For the section at the support, $A_s = 3.8\ \text{in.}^2 + 0.38 = 4.18\ \text{in.}^2$ Choose two no. 10 and two no. 9 bars ($A_s = 4.53\ \text{in.}^2$) as top bars. At middepth, use two no. 4 bars ($A_s = 0.4\ \text{in.}^2$). Extend two no. 9 bars of the midspan section to the support. At middepth use two no. 4 bars ($A = 0.4\ \text{in.}^2$). Details of the section are shown in Fig. 21.3.

Circular beams in an office building.

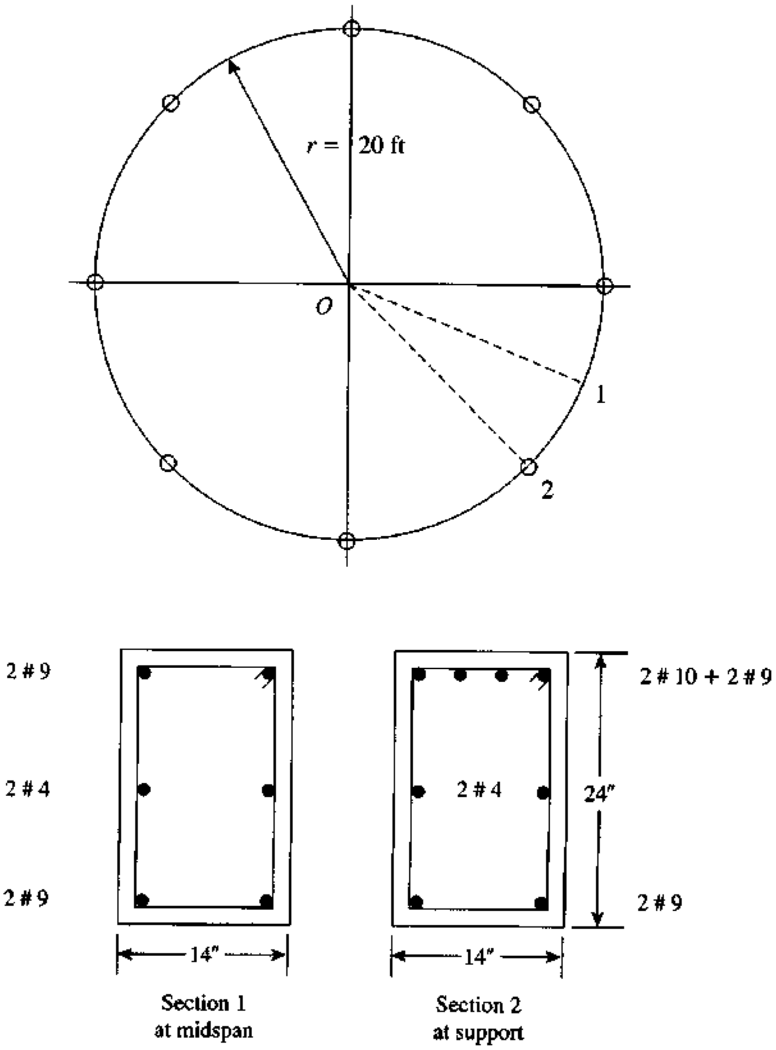


Figure 21.3 Example 21.1.

21.3 SEMICIRCULAR BEAM FIXED AT END SUPPORTS

If a semicircular beam supports a concrete slab, as shown in Fig. 21.4, the ratio of the length to the width of the slab is $2r/r = 2$, and the slab is considered a one-way slab. The beam will be subjected to a distributed load, which causes torsional moments in addition to the bending moments and shearing forces. The structural analysis of the curved beam can be performed in steps as follows.

1. Load on beam: The load on the curved beam will be proportional to its distance from the support AB. If the uniform load on the slab equals w psf, the load on the curved beam at any section N is equal to half the load on the area $NCDE$ (Fig. 21.4). The lengths are $CN = r\sin\theta$, $OC = r\cos\theta$, and $CD = (d/d\theta)(r\cos\theta) = (r\sin\theta\, d\theta)$, and the arc NE is $r\,d\theta$. The load on the curved beam per unit length is equal to

$$w' = \frac{w(r\sin\theta)r\sin\theta\, d\theta}{2(r\,d\theta)} = \frac{wr\sin^2\theta}{2} \tag{21.10}$$

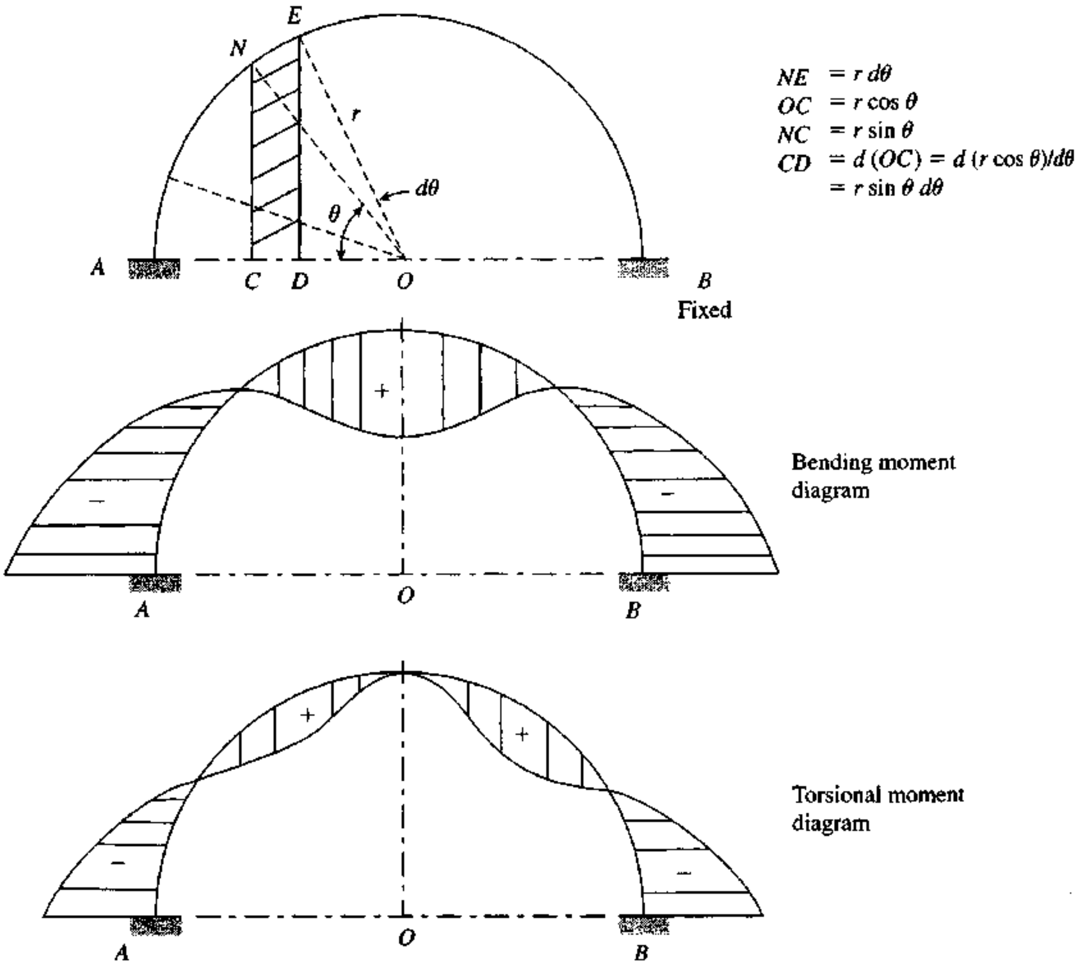


Figure 21.4 Semicircular beam fixed at the supports.

2. Shearing force at A: For a uniform symmetrical load on the slab, the shearing force at A is equal to

$$V_A = V_B = \int_0^{\pi/2} \left(\frac{wr}{2}\sin^2\theta\right)(r\,d\theta) = \frac{wr^2}{2}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right] \tag{21.11}$$

$$= \left(\frac{\pi}{8}\right) wr^2 = 0.39wr^2$$

3. Bending moment at A: Taking moments about the line AB, the bending moment at A is equal to

$$M_A = M_B = \int_0^{\pi/2} w'(r\,d\theta) \times (r\,\sin\theta) \tag{21.12}$$

$$= \int_0^{\pi/2} \left(\frac{wr}{2}\sin^2\theta\right)(r\,\sin\theta)(r\,d\theta) = -\frac{wr^3}{3}$$

4. Torsional moment at support A: T_A can be obtained by differentiating the strain energy of the beam with respect to T_A and equating it to 0. Considering that T_A is acting clockwise

at A, then the bending moment at any section N is calculated as follows:

$$M_N = V_A(r \sin\theta) - M_A \cos\theta + T_A \sin\theta - \int_0^{\theta} \left(\frac{wr}{2}\sin^2\theta\right)(r\,d\alpha) \times r \sin(\theta - \alpha)$$

$$M_N = wr^3\left[\frac{\pi}{8}\sin\theta - \left(\frac{1}{6}\right)(1+\cos^2\theta)\right] + T_A \sin\theta \tag{21.13}$$

The torsional moment at any station N on the curved beam is equal to

$$T_n = -V_A r(1-\cos\theta) + M_A \sin\theta + T_A \cos\theta + \int_0^{\pi/2} \left(\frac{wr}{2}\sin^2\alpha\right)(r\,d\alpha)$$
$$\times\, r[1-\cos(\theta - \mathrm{a})]$$

$$T_N = wr^3\left[\frac{\pi}{8}(\cos\theta - 1) + \frac{\theta}{4} + \frac{1}{24}\sin 2\theta\right] + T_A \cos\theta \tag{21.14}$$

The strain energy is

$$U = \int \frac{M_N^2\,ds}{2\mathrm{EI}} + \int \frac{T_N^2\,ds}{2\mathrm{GJ}} \tag{21.15}$$

where

$ds = r\,d\theta$
G = modulus of rigidity
E = modulus of elasticity
I = moment of inertia of the section
J = rotational constant of the section
= polar moment of inertia

To obtain T_A, differentiate U with respect to T_A

$$\frac{\delta_U}{\delta T_A} = \int \frac{M_N}{EI} \times \frac{dM_N}{dT_A}(r\,d\theta) + \int \frac{T_N}{GJ} \times \frac{dT_N}{dT_A} \times (r\,d\theta) = 0$$

$$\frac{dM_N}{dT_A} = \sin\theta \quad \text{and} \quad \frac{dT_N}{dT_A} = \cos\theta$$

Therefore,

$$\frac{\delta_U}{\delta T_A} = \frac{r}{EI}\int_0^{\pi/2} \sin\theta\left\{wr^2\left[\frac{\pi}{8}\sin\theta - \frac{1}{6}(1+\cos^2\theta)\right] + T_A \sin\theta\right\} d\theta$$
$$+ \frac{r}{GJ}\int_0^{\pi/2}\left\{wr^3\left[\frac{\pi}{8}(\cos\theta - 1) + \frac{\theta}{4} + \frac{1}{24}\sin 2\theta\right] + T_A \cos\theta\right\}\cos\theta \times d\theta = 0$$

$$\frac{r}{EI}\left[wr^3\left(\frac{\pi^2}{32} - \frac{2}{9}\right) + T_A\left(\frac{\pi}{4}\right)\right] + \frac{r}{GJ}\left[wr^3\left(\frac{\pi^2}{32} - \frac{2}{9}\right) + T_A\left(\frac{\pi}{4}\right)\right] = 0$$

Let $EI/GJ = \lambda$; then

$$T_A\left(\frac{\pi}{4}\right)(1+\lambda) = wr^3\left[\left(\frac{2}{9}-\frac{\pi^2}{32}\right)+\lambda\left(\frac{2}{9}-\frac{\pi^2}{32}\right)\right]$$

$$= wr^3\left(\frac{2}{9}-\frac{\pi^2}{32}\right)(1+\lambda) = -0.0862wr^3(1+\lambda)$$

Therefore,

$$T_A = -0.11wr^3 \qquad (21.16)$$

Substituting the value of T_A in Eq. 21.13, the bending moment at any point N is equal to

$$M_N = wr^3\left[\frac{\pi}{8}\sin\theta - \frac{1}{6}(1+\cos^2\theta) - 0.11\ \sin\theta\right] \qquad (21.17)$$

Substituting the value of T_A in Eq. 21.14,

$$T_N = wr^3\left[\frac{\pi}{8}(\cos\theta - 1) + \frac{\theta}{4} + \frac{1}{24}\sin 2\theta - 0.11\ \cos\theta\right] \qquad (21.18)$$

5. The value of G/E for concrete may be assumed to be equal to 0.43. The value of J for a circular section is $(\pi/2)r^4$, whereas J for a square section of side x is equal to $0.141x^4$. For a rectangular section with short and long sides x and y, respectively, J can be calculated as follows:

$$J = K' \times y^3 \qquad (21.19)$$

The values of K' are calculated as follows:

$$K' = \frac{1}{16}\left[\frac{16}{3} - 3.36\frac{x}{y}\left(1-\frac{x^4}{12y^4}\right)\right] \qquad (21.20)$$

whereas

$$\lambda = \frac{EI}{GJ} = \left(\frac{1}{0.43}\right)\left(\frac{xy^3}{12}\right)\left(\frac{1}{K'yx^3}\right) = \frac{1}{5.16K'}\left(\frac{y}{x}\right)^2$$

Values of K' and λ are both shown in Table 21.2.

Example 21.2

Determine the factored bending and torsional moments in sections C and D of the 10-ft-radius semicircular beam $ADCB$ shown in Fig. 21.5. The beam is part of a floor slab that carries a uniform factored load of 304 psf (including self-weight).

Table 21.2 Values of K' and λ for Different Values of y/x

y/x	0.5	1.0	1.1	1.2	1.25	1.3	1.4	1.5	1.6
K'	0.473	0.141	0.154	0.166	0.172	0.177	0.187	0.196	0.204
λ	0.102	1.37	1.52	1.68	1.76	1.85	2.03	2.22	2.43
y/x	1.7	1.75	2.0	2.5	3.0	4.0	5.0	6.0	10
K'	0.211	0.214	0.229	0.249	0.263	0.281	0.291	0.300	0.312
λ	2.65	2.77	3.39	4.86	6.63	11.03	16.5	23.3	62.1

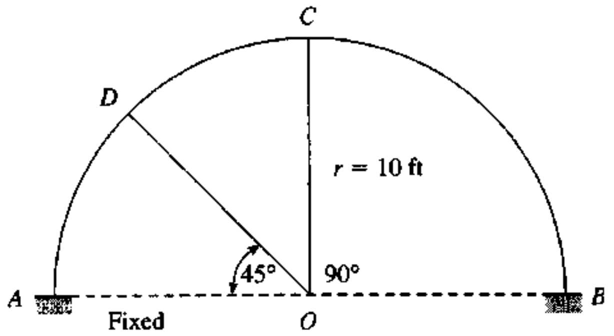


Figure 21.5 Example 21.2.

Solution

1. Factored load $w_u = 304\,\text{psf}$.
2. For the section at C, $\theta = \pi/2$ and $w_u\, r^3 = 0.304(10)^3 = 304$. From Eq. 21.17,

$$M_c = 304\left[\frac{\pi}{8}\sin\frac{\pi}{2} - \frac{1}{6}\left(1 + \cos^2\frac{\pi}{2}\right) - 0.11\sin\frac{\pi}{2}\right] = 35.3\ \text{K·ft}$$

From Eq. 21.18,

$$T_c = 304\left[\frac{\pi}{8}\left(\cos\frac{\pi}{2} - 1\right) + \frac{\pi}{8} + \frac{1}{24}\sin\pi - 0.11\cos\frac{\pi}{2}\right] = 0$$

3. For the section at D, $\theta = \pi/4$.

$$M_D = 304\left[\frac{\pi}{8}\sin\frac{\pi}{4} - \frac{1}{6}\left(1 + \cos^2\frac{\pi}{4}\right) - 0.11\sin\frac{\pi}{4}\right] = -15.2\ \text{K·ft}$$

$$T_D = 304\left[\frac{\pi}{8}\left(\cos\frac{\pi}{4} - 1\right) + \frac{\pi}{16} + \frac{1}{24}\sin\frac{\pi}{2} - 0.11\cos\frac{\pi}{4}\right] = 13.7\ \text{K·ft}$$

4. Maximum shearing force occurs at the supports.

$$V_A = 0.39 w_u r^2 = 0.39(0.304)(100) = 11.9\ \text{K}$$

Maximum positive moment occurs at C, whereas the maximum negative moment occurs at the supports.

$$M_A = -\frac{w_u r^3}{3} = -\frac{304}{3} = 101.3\ \text{K·ft}$$

5. Design the critical sections for shear, bending, and torsional moments, as explained in Example 21.1.

21.4 FIXED-END SEMICIRCULAR BEAM UNDER UNIFORM LOADING

The previous section dealt with a semicircular beam fixed at both ends and subjected to a variable distributed load. If the load is uniform, then the beam will be subjected to a uniformly distributed load w K/ft, as shown in Fig. 21.6. The forces in the curved beam can be determined as follows:

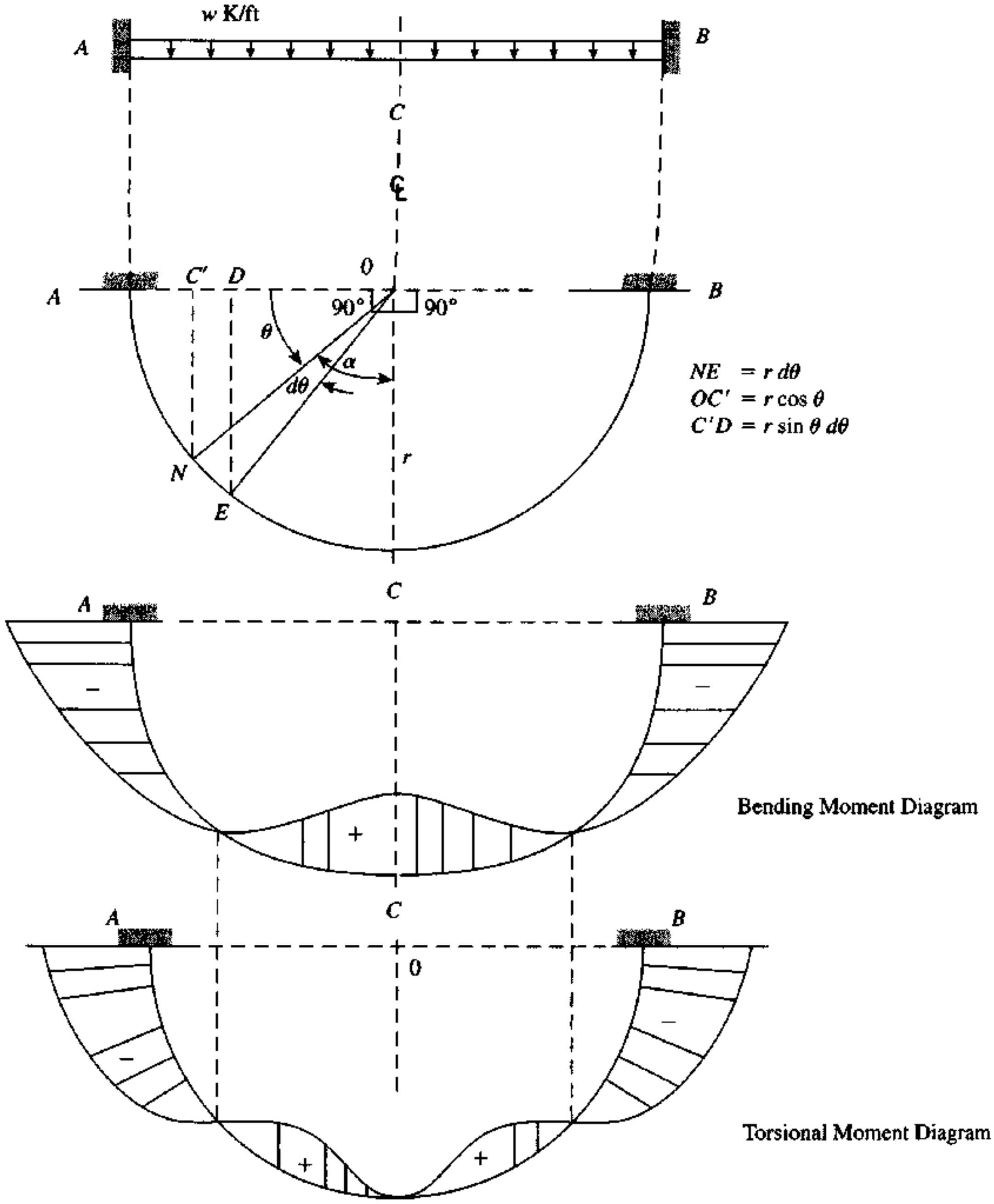


Figure 21.6 Semicircular beam under uniform load.

1. Shearing force at A:

$$V_A = V_B = \int_0^{\pi/2} wr\, d\theta = wr\frac{\pi}{2} = 1.57wr \tag{21.21}$$

2. Bending moment at A:

$$M_A = M_B = \int_0^{\pi/2} w(r\, d\theta) \times (r\ \sin\theta) = wr^2 \tag{21.22}$$

3. Bending moment at any section N on the curved beam when the torsional moment at A (T_A) acts clockwise:

$$M_N = V_A\ (r\ \sin\theta) - M_A\ \cos\theta + T_A\ \sin\theta - \int_0^\theta (wr\, d\alpha)[r\ \sin(\theta - a)]$$

$$= \frac{\pi}{2} wr^2 \sin\theta - wr^2 \cos\theta + T_A\ \sin\theta - [wr^2 - wr^2 \cos\theta]$$

$$M_N = wr^2 \left[\frac{\pi}{2} \sin\theta - 1\right] + T_A\ \sin\theta \tag{21.23}$$

4. Torsional moment at any section N:

$$T_N = -V_A\ r(1 - \cos\theta) + M_A\ \sin\theta + T_A\ \cos\theta + \int_0^\theta (wr\, d\alpha) r[1 - \cos(\theta - \alpha)]$$

$$= -\frac{\pi}{2} wr^2 + \frac{\pi}{2} wr^2 \cos\theta + T_A\ \cos\theta + M_A\ \sin\theta + wr^2\theta - wr^2 \sin\theta$$

Substitute $M_A = wr^2$:

$$T_N = wr^2 \left[\frac{\pi}{2} \cos\theta - \frac{\pi}{2} + \theta\right] + T_A\ \cos\theta \tag{21.24}$$

5. The strain energy expression was given in the previous section:

$$U = \int \frac{M_N^2\, ds}{2EI} + \int \frac{T_N^2\, ds}{2GJ} \tag{21.25}$$

To obtain T_A, differentiate U with respect to T_A.

$$\frac{\delta_U}{\delta T_A} = \int \frac{M_N}{EI} \times \frac{dM_N}{dT_A} (r\, d\theta) + \int \frac{T_N}{GJ} \times \frac{dT_N}{dT_A} \times (r\, d\theta) = 0$$

$$\frac{dM_N}{dT_A} = \sin\theta \quad \text{and} \quad \frac{dT_N}{dT_A} = \cos\theta \quad \text{(from the preceding equations)}$$

$$\frac{\delta_U}{\delta T_A} = \frac{r}{EI} \int_0^{\pi/2} \left[wr^2 \left(\frac{\pi}{2} \sin -1\right) + T_A\ \sin\theta\right] \sin\theta\ d\theta$$

$$+ \frac{r}{GJ} \int_0^{\pi/2} \left[wr^2 \left(\frac{\pi}{2} \cos\theta - \frac{\pi}{2} + \theta\right) + T_A\ \cos\theta\right] \cos\theta\ d\theta = 0$$

The integration of the preceding equation produces the following:

$$\frac{\delta U}{\delta T_A} = \frac{r}{EI} \left[wr^2 \left(\frac{\pi^2}{8} - 1\right) + \frac{\pi}{4} T_A\right] + \frac{r}{GJ} \left[wr^2 \left(\frac{\pi^2}{8} - 1\right) + \frac{\pi}{4} T_A\right] = 0$$

$$r \left[wr^2 \left(\frac{\pi^2}{8} - 1\right) + \frac{\pi}{4} T_A\right] \left(\frac{EI}{GJ} + 1\right) = 0$$

Because EI/GJ is not equal to zero,

$$wr^2 \left(\frac{\pi^2}{8} - 1\right) + \frac{\pi}{4} T_A = 0$$

$$T_A = -wr^2 \left(\frac{4}{\pi}\right) \left(\frac{\pi^2}{8} - 1\right) = -0.3wr^2 \tag{21.26}$$

6. Substitute T_A in Eq. 21.23:

$$M_N = wr^2 \left[\left(\frac{\pi}{2} \sin\theta - 1 \right) - \left(\frac{\pi}{2} - \frac{4}{\pi} \right) \sin\theta \right]$$

$$= wr^2 \left(\frac{4}{\pi} \sin\theta - 1 \right) \qquad (21.27)$$

$$T_N = wr^2 \left[\left(\frac{\pi}{2} \cos\theta + \theta - \frac{\pi}{2} \right) - \left(\frac{\pi}{2} - \frac{4}{\pi} \right) \cos\theta \right]$$

$$= wr^2 \left(\theta - \frac{\pi}{2} + \frac{4}{\pi} \cos\theta \right) \qquad (21.28)$$

The values of the bending and torsional moments at any section N are independent of λ ($1 = EI/GJ$).

7. Bending and torsional moments at midspan, section C, can be found by substituting $\theta = \pi/2$ in Eqs. 21.27 and 21.28:

$$M_c = wr^2 \left(\frac{4}{\pi} - 1 \right) = 0.273wr^2 \qquad (21.29)$$

$$T_c = wr^2 \left(\frac{\pi}{2} - \frac{\pi}{2} + 0 \right) = 0 \qquad (21.30)$$

21.5 CIRCULAR BEAM SUBJECTED TO UNIFORM LOADING

The previous section dealt with a semicircular beam subjected to a uniformly distributed load. The forces acting on the beam at any section vary with the intensity of load, the span (or the radius of the circular beam), and the angle α measured from the centerline axis of the beam.

Considering the general case of a circular beam fixed at both ends and subjected to a uniform load w (K/ft), as shown in Fig. 21.7, the bending and torsional moments can be calculated from the following expressions:

1. The moment at the centerline of the beam, M_c, can be derived using the strain energy expression, equation 21.25, and can be expressed as follows:

$$M_c = \frac{wr^2}{K_4} [\lambda(K_1 + K_2 - K_3) + (K_1 - K_2)] \qquad (21.31)$$

where

$\lambda = \dfrac{EI}{GJ}$

$K_1 = 2(2 \sin\theta - \theta)$

$K_2 = 2 \sin\theta \cos\theta = \sin 2\theta$

$K_3 = 4\theta \cos\theta$

$K_4 = 2\theta(\lambda + 1) - (\lambda - 1) \sin 2\theta$

2θ = total central angle of the ends of the beam, angle AOB (Fig. 19.7)

The torsional moment at the centerline section, T_c, is 0.

Curved-beam bridge, Washington, D.C.

2. The moment at any section N on the curved beam where ON makes an angle α with the centerline axis (Fig. 21.7) is

$$M_N = M_c \cos\alpha - wr^2(1 - \cos\alpha) \tag{21.32}$$

3. The torsional moment at any section N on the curved beam as a function of the angle α was derived earlier:

$$T_N = M_c \sin\alpha - wr^2(\alpha - \sin\alpha) \tag{21.33}$$

4. To compute the bending moment and torsional moment at the supports, substitute θ for α in the preceding equations:

$$M_A = M_c \cos\theta - wr^2(1 - \cos\theta) \tag{21.34}$$

$$T_A = M_c \sin\theta - wr^2(\theta - \sin\theta) \tag{21.35}$$

Example 21.3

A curved beam has a quarter-circle shape in plan with a 10-ft radius. The beam has a rectangular section with the ratio of the long to the short side of 2.0 and is subjected to a factored load of 8 K/ft. Determine the bending and torsional moments at the centerline of the beam, supports, and maximum values.

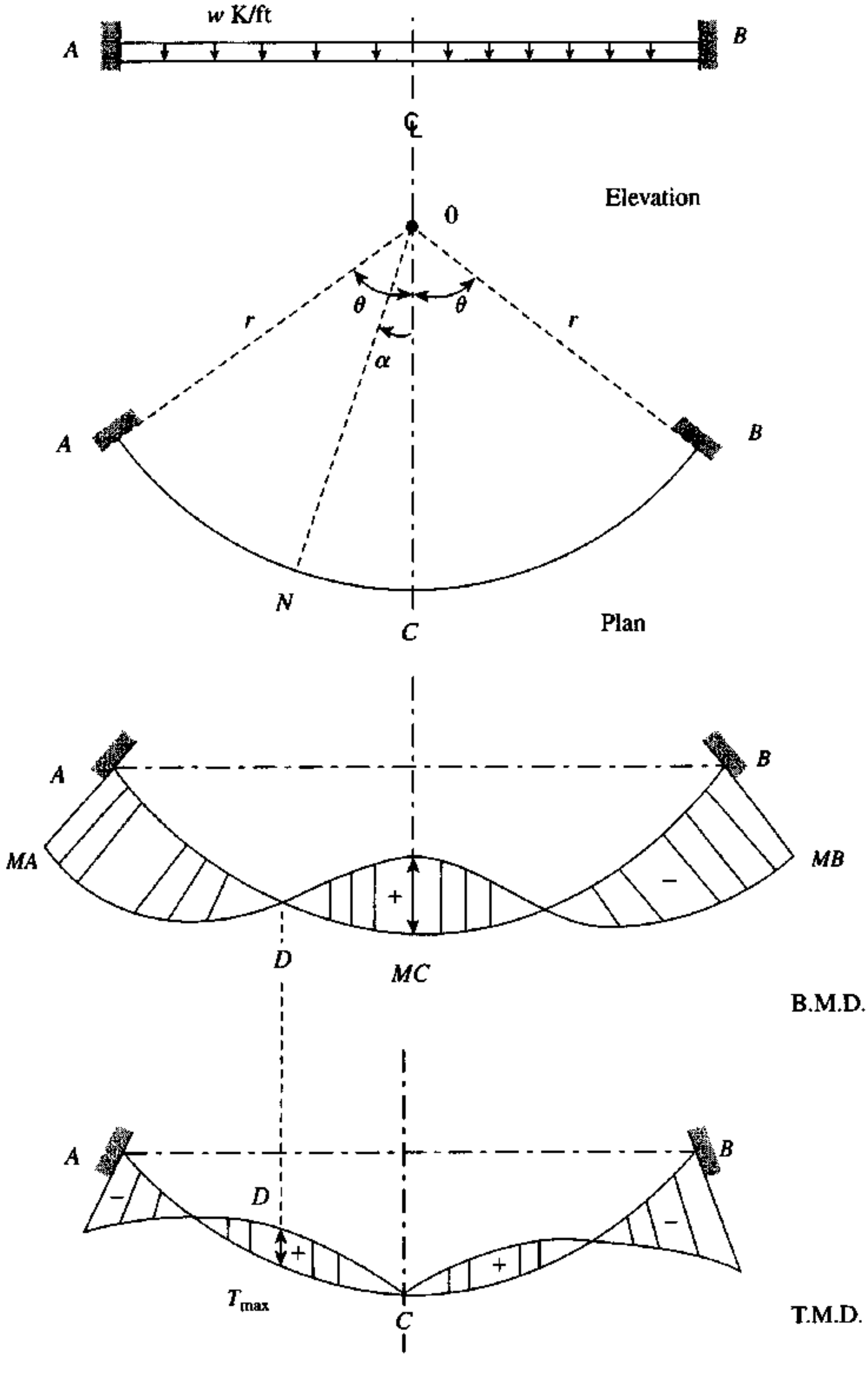


Figure 21.7 Circular beam subjected to uniform load, showing the bending moment diagram (BMD) and the torsional moment diagram (TMD).

Solution

1. For a rectangular section with $y/x = 2$, $\lambda = EI/GJ = 3.39$ (Table 21.2).
2. The bending and torsional moments can be calculated using Eqs. 21.31 through 21.35 for $\theta = \pi/4$. From Eq. 21.31,

$$K_1 = 2\left(2\sin\frac{\pi}{4} - \frac{\pi}{4}\right) = 1.2576, \quad K_2 = \sin\frac{\pi}{2} = 1.0$$

$$K_3 = 4\left(\frac{\pi}{4}\right)\cos\frac{\pi}{4} = 2.2214$$

$$K_4 = 2\left(\frac{\pi}{4}\right)(3.39 + 1) - (3.39 - 1)\sin\frac{\pi}{2} = 4.506$$

$$M_c = \frac{wr^2}{4.506}[3.39(1.2576 + 1.0 - 2.2214) + (1.2576 - 1.0)]$$

$$= 0.0844wr^2$$

For $w = 8$ K·ft and $r = 10$ ft, $M_c = 64$ K·ft; $T_c = 0$.

3. $M_N = M_c \cos\alpha - wr^2(1 - \cos\alpha) = wr^2(1.08\cos\alpha - 1)$

$$T_N = M_c \sin\alpha - wr^2(\alpha - \sin\alpha) = wr^2(1.08\ \sin\alpha - \alpha)$$

For the moments at the supports, $\alpha = \theta = \pi/4$.

$$M_A = wr^2\left(1.08\cos\frac{\pi}{4} - 1\right) = -0.236wr^2$$

$$= -0.236 \times 8 \times (10)^2 = -189 \text{ K·ft}$$

$$T_A = wr^2\left(1.08\sin\frac{\pi}{4} - \frac{\pi}{4}\right) = 0.022wr^2 = -17.4 \text{ K·ft}$$

For $M_N = 0$, $1.08\cos\alpha - 1 = 0$, or $\cos\alpha = 0.926$ and $\alpha = 22.2° = 0.387$ rad. To calculate $T_{N\,\max}$, let $dT_N/d\alpha = 0$, or $(1.08\cos\alpha - 1) = 0$. Then $\cos\alpha = 0.926$ and $\alpha = 22.2°$.

$$T_N(\max) = wr^2(1.08\ \sin 22.2 - 0.387) = 0.0211wr^2$$

$$T_{N\,\max} = 0.0211 - 800 = 16.85 \text{ K·ft}$$

21.6 CIRCULAR BEAM SUBJECTED TO A CONCENTRATED LOAD AT MIDSPAN

If a concentrated load is applied at the midspan of a circular beam, the resulting moments vary with the magnitude of the load, the span, and the coefficient $\lambda = EI/GJ$. Considering the general case of a circular beam fixed at both ends and subjected to a concentrated load P at midspan (Fig. 21.8), the bending and torsional moments can be calculated from the following expressions:

1. The moment at the centerline of the beam, section C, can be expressed as follows:

$$M_c = \frac{\lambda(2 - 2\ \cos\theta - \sin^2\theta) + \sin^2\theta}{2\theta(\lambda + 1) - (\lambda - 1)\sin 2\theta}(Pr) \tag{21.36}$$

$$M_c = \frac{Pr}{K_3}(\lambda K_1 + K_2)$$

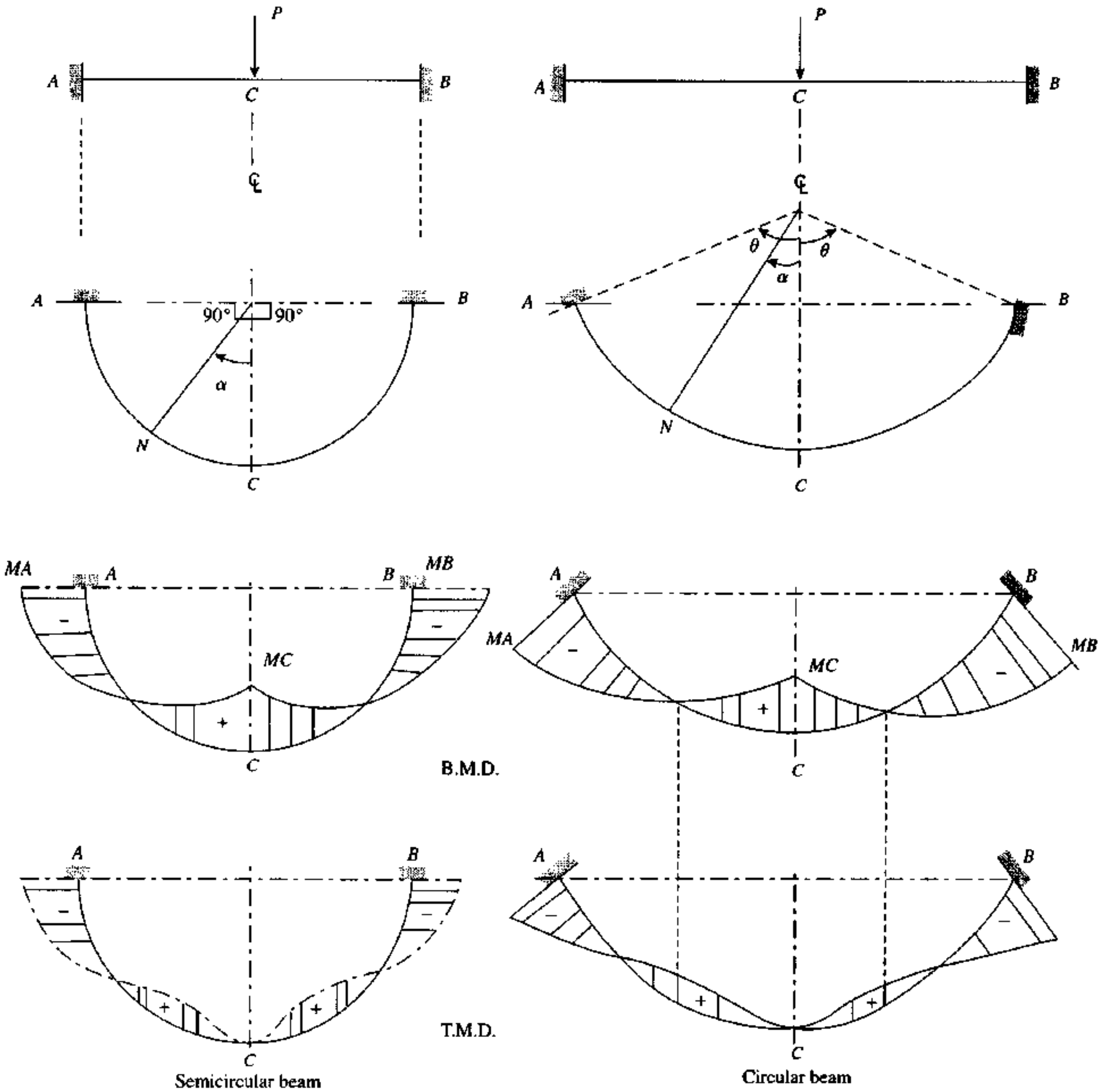


Figure 21.8 Circular beam subjected to a concentrated load at midspan, showing the bending moment diagram (BMD) and the torsional moment diagram (TMD).

where

$$\lambda = \frac{EI}{GJ}$$
$$K_1 = (2 - 2\cos\theta - \sin^2\theta)$$
$$K_2 = \sin^2\theta$$
$$K_3 = 2\theta\ (\lambda + 1) - (\lambda - 1)\sin^2\theta$$

The torsional moment at the centerline is $T_c = 0$.

2. The bending and torsional moments at any section N on the curved beam where ON makes an angle α with the centerline axis are calculated as follows:

$$M_N = M_c \cos\alpha - \left(\frac{P}{2}r\right)\sin\alpha \qquad (21.37)$$

$$T_N = M_c \sin\alpha - \left(\frac{P}{2}r\right)(1-\cos\alpha) \qquad (21.38)$$

3. To compute the bending and torsional moments at the supports, substitute θ for α.

$$M_A = M_c \cos\theta - \left(\frac{P}{2}r\right)\sin\theta \qquad (21.39)$$

$$T_A = M_c \sin\theta - \left(\frac{P}{2}r\right)(1-\cos\theta) \qquad (21.40)$$

Example 21.4

Determine the bending and torsional moments of the quarter-circle beam of Example 21.3 if $\lambda = 1.0$ and the beam is subjected to a concentrated load at midspan of $P = 20\,\mathrm{K}$.

Solution

1. Given: $\lambda = 1.0$ and $\theta = \pi/4$. Therefore,

$$M_c = \left(\frac{Pr}{2}\right)\left(\frac{1-\cos\theta}{\theta}\right)$$

(Eq. 21.36) and $T_c = 0$. For $\theta = \pi/4$,

$$M_c = 0.187Pr = 0.187(20 \times 10) = 37.4\ \mathrm{K{\cdot}ft}$$

2. From Eqs. 21.39 and 21.40,

$$M_A = 0.187Pr\cos\frac{\pi}{4} - \frac{Pr}{2}\sin\frac{\pi}{4} = -0.22\ Pr$$
$$= -0.22 \times (200) = -44\ \mathrm{K{\cdot}ft}$$
$$T_A = 0.187Pr\sin\frac{\pi}{4} - 0.5Pr\left(1-\cos\frac{\pi}{4}\right) = -0.0142Pr$$
$$= -0.0142 \times 200 = -2.84\ \mathrm{K{\cdot}ft}$$

3. $M_N = 0$ when

$$M_c\cos\alpha - \frac{Pr}{2}\sin\alpha = 0 \qquad \text{(Eq. 21.37)}$$
$$0.187Pr\cos\alpha - 0.5Pr\sin\alpha = 0$$
$$\tan\alpha = 0.374 \quad \text{and} \quad \alpha = 20.5^\circ$$

$T_n = 0$ when M_c sin α − (P/2) r(1 − cos α) = 0 (Eq. 21.38), from which $\alpha = 37.7^\circ$.

4. To compute T_{max}, let $dT_N/d\alpha = 0$ (equation (21.38)).

$$0.187Pr\cos\alpha - 0.5Pr\sin\alpha = 0, \quad \tan\alpha = 0.374$$

and $\alpha = 20.5^\circ$. Substitute $\alpha = 20.5^\circ$ in Eq. (21.38) to get $T_{max} = 0.035Pr = 7$ K·ft.

21.7 V-SHAPE BEAMS SUBJECTED TO UNIFORM LOADING

Beams that have a V shape in plan and are subjected to loads normal to the plane of the beam may be analyzed using the *strain-energy principles*. Fig. 21.9 shows typical bending moment diagram for a V-shape beam subjected to a uniform load w. Considering the general case of a V-shape beam fixed at both ends and subjected to a uniform load w (K/ft), the bending and torsional moments can be calculated from the following expressions:

1. The moment at the centerline of the beam, section C, is calculated as follows:

$$M_c = (wa^2)\left[\frac{\sin^2\theta}{6(\sin^2\theta + \lambda\ \cos^2\theta)}\right] \tag{21.41}$$

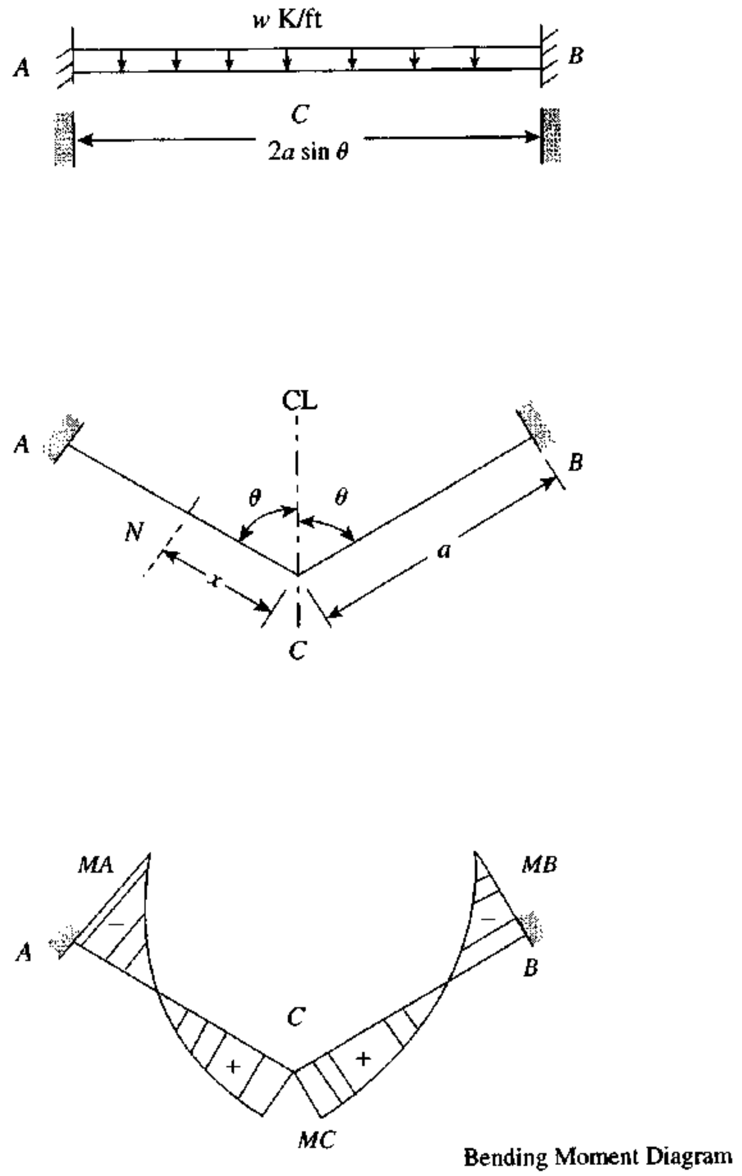


Figure 21.9 V-shape beam under uniform load.

90° V-shape beams, London, Ontario, Canada.

where

$$\lambda = \frac{EI}{GJ}$$

a = half the total length of the beam (length AC)

θ = half the angle between the two sides of the V-shape beam.

The torsional moment at the centerline section is

$$T_c = \frac{M_c}{\sin\theta} \times \cos\theta = M_c \cot\theta \tag{21.42}$$

2. The bending and torsional moments at any section N along half the beam AC or BC at a distance x measured from section C are calculated as follows:

$$M_N = M_c - w\frac{x^2}{2} \tag{21.43}$$

$$T_N = T_c = \frac{M_c}{\sin\theta} \times \cos\theta = M_c \cot\theta \tag{21.44}$$

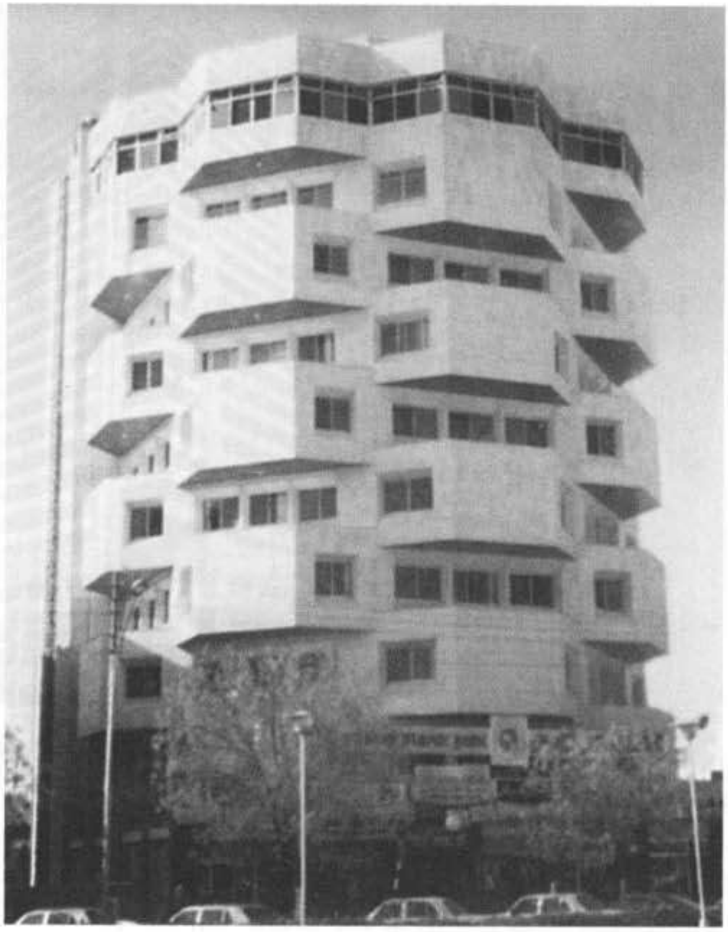
To compute the moments at the supports, let $x = a$. Then

$$M_A = M_c - w\frac{a^2}{2}$$

$$T_A = T_c = M_c \cot\theta$$

Example 21.5

Determine the bending and torsional moments in a V-shape beam subjected to a uniform load of 6 K/ft. The length of half the beam is $a = 10$ ft and the angle between the V-shape members is $2\theta = \pi/2$. The beam section is rectangular with a ratio of long side to short side of 2.



Apartment building.

Solution

1. For a rectangular section with the sides ratio, $y/x = 2$, $\lambda = 3.39$. For this beam $\theta = \pi/4$.

2. $$M_c = \frac{wa^2}{6}\left[\frac{\sin^2\theta}{(\sin^2\theta + \lambda\cos^2\theta)}\right] \quad \text{(Eq. 21.41)}$$

$$M_c = \frac{wa^2}{6}\left(\frac{0.5}{(0.5 + 3.39 \times 0.5)}\right) = 0.038wa^2$$

$$= 0.038 \times 6(10)^2 = 22.8 \text{ K·ft}$$

$$M_A = M_c - w\frac{a^2}{2} = 0.038wa^2 - 0.5wa^2 = -0.462wa^2$$

$$= -277.2 \text{ K·ft}$$

$$M_N = 0 \quad \text{when} \quad M_c - w\frac{x^2}{2} = 0$$

or $0.038wa^2 - 0.5wx^2 = 0$, so $x = 0.276\text{a} = 2.76$ ft measured from c.

3. $$T_A = T_C = M_C \cot\theta$$

$$= 0.038wa^2 = 0.038 \times 600 = 22.8 \text{ K·ft}$$

21.8 V-SHAPE BEAMS SUBJECTED TO A CONCENTRATED LOAD AT THE CENTERLINE OF THE BEAM

The general equations for computing the bending and torsional moments in a V-shape beam fixed at both ends and subjected to a concentrated load P at the centerline of the beam (Fig. 21.10) are as follows:

1. The moment at the centerline of the beam, section C, for any value of λ, is

$$M_c = \left(\frac{Pa}{4}\right)\left(\frac{\sin^2\theta}{(\sin^2\theta + \lambda\cos^2\theta)}\right) \tag{21.45}$$

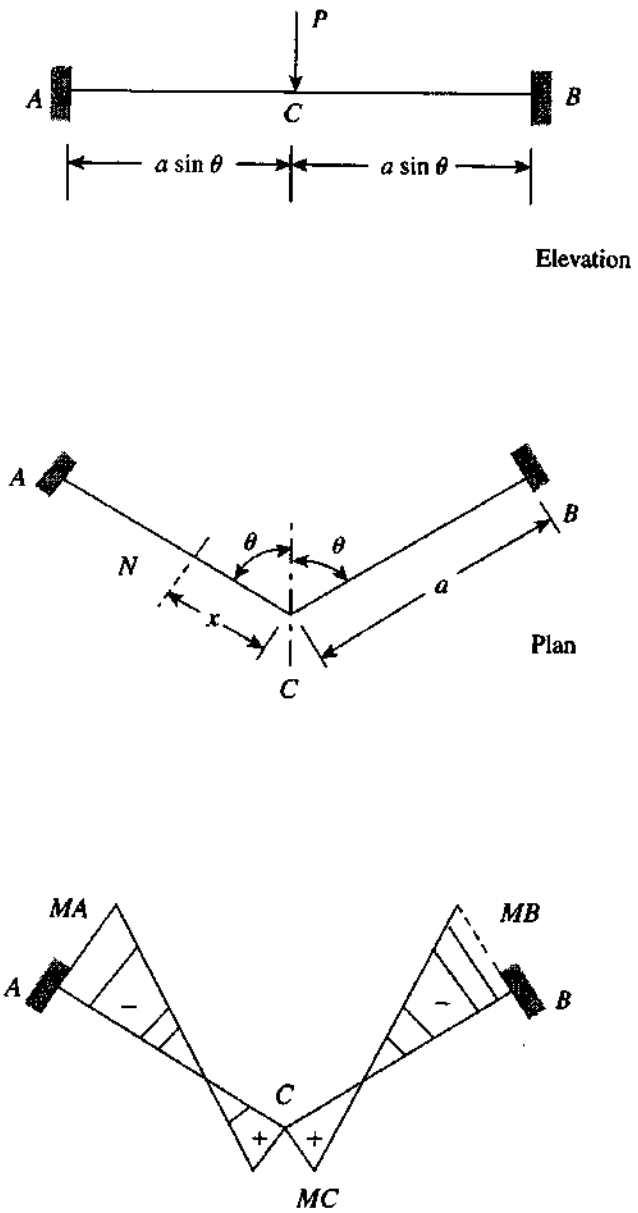


Figure 21.10 V-shape beam under concentrated load.

where

$$\lambda = \frac{EI}{GJ}$$

a = half the total length of the beam (part AB or BC)

θ = half the angle between the two sides of the V-shape beam

The torsional moment at the centerline section is

$$T_c = \frac{M_c}{\sin\theta}\cos\theta = M_c \cot\theta \tag{21.46}$$

2. The bending and torsional moments at any section N along half the beam AC or BC at a distance x measured from C are calculated as follows:

$$M_N = M_c - \frac{Px}{2} \tag{21.47}$$

$$T_N = T_c = M_c \cot\theta \tag{21.48}$$

The moments at the supports are determined by assuming $x = a$:

$$M_A = M_c - \frac{Pa}{2} \tag{21.49}$$

$$T_A = T_c = M_c \cot\theta \tag{21.50}$$

Example 21.6

Determine the bending and torsional moments in a V-shape beam subjected to a concentrated load $P = 30\,\text{K}$ acting at the centerline of the beam. Given: $\theta = \pi/4$, $y/x = 2.0$, and a $= 12$ ft.

Solution

1. For a rectangular section with $y/x = 2.0$, $\lambda = 3.39$.

2. $$M_c = \frac{Pa}{4}\left(\frac{\sin^2\pi/4}{\sin^2\pi/4 + 3.39\cos^2\pi/4}\right) = 0.057(Pa)$$

$$= 0.057 \times 30 \times 12 = 20.5\,\text{K·ft}$$

$$M_A = M_c - \frac{Pa}{2} = (0.057 - 0.5)Pa = -0.443(Pa)$$

$$= -0.0443 \times 360 = -159.5\,\text{K·ft}$$

$$M_N = 0 \quad \text{when} \quad M_c - \frac{Px}{2} = 0$$

Hence $0.057Pa - 0.5\ Px = 0$ and $x = 0.114a = 0.114 \times 12 = 1.37$ ft measured from c.

3. $T_A = T_c = T_N = M_c \cot\frac{\pi}{4} = 0.057(Pa) = 20.5\,\text{K·ft}$

Example 21.7

Determine the bending and torsional moments in the beam of Example 21.6 if the angle θ is $\pi/2$ (a straight beam fixed at both ends).

Solution

Given $\theta = \pi/2$ and the span $L = 2a$ = the distance between the two supports. The bending moment at the centerline is

$$M_c = \frac{Pa}{4}\left(\frac{1}{1}\right) = \frac{Pa}{4} = \frac{PL}{8} = +90 \text{ K·ft}$$

$$M_A = M_c - \frac{Pa}{2} = \frac{PL}{8} - \frac{P}{2}\left(\frac{L}{2}\right) = -\frac{PL}{8} = -90 \text{ K·ft}$$

$$T_A = T_c = 0$$

These values are similar to those obtained from the structural analysis of the fixed-end beam subjected to a concentrated load at midspan.

Example 21.8

The beam shown in Fig. 21.11 has a V shape in plan and carries a uniform dead load of 3.5 K/ft and a live load of 3 K/ft. The inclined length of half the beam is $a = 10$ ft and $\theta = 60°$. Design the beam for shear, bending, and torsional moments using $f'_c = 4$ Ksi and $f_y = 60$ Ksi.

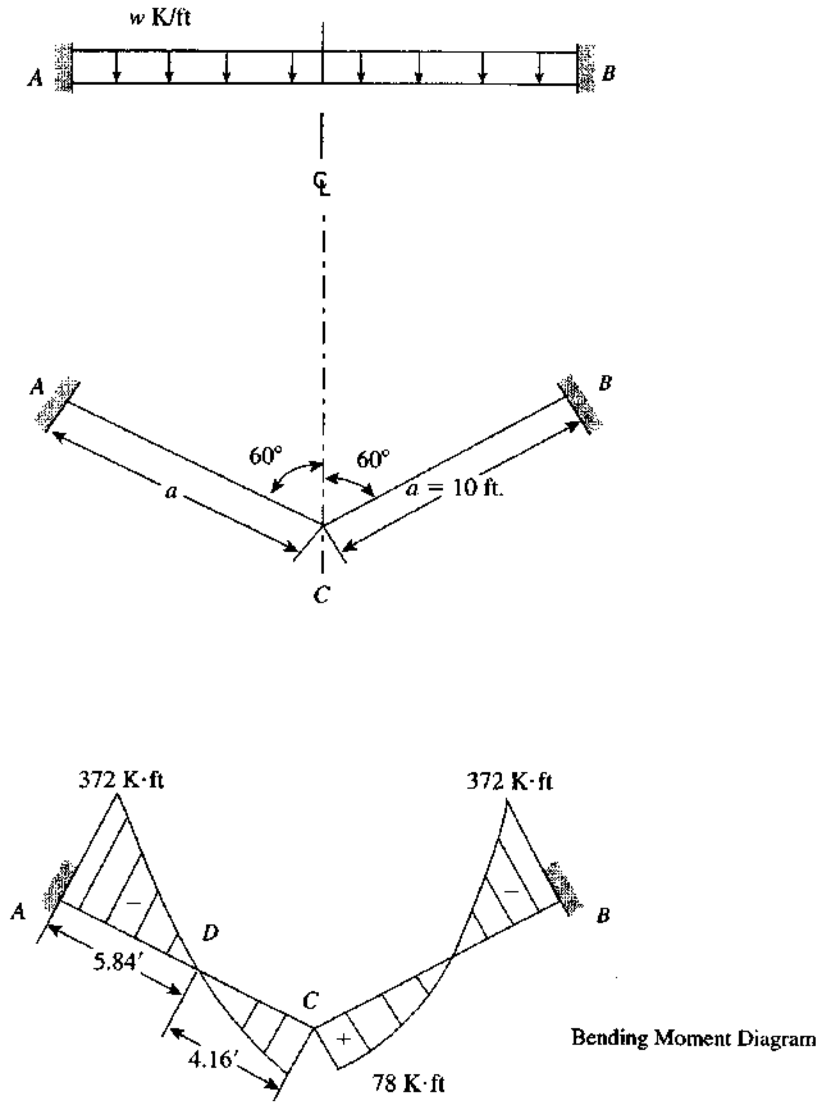


Figure 21.11 Example 21.8.

Solution

1. $w_u = 1.2D + 1.6L = 1.2 \times 3.5 + 1.6 \times 3 = 9.0$ K/ft.
2. Assuming a rectangular section with a ratio of long to short side of $y/x = 1.75$, the value of λ is 2.77 (from Table 21.2). For $\theta = 60° = \pi/3$,

$$M_c = \frac{w_u a^2 \sin^2\theta}{6(\sin^2\theta + \lambda\cos^2\theta)} = \frac{9(100)(0.75)}{6(0.75 + 2.77 \times 0.25)} = +\ 78 \text{ K·ft}$$

$$M_A = M_c - w_u\frac{a^2}{2} = 78 - 9\left(\frac{100}{2}\right) = -372 \text{ K·ft}$$

$$T_A = M_c \cot\theta = 78 \times 0.577 = 45 \text{ K·ft} = 540 \text{ K·in.}$$

$$T_c \text{ (at } x = 0) = M_c \cot\theta = 45 \text{ K·ft} = 540 \text{ K·in.}$$

$$V_A = 9 \times 10 = 90 \text{ K}$$

The bending moment is zero at $M_N = 0 = M_c - w_u\, x^2/2$. Hence, $78 - \frac{9}{2}x^2 = 0$ and x = 4.16 ft measured from c. The bending moment diagram is shown in Fig. 21.11.

3. Design for a bending moment, M_u, equal to -372 K·ft.
   a. For $f'_c = 4$ Ksi, $f_y = 60$ Ksi, $\rho_{max} = 0.0018$, choose $\rho = 0.015$, $R_u = 702$ psi and $\phi = 0.9$. (Appendix A)

$$bd^2 = \frac{M_u}{R_u} = \frac{372 \times 12}{0.705} = 6332 \text{ in.}^3$$

   For a ratio,

$$\frac{y}{x} = \frac{(d+3)}{b} = 1.75$$

   as assumed, then $d = 21.4$ in. and $b = 13.8$ in. Use a section 14×24 in.

$$A_s = \rho_{max} bd = 0.015(14 \times 21.4) = 4.5 \text{ in.}^2$$

   b. For the section at midspan, $M_u = 78$ K·ft and actual $d = 21.5$ in.

$$R_u = \frac{M_u}{bd^2} = \frac{78{,}000 \times 12}{14 \times (21.5)} = 145 \text{ psi}$$

$$\rho < \rho_{min} = 0.0033$$

   Use $A_s = 0.0033 \times 14 \times 21.5 = 1.0 \text{ in.}^2$
   c. Design for torsional moment and shear: $T_u = 45$ K·ft for all sections.

$$V_u(\text{at distance } d) = 90 - \frac{21.5}{12} \times 9 = 74.0 \text{ K}$$

   The design procedure will be similar to that of Example 21.1. Details of the final section are shown in Fig. 21.12.

SUMMARY

Sections 21.1–21.5

In a curved beam in plan, the center of gravity of normal loads lies outside the line joining the supports developing torsional moments. The analysis of uniformly loaded circular beams is presented in these sections.

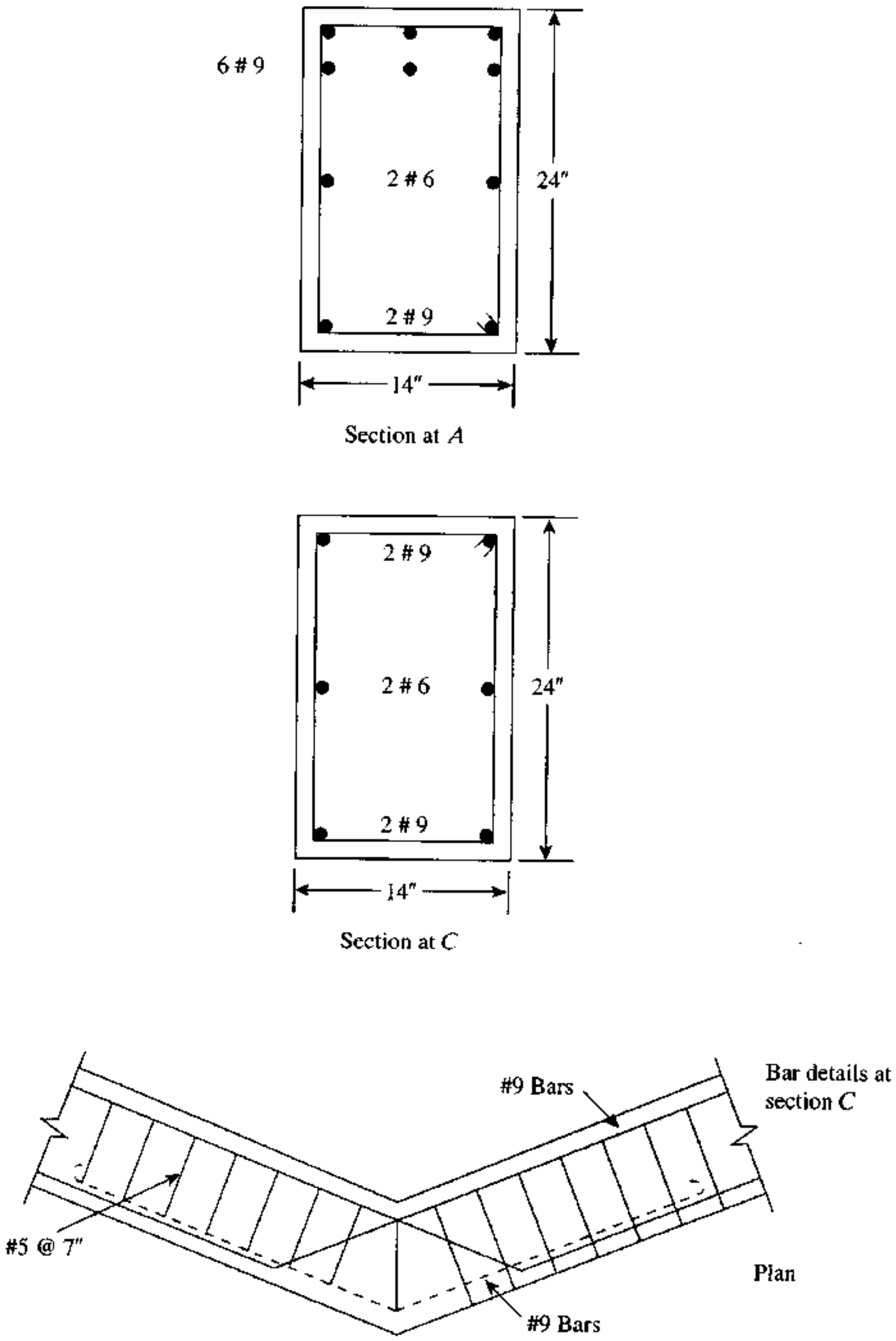


Figure 21.12 Example 21.8.

Section 21.6

The analysis of circular beams subjected to concentrated loads is presented in this section.

Section 21.7

V-shape beams subjected to gravity loads may be analyzed using the strain energy principles. Equations to calculate the torsional moments of these types of beams are presented.

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PROBLEMS

21.1 A circular beam is supported on six equally spaced columns, and its centerline lies on a circle 20 ft in diameter. The beam carries a uniform dead load of 9.8 K/ft and a live load of 5 K/ft. Design the beam using $f'_c = 4$Ksi, $f_y = 60$ Ksi, and $b = 14$ in.

21.2 Design a semicircular beam fixed on both ends. The center of columns lies on a circle 12 ft in diameter. The beam carries uniform dead and live loads of 4.9 K/ft and 3 K/ft, respectively. Use $f'_c = 4$ Ksi, $f_y = 60$ Ksi, and $b = 20$ in.

21.3 Determine the factored bending and torsional moment at sections C and D of the fixed-end beam shown in Fig. 21.5 if the diameter of the circle is 30 ft. The beam is part of a floor slab that carries a uniform dead load (including its own weight) of 126 psf and a live load of 120 psf.

21.4 A quarter-circle cantilever beam has a radius of 8 ft and carries a uniform dead load of 6.4 K/ft and a concentrated live load of 4.25 K at its free end. Design the beam using $f'_c = 4$ Ksi, $f_y = 60$ Ksi, and $b = 14$ in.

21.5 Design the beam shown in Fig. 21.11 if the inclined length of half the beam is $a = 8$ ft. The beam has a 60° V shape in plan and carries uniform dead and live loads of 3.8 K/ft and 4 K/ft. Assume the ratio of the long to the short side of the rectangular section is 2. Use $f'_c = 4$ Ksi, and $f_y = 60$ Ksi.

APPENDIX A

DESIGN TABLES (U.S. CUSTOMARY UNITS)

Table A.1 Values of R_u and a/d for $f'_c = 3000$ psi ($\varepsilon_t \geq 0.005$, $\phi = 0.9$ and $d = d_t$)

100	$f_y = 40$ ksi		$f_y = 50$ ksi		$f_y = 60$ ksi		$f_y = 75$ ksi	
ρ	R_u	a/d	R_u	a/d	R_u	a/d	R_u	a/d
0.2	71	0.031	88	0.039	106	0.047	131	0.059
0.3	105	0.047	131	0.059	156	0.071	192	0.089
0.4	140	0.062	173	0.078	206	0.094	254	0.118
0.5	173	0.078	214	0.098	254	0.118	310	0.148
0.6	206	0.094	254	0.118	301	0.141	368	0.177
0.7	238	0.110	293	0.138	347	0.165	421	0.207
0.8	270	0.126	332	0.157	391	0.189	475	0.236
0.9	301	0.142	369	0.177	434	0.213	524	0.266
1.0	332	0.157	406	0.196	476	0.238	572	0.295
1.1	362	0.173	441	0.216	517	0.260	620	0.325
1.2	390	0.188	476	0.235	556	0.282	615	0.319
1.3	420	0.204	510	0.255	594	0.306	($\rho_{max} = 1.082$)	
1.4	450	0.220	543	0.274	631	0.330		
1.5	476	0.236	575	0.294	667	0.353		
1.6	504	0.252	607	0.314	700	0.376		
1.7	530	0.267			615	0.319		
1.8	556	0.282			($\rho_{max} = 1.356$)			
1.9	582	0.298						
2.0	607	0.314						
2.1	630	0.330						
			615	0.319				
			($\rho_{max} = 1.624$)					
	615	0.319						
	($\rho_{max} = 2.031$)							

Note: Last values are the maximum for $\varepsilon_t = 0.005$.

Table A.2 Values of R_u and a/d for $f'_c = 4000$ psi ($\varepsilon_t \geq 0.005$, $\phi = 0.9$ and $d = d_t$)

100	$f_y = 40$ ksi		$f_y = 50$ ksi		$f_y = 60$ ksi		$f_y = 75$ ksi	
ρ	R_u	a/d	R_u	a/d	R_u	a/d	R_u	a/d
0.2	71	0.024	89	0.029	106	0.035	132	0.044
0.3	106	0.036	132	0.044	158	0.053	194	0.066
0.4	140	0.047	175	0.059	208	0.071	257	0.088
0.5	175	0.059	217	0.074	258	0.089	317	0.110
0.6	208	0.071	260	0.088	307	0.106	378	0.132
0.7	242	0.083	300	0.103	355	0.123	434	0.154
0.8	274	0.094	340	0.118	400	0.141	490	0.176
0.9	307	0.106	378	0.132	447	0.158	545	0.198
1.0	340	0.118	419	0.147	492	0.176	600	0.220
1.1	370	0.130	455	0.161	536	0.194	650	0.242
1.2	400	0.141	492	0.176	580	0.212	702	0.264
1.3	432	0.153	530	0.191	620	0.230	752	0.286
1.4	462	0.165	565	0.206	662	0.247	801	0.308
1.5	492	0.177	600	0.221	700	0.265		
1.6	522	0.188	635	0.236	742	0.282	820	0.319
							($\rho_{max} = 1.445$)	
1.7	550	0.200	670	0.250	780	0.300		
1.8	580	0.212	702	0.265	818	0.318		
1.9	607	0.224	735	0.280				
2.0	635	0.236	768	0.294				
2.1	662	0.248	800	0.309				
2.2	690	0.260			820	0.319		
2.3	717	0.271			($\rho_{max} = 1.806$)			
2.4	742	0.282						
2.5	767	0.294						
2.6	792	0.306						
2.7	817	0.318						
			820	0.319				
			($\rho_{max} = 2.167$)					
	820	0.319						
	($\rho_{max} = 2.715$)							

Note: Last values are the maximum for $\varepsilon_t = 0.005$.

Table A.3 Values of R_u and a/d for $f'_c = 5000$ psi ($\varepsilon_t \geq 0.005$, $\phi = 0.9$ and $d = d_t$)

100	$f_y = 40$ ksi		$f_y = 50$ ksi		$f_y = 60$ ksi		$f_y = 75$ ksi	
ρ	R_u	a/d	R_u	a/d	R_u	a/d	R_u	a/d
0.2	71	0.019	89	0.024	106	0.028	132	0.035
0.3	106	0.029	133	0.036	159	0.042	196	0.052
0.4	141	0.038	176	0.047	210	0.056	260	0.070
0.5	176	0.047	218	0.060	260	0.070	322	0.088
0.6	210	0.056	260	0.071	310	0.085	384	0.106
0.7	244	0.066	302	0.083	360	0.100	442	0.123
0.8	277	0.075	343	0.094	408	0.113	500	0.141
0.9	310	0.085	383	0.106	455	0.127	556	0.159
1.0	343	0.094	424	0.118	502	0.141	612	0.177
1.1	375	0.104	463	0.130	550	0.155	667	0.195
1.2	408	0.113	500	0.141	593	0.169	722	0.212
1.3	440	0.123	540	0.153	637	0.183	776	0.230
1.4	470	0.132	578	0.165	681	0.198	830	0.247
1.5	502	0.141	615	0.177	724	0.212	875	0.265
1.6	532	0.150	652	0.188	766	0.226	920	0.282
1.7	563	0.160	688	0.200	808	0.240	970	0.300
1.8	593	0.169	724	0.212	848	0.254		
1.9	623	0.179	760	0.224	890	0.268	975	0.300
2.0	652	0.188	794	0.235	927	0.282	($\rho_{max} = 1.704$)	
2.1	681	0.198	830	0.247	965	0.292		
2.2	710	0.207	862	0.259	1003	0.311		
2.3	738	0.217	894	0.271				
2.4	766	0.226	927	0.282				
2.5	794	0.235	958	0.294				
2.6	821	0.244	990	0.306	975	0.300		
					($\rho_{max} = 2.123$)			
2.7	848	0.254						
2.8	875	0.263						
2.9	900	0.272						
3.0	127	0.282						
3.1	952	0.292						
			975	0.300				
			($\rho_{max} = 2.551$)					
	975	0.300						
	($\rho_{max} = 3.18$)							

Note: Last values are the maximum for $\varepsilon_t = 0.005$.

Table A.4 Values of ρ_{max}, $R_{u\,max}$, ρ_b, ρ_{min}

$\rho_b = 0.85\beta_1(f'_c/f_y)[87/(87+f_y)]$ $\rho_{max} = (0.003 + f_y/E_s)\rho_b/0.008$ $R_u = \phi\rho f_y[1 - \rho f_y/1.7f'_c]$

f'_c psi	$f_y = 40$ ksi $100\rho_{max}$	$R_{u\,max}$ psi	$100\rho_b$	$100\rho_{min}$	$f_y = 50$ ksi $100\rho_{max}$	$R_{u\,max}$ psi	$100\rho_b$	$100\rho_{min}$
3000	2.031	615	3.71	0.50	1.624	615	2.75	0.40
4000	2.715	820	4.96	0.50	2.167	820	3.67	0.40
5000	3.180	975	5.81	0.53	2.551	975	4.32	0.42
6000	3.575	1108	6.53	0.58	2.864	1108	4.85	0.47

f'_c psi	$f_y = 60$ ksi $100\rho_{max}$	$R_{u\,max}$ psi	$100\rho_b$	$100\rho_{min}$	$f_y = 75$ ksi $100\rho_{max}$	$R_{u\,max}$ psi	$100\rho_b$	$100\rho_{min}$
3000	1.356	615	2.14	0.33	1.082	615	1.55	0.27
4000	1.806	820	2.85	0.33	1.445	820	2.07	0.27
5000	2.123	975	3.35	0.35	1.704	975	2.44	0.28
6000	2.389	1108	3.77	0.39	1.920	1108	2.75	0.31

Note: ρ_{max} values are for $\varepsilon_t = 0.005$ and $\phi = 0.9$.

Table A.5 Suggested Design Steel Ratios, ρ_s, and Comparison With Other Steel Ratios

f'_c psi	f_y ksi	$100\rho_b$	$100\rho_{max}$	$100\rho_s$	R_u for ρ_s (psi)	Ratio ρ_s/ρ_b	Ratio ρ_s/ρ_{max}	Weight of ρ_s (lb/ft^3 of concrete)
3000	40	3.71	2.031	1.4	450	0.377	0.689	7
	50	2.75	1.624	1.2	476	0.436	0.739	6
	60	2.15	1.356	1.2	556	0.558	0.885	6
4000	40	4.96	2.715	1.4	462	0.282	0.516	7
	50	3.67	2.167	1.4	565	0.381	0.646	7
	60	2.85	1.806	1.4	662	0.491	0.775	7
5000	40	5.81	3.180	1.6	532	0.275	0.503	8
	50	4.32	2.551	1.6	652	0.370	0.627	8
	60	3.35	2.123	1.6	766	0.478	0.754	8

Note: ρ_{max} values are for $\varepsilon_t = 0.005$ and $\phi = 0.9$.

Table A.6 Minimum Thickness of Beams and One-Way Slabs

Member	Yield Strength f_y (ksi)	Simply Supported	One End Continuous	Both Ends Continuous	Cantilevel
Solid one-way slabs	40	L/25	L/30	L/35	L/12.5
	50	L/22	L/27	L/31	L/11
	60	L/20	L/24	L/8	L/10
Beams or ribbed one-way slabs	40	L/20	L/23	L/26	L/10
	50	L/18	L/20.5	L/23.5	L/9
	60	L/16	L/18.5	L/21	L/8

Table A.7 Minimum Beam Width (in.) (Using Stirrups)

Size of Bars	Number of Bars in Single Layer of Reinforcement							Add For Each Added Bar (in.)
	2	3	4	5	6	7	8	
No. 4	6.1	7.6	9.1	10.6	12.1	13.6	15.1	1.50
No. 5	6.3	7.9	9.6	11.2	12.8	14.4	16.1	1.63
No. 6	6.5	8.3	10.0	11.8	13.5	15.3	17.0	1.75
No. 7	6.7	8.6	10.5	12.4	14.2	16.1	18.0	1.88
No. 8	6.9	8.9	10.9	12.9	14.9	16.9	18.9	2.00
No. 9	7.3	9.5	11.8	14.0	16.3	18.6	20.8	2.26
No. 10	7.7	10.2	12.8	15.3	17.8	20.4	22.9	2.54
No. 11	8.0	10.8	13.7	16.5	19.3	22.1	24.9	2.82
No. 14	8.9	12.3	15.6	19.0	22.4	25.8	29.2	3.39
No. 18	10.5	15.0	19.5	24.0	28.6	33.1	37.6	4.51

Table A.8 Values of bd^2 (in.^{3}) $bd^2 = \left[\frac{M_u}{R_u}\left(\frac{\text{lb} \cdot \text{in.}}{\text{psi}}\right)\right]$

d (in.)	Values of *b* (in.)											
	6	7	8	9	10	11	12	13	14	15	16	20
4	96	112	128	144	160	176	192	208	224	240	256	320
4.5	122	142	162	182	202	223	244	264	284	305	325	405
5	150	175	200	225	250	275	300	325	350	375	400	500
5.5	182	212	242	273	303	333	364	394	424	455	485	605
6	216	252	288	324	360	396	432	468	504	540	576	720
6.5	255	297	340	382	425	467	510	552	595	637	680	850
7	294	343	392	441	490	539	588	637	686	735	784	980
8	384	448	512	576	640	704	768	832	896	960	1024	1280
9	486	567	648	729	810	891	972	1053	1134	1215	1296	1620
10	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	2000
11	726	847	968	1089	1210	1331	1452	1573	1694	1815	1936	2420
12	864	1008	1152	1296	1440	1584	1728	1872	2016	2160	2304	2880
13	1014	1183	1352	1521	1690	1859	2028	2197	2366	2535	2704	3380
14	1176	1372	1568	1764	1960	2156	2352	2548	2744	2940	3136	3920
15	1350	1575	1800	2025	2250	2475	2700	2925	3150	3375	3600	4500
16	1536	1792	2048	2304	2560	2816	3072	3328	3584	3840	4096	5120
17	1734	2023	2312	2601	2890	3179	3468	3757	4046	4335	4624	5780
18	1944	2268	2592	2916	3240	3564	3888	4212	4536	4860	5184	6480
19	2166	2527	2888	3249	3610	3971	4332	4693	5054	5415	5776	7220
20	2400	2800	3200	3600	4000	4400	4800	5200	5000	6000	6400	8000
21	2646	3087	3528	3969	4410	4851	5292	5733	6174	6615	7056	8820
22	2904	3388	3872	4356	4840	5324	5808	6292	6776	7260	7744	9680
23	3174	3703	4232	4761	5290	5819	6348	6877	7406	7935	8464	10,580
24	3456	4032	4608	5184	5760	6336	6912	7488	8064	8640	9216	11,520
28	4704	5488	6272	7056	7840	8624	9408	10,192	10,976	11,760	12,544	15,680
30	5400	6300	7200	8100	9000	9900	10,800	11,700	12,600	13,500	14,400	18,000
34	6936	8092	9248	10,404	11,560	12,716	13,872	15,028	16,184	17,340	18,496	23,120
40	9600	11,200	12,800	14,400	16,000	17,600	19,200	20,800	22,400	24,000	25,600	32,000

Table A.9 Rectangular Sections with Compression Steel Minimum Steel Percentage $100(\rho - \rho')$ for Compression Steel to Yield

$$(\rho - \rho') \geq 0.85\ \beta_1 \frac{f'_c}{f_y} \times \frac{d'}{d} \times \frac{87}{(87 - f_y)} \quad (f_c \text{ and } f_y \text{ in ksi})$$

f'_c (psi)	β_1	d'/d	f_y 40 ksi	f_y 50 ksi	f_y 60 ksi	f_y 75 ksi
3000	0.85	0.10	1.00	1.02	1.16	2.09
4000	0.85	0.10	1.33	1.35	1.55	2.78
5000	0.80	0.10	1.57	1.59	1.81	3.27
6000	0.75	0.10	1.78	1.81	2.06	3.71
3000	0.85	0.12	1.20	1.22	1.39	2.51
4000	0.85	0.12	1.60	1.62	1.86	3.34
5000	0.80	0.12	1.88	1.91	2.17	3.92
6000	0.75	0.12	2.14	2.17	2.47	4.45
3000	0.85	0.15	1.50	1.53	1.74	3.14
4000	0.85	0.15	2.00	2.03	2.33	4.17
5000	0.80	0.15	2.36	2.39	2.72	4.91
6000	0.75	0.15	2.67	2.72	3.09	5.57

Note: Minimum $(\rho - \rho')$ for any value of $d'/d = 10 \times (d'/d) \times$ value shown in table with $d'/d = 0.10$.

Table A.10 Modulus of Elasticity of concrete, E_c (Ksi)

Concrete Cylinder Strength (f'_c)	Unit Weight of Concrete (psi) 90	100	110	125	145
3000	1540	1800	2080	2520	3150
4000	1780	2090	2410	2920	3640
5000	1990	2330	2690	3260	4060
6000	2185	2560	2950	3580	4500
7000	2360	2760	3190	3870	4800
8000	2520	2950	3410	4130	5200

Note: $E_c = 33\ W^{1.5}\sqrt{f'_c}$

$E_c = 57,000\sqrt{f'_c} = W = 145$ psf (normal-weight concrete)

Table A.11(a) Values of ℓ_d/d_b for Various Values of f'_c and f_y (Tension Bars)

	$f_y = 40$ ksi				$f_y = 60$ ksi			
	≤No. 6 Bars		≥No. 7 Bars		≤No. 6 Bars		≥No. 7 Bars	
f'_c (ksi)	Conditions Met	Others	Conditions Met	Others	Conditions Met	Others	Conditions Met	Others
3	29.3	43.9	36.6	54.8	43.9	65.8	54.8	82.2
4	25.3	38.0	31.7	47.5	38.0	57.0	47.5	71.2
5	22.7	34.0	28.3	42.5	34.0	51.0	42.5	63.7
6	20.7	31.0	25.9	38.8	31.0	46.5	38.8	58.1

Table A.11(b) Development Length ℓ_d for Tension Bars and $f_y = 60$ Ksi ($\psi_t = \psi_e = \lambda = 1$)

Bar Number	Bar Diameter (in.)	Development Length ℓ_d (in.) — Tension Bars			
		$f'_c = 3$ ksi		$f'_c = 4$ ksi	
		Conditions Met	Others	Conditions Met	Others
3	0.375	17	25	15	21
4	0.500	22	33	19	29
5	0.625	28	41	24	36
6	0.750	33	50	29	43
7	0.875	48	72	42	63
8	1.000	55	83	48	72
9	1.128	62	93	54	81
10	1.270	70	105	61	92
11	1.410	78	116	68	102

Table A.12 Designations, Areas, Perimeters, and Weights of Standard U.S. Bars

Bar No.	Diameter (in.)	Cross-Sectional Area (in.^{2})	Perimeter (in.)	Unit Weight per Foot (lb)	Diameter (mm)	Area (mm^2)
2	$\frac{1}{4} = 0.250$	0.05	0.79	0.167	6.4	32
3	$\frac{3}{8} = 0.375$	0.11	1.18	0.376	9.5	71
4	$\frac{1}{2} = 0.500$	0.20	1.57	0.668	12.7	129
5	$\frac{5}{8} = 0.625$	0.31	1.96	1.043	15.9	200
6	$\frac{3}{4} = 0.750$	0.44	2.36	1.502	19.1	284
7	$\frac{7}{8} = 0.875$	0.60	2.75	2.044	22.2	387
8	$1 = 1.000$	0.79	3.14	2.670	25.4	510
9	$1\frac{1}{8} = 1.128$	1.00	3.54	3.400	28.7	645
10	$1\frac{1}{4} = 1.270$	1.27	3.99	4.303	32.3	820
11	$1\frac{3}{8} = 1.410$	1.56	4.43	5.313	35.8	1010
14	$1\frac{3}{4} = 1.693$	2.25	5.32	7.650	43.0	1450
18	$2\frac{1}{4} = 2.257$	4.00	7.09	13.600	57.3	2580

Table A.13 Areas of Groups of Standard U.S. Bars in Square Inches

Bar Number	Number of Bars											
	1	2	3	4	5	6	7	8	9	10	11	12
3	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	1.00	1.10	1.21	1.32
4	0.20	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36
5	0.31	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68
6	0.44	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.84	5.30
7	0.60	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22
8	0.79	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43
9	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00
10	1.27	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19
11	1.56	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75
14	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00
18	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00

Table A.14 Areas of Bars in Slabs (square inches per foot)

Spacing (in.)	Bar Number								
	3	4	5	6	7	8	9	10	11
3	0.44	0.78	1.23	1.77	2.40	3.14	4.20	5.06	6.25
$3\frac{1}{2}$	0.38	0.67	1.05	1.51	2.06	2.69	3.43	4.34	5.36
4	0.33	0.59	0.92	1.32	1.80	2.36	3.00	3.80	4.68
$4\frac{1}{2}$	0.29	0.52	0.82	1.18	1.60	2.09	2.67	3.37	4.17
5	0.26	0.47	0.74	1.06	1.44	1.88	2.40	3.04	3.75
$5\frac{1}{2}$	0.24	0.43	0.67	0.96	1.31	1.71	2.18	2.76	3.41
6	0.22	0.39	0.61	0.88	1.20	1.57	2.00	2.53	3.12
$6\frac{1}{2}$	0.20	0.36	0.57	0.82	1.11	1.45	1.85	2.34	2.89
7	0.19	0.34	0.53	0.76	1.03	1.35	1.71	2.17	2.68
$7\frac{1}{2}$	0.18	0.31	0.49	0.71	0.96	1.26	1.60	2.02	2.50
8	0.17	0.29	0.46	0.66	0.90	1.18	1.50	1.89	2.34
9	0.15	0.26	0.41	0.59	0.80	1.05	1.33	1.69	2.08
10	0.13	0.24	0.37	0.53	0.72	0.94	1.20	1.52	1.87
12	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56

Table A.15 Common Styles of Welded Wire Fabric

Style Designation	Steel Area (in.^{2} ft) Longitudinal	Steel Area (in.^{2} ft) Transverse	Weight Approx. lb/100 ft^2
6 × 6—W1.4 × W1.4	0.03	0.03	21
6 × 6—W2 × W2	0.04	0.04	29
6 × 6—W2.9 × W2.9	0.06	0.06	42
6 × 6—W4 × W4	0.08	0.08	58
6 × 6—W5.5 × W5.5	0.11	0.11	80
4 × 4—W1.4 × W1.4	0.04	0.04	31
4 × 4—W2 × W2	0.06	0.06	43
4 × 4—W2.9 × W2.9	0.09	0.09	62
4 × 4—W4 × W4	0.12	0.12	86

Table A.16 Size and Pitch of Spirals

f_y (ksi)	Diameter of Column (in.)	Outside to Outside of Spiral (in.)	f'_c (psi) 3000	f'_c (psi) 4000	f'_c (psi) 5000
40	14, 15	11, 12	$\frac{3}{8} - 1\frac{3}{4}$	$\frac{1}{2} - 2\frac{1}{2}$	$\frac{1}{2} - 1\frac{3}{4}$
	16	13	$\frac{3}{8} - 1\frac{3}{4}$	$\frac{1}{2} - 2\frac{1}{2}$	$\frac{1}{2} - 2$
	17–19	14–16	$\frac{3}{8} - 1\frac{3}{4}$	$\frac{1}{2} - 2\frac{1}{2}$	$\frac{1}{2} - 2$
	20–23	17–20	$\frac{3}{8} - 1\frac{3}{4}$	$\frac{1}{2} - 2\frac{1}{2}$	$\frac{1}{2} - 2$
	24–30	21–27	$\frac{3}{8} - 2$	$\frac{1}{2} - 2\frac{1}{2}$	$\frac{1}{2} - 2$
60	14, 15	11, 12	$\frac{3}{8} - 2\frac{3}{4}$	$\frac{3}{8} - 2$	$\frac{1}{2} - 2\frac{3}{4}$
	16–23	13–20	$\frac{3}{8} - 2\frac{3}{4}$	$\frac{3}{8} - 2$	$\frac{1}{2} - 2\frac{3}{4}$
	24–29	21–26	$\frac{3}{8} - 3$	$\frac{3}{8} - 2\frac{1}{4}$	$\frac{1}{2} - 3$
	30	27	$\frac{3}{8} - 3$	$\frac{3}{8} - 2\frac{1}{4}$	$\frac{1}{2} - 3\frac{1}{4}$

APPENDIX B

DESIGN TABLES (SI UNITS)

Table B.1 Values of R_u and a/d for $f'_c = 21$ MPa (R_u in MPa), ($\varepsilon_t \geq 0.005$, $\phi = 0.9$ and $d = d_t$)

100	$f_y = 280$ MPa		$f_y = 350$ MPa		$f_y = 420$ MPa		$f_y = 520$ MPa	
ρ	R_u	a/d	R_u	a/d	R_u	a/d	R_u	a/d
0.2	0.50	0.031	0.62	0.039	0.75	0.047	0.92	0.059
0.3	0.74	0.046	0.92	0.059	1.10	0.071	1.35	0.089
0.4	0.98	0.062	1.22	0.078	1.45	0.094	1.79	0.118
0.5	1.21	0.078	1.50	0.098	1.79	0.118	2.18	0.148
0.6	1.45	0.094	1.79	0.118	2.12	0.141	2.59	0.177
0.7	1.68	0.110	2.06	0.138	2.44	0.165	2.96	0.207
0.8	1.90	0.126	2.33	0.157	2.75	0.189	3.34	0.236
0.9	2.12	0.142	2.59	0.177	3.05	0.213	3.68	0.266
1.0	2.33	0.157	2.84	0.196	3.35	0.238	4.02	0.295
1.1	2.55	0.173	3.10	0.216	3.64	0.260	4.36	0.325
1.2	2.74	0.188	3.35	0.235	3.91	0.280	4.32	0.319
1.3	2.95	0.204	3.59	0.255	4.18	0.306	($\rho_{max} = 1.085$)	
1.4	3.16	0.220	3.82	0.274	4.44	0.330		
1.5	3.35	0.236	4.04	0.294				
1.6	3.54	0.252	4.27	0.314				
1.7	3.73	0.267			4.32	0.319		
1.8	3.91	0.282			($\rho_{max} = 1.37$)			
1.9	4.09	0.298						
2.0	4.27	0.314						
2.1	4.43	0.330	4.32	0.319				
			($\rho_{max} = 1.63$)					
	4.32	0.319						
	($\rho_{max} = 2.04$)							

Note: Last values are the maximum for $\varepsilon_t = 0.005$.

Table B.2 Values of R_u and a/d for $f'_c = 28$ MPa (R_u in MPa), ($\varepsilon_t \geq 0.005$, $\phi = 0.9$ and $d = d_t$)

100	$f_y = 280$ MPa		$f_y = 350$ MPa		$f_y = 420$ MPa		$f_y = 520$ MPa	
ρ	R_u	a/d	R_u	a/d	R_u	a/d	R_u	a/d
0.2	0.50	0.024	0.63	0.029	0.75	0.025	0.93	0.044
0.3	0.74	0.036	0.93	0.044	1.11	0.053	1.36	0.066
0.4	0.98	0.047	1.23	0.059	1.46	0.071	1.81	0.088
0.5	1.23	0.059	1.53	0.074	1.81	0.089	2.23	0.110
0.6	1.46	0.071	1.83	0.088	2.16	0.106	2.66	0.132
0.7	1.70	0.083	2.11	0.103	2.50	0.123	3.05	0.154
0.8	1.93	0.094	2.39	0.118	2.81	0.141	3.45	0.176
0.9	2.16	0.106	2.66	0.132	2.14	0.158	3.83	0.198
1.0	2.39	0.118	2.95	0.147	3.46	0.176	4.22	0.220
1.1	2.60	0.130	3.20	0.161	3.77	0.194	4.57	0.242
1.2	2.81	0.141	3.46	0.176	4.08	0.212	4.94	0.264
1.3	3.04	0.153	3.73	0.191	4.36	0.230	5.29	0.286
1.4	3.25	0.165	3.97	0.206	4.65	0.247		
1.5	3.46	0.177	4.22	0.221	4.92	0.265		
1.6	3.67	0.188	4.46	0.236	5.22	0.282	5.77	0.319
							($\rho_{max} = 1.45$)	
1.7	3.87	0.200	4.71	0.250	5.48	0.300		
1.8	4.08	0.212	4.94	0.265	5.75	0.318		
1.9	4.27	0.224	5.17	0.280				
2.0	4.46	0.236	5.40	0.294				
2.1	4.65	0.248	5.62	0.309				
2.2	4.85	0.260			5.77	0.319		
2.3	5.04	0.271			($\rho_{max} = 1.82$)			
2.4	5.22	0.282						
2.5	5.39	0.294						
2.6	5.57	0.306						
2.7	5.74	0.318						
2.8	5.92	0.330	5.77	0.319				
			($\rho_{max} = 2.18$)					
	5.77	0.319						
	($\rho_{max} = 2.73$)							

Note: Last values are the maximum for $\varepsilon_t = 0.005$.

Table B.3 Values of R_u and a/d for $f'_c = 35$ MPa (R_u in MPa), ($\varepsilon_t \geq 0.005$, $\phi = 0.9$ and $d = d_t$)

100	$f_y = 350$ MPa		$f_y = 420$ MPa		$f_y = 520$ MPa	
ρ	R_u	a/d	R_u	a/d	R_u	a/d
0.2	0.63	0.024	0.75	0.028	0.93	0.035
0.3	0.93	0.036	1.12	0.042	1.38	0.052
0.4	1.24	0.047	1.48	0.056	1.83	0.070
0.5	1.53	0.060	1.83	0.070	2.26	0.088
0.6	1.83	0.071	2.18	0.085	2.70	0.106
0.7	2.12	0.083	2.53	0.100	3.11	0.123
0.8	2.41	0.094	2.87	0.113	3.52	0.141
0.9	2.69	0.106	3.20	0.127	3.91	0.159
1.0	2.98	0.118	3.53	0.141	4.30	0.177
1.1	3.26	0.130	3.87	0.155	4.69	0.195
1.2	3.52	0.141	4.17	0.169	5.08	0.212
1.3	3.80	0.153	4.48	0.183	5.46	0.230
1.4	4.06	0.165	4.79	0.198	5.84	0.247
1.5	4.32	0.177	5.09	0.212	6.15	0.265
1.6	4.58	0.188	5.39	0.226	6.47	0.282
1.7	4.84	0.200	5.68	0.240	6.82	0.300
1.8	5.09	0.212	5.96	0.254		
1.9	5.34	0.224	6.26	0.268		
2.0	5.58	0.235	6.52	0.282		
2.1	5.84	0.247	6.78	0.296		
2.2	6.06	0.259				
2.3	6.29	0.271				
2.4	6.52	0.282				
2.5	6.74	0.294				
					6.85	0.3192
					($\rho_{max} = 1.71$)	
			6.85	0.319		
			($\rho_{max} = 2.16$)			
	6.85	0.319				
	($\rho_{max} = 2.57$)					

Note: Last values are the maximum for $\varepsilon_t = 0.005$.

Table B.4 Values of ρ_{max}, $R_{u\,max}$, ρ_b, ρ_{min}

$\rho_b = 0.85\beta_1(f'_c/f_y)[87/(87+f_y)]$ $\rho_{max} = (0.003 + f_y/E_s)\rho_b/0.008$ $R_u = \phi\rho f_y[1 - \rho f_y/1.7f'_c]$

	$f_y = 280$ MPa				$f_y = 350$ MPa			
f'_c MPa	100 ρ_{max}	$R_{u\,max}$ MPa	100 ρ_b	100 ρ_{min}	100 ρ_{max}	$R_{u\,max}$ MPa	100 ρ_b	100 ρ_{min}
21	2.031	4.32	3.71	0.50	1.624	4.32	2.75	0.40
28	2.715	5.77	4.96	0.50	2.167	5.77	3.67	0.40
35	3.180	6.85	5.81	0.53	2.551	6.85	4.32	0.42
42	3.575	7.78	6.53	0.58	2.864	7.78	4.85	0.47
	$f_y = 420$ MPa				$f_y = 525$ MPa			
f'_c MPa	100 ρ_{max}	$R_{u\,max}$ MPa	100 ρ_b	100 ρ_{min}	100 ρ_{max}	$R_{u\,max}$ MPa	100 ρ_b	100 ρ_{min}
21	1.356	4.32	2.14	0.33	1.082	4.32	1.55	0.27
28	1.806	5.77	2.85	0.33	1.445	5.77	2.07	0.27
35	2.123	6.85	3.35	0.35	1.704	6.85	2.44	0.28
42	2.389	7.78	3.77	0.39	1.920	7.78	2.75	0.31

Note: ρ_{max} values are for $\varepsilon_t = 0.005$ and $\phi = 0.9$.

Table B.5 Suggested Design Steel Ratios, ρ_s, and Comparison with Other Steel Ratios

f'_c MPa	f_y MPa	100 ρ_b	100 ρ_{max}	100 ρ_s	R_u for ρ_s (MPa)	Ratio ρ_s/ρ_b	Ratio ρ_s/ρ_{max}	Weight of ρ_s (kg/m^3 of concrete)
21	280	3.71	2.04	1.4	3.16	0.377	0.689	112
	350	2.75	1.63	1.2	3.35	0.436	0.739	96
	420	2.15	1.37	1.2	3.91	0.558	0.885	96
28	280	4.96	2.73	1.4	3.25	0.282	0.516	112
	350	3.67	2.18	1.4	3.97	0.381	0.646	112
	420	2.85	1.81	1.4	4.65	0.491	0.775	112
35	280	5.81	3.20	1.6	3.72	0.275	0.503	128
	350	4.32	2.57	1.6	4.58	0.370	0.627	128
	420	3.35	2.16	1.6	5.39	0.478	0.754	128

Note: ρ_{max} values are for $\varepsilon_t = 0.005$ and $\phi = 0.9$.

Table B.6 Minimum Thickness of Beams and One-Way Slabs

Member	Yield Strength f_y (MPa)	Simply Supported	One End Continuous	Both Ends Continuous	Cantilever
Solid one-way slabs	280	*L*/25	*L*/30	*L*/35	*L*/12.5
	350	*L*/22	*L*/27	*L*/31	*L*/11
	420	*L*/20	*L*/24	*L*/28	*L*/10
Beams or ribbed one-way slabs	280	*L*/20	*L*/23	*L*/26	*L*/10
	350	*L*/18	*L*/20.5	*L*/23.5	*L*/9
	420	*L*/16	*L*/18.5	*L*/21	*L*/8

Table B.7 Rectangular Sections with Compression Steel. Minimum Steel Percentage 100 $(\rho - \rho')$ for Compression Steel to Yield

$$(\rho - \rho') \geq 0.85\beta_1\left(\frac{f'_c}{f_y}\right) \times \left(\frac{d'}{d}\right) \times \frac{600}{(600 - f_y)}, (f_y, f'_c \text{ in MPa})$$

f'_c MPa	β_1	d'/d	f_y 300 MPa	f_y 400 MPa	f_y 500 MPa
21	0.85	0.10	1.20	1.35	2.16
28	0.85	0.10	1.45	1.63	2.60
35	0.80	0.10	1.59	1.80	2.85
42	0.75	0.10	1.70	1.91	3.06
21	0.85	0.12	1.45	1.63	2.60
28	0.85	0.12	1.73	1.95	3.12
35	0.80	0.12	2.02	2.27	3.64
42	0.75	0.12	2.04	2.29	3.67
21	0.85	0.15	1.81	2.03	3.25
28	0.85	0.15	2.17	2.44	3.90
35	0.80	0.15	2.38	2.68	4.28
42	0.75	0.15	2.55	2.87	4.59

Note: Minimum $(\rho - \rho')$ for any value of $d'/d = 10 \times (d'/d) \times$ value shown in table with $d'/d = 0.10$.

Table B.8 Modulus of Elasticity of Normal-Weight Concrete

General: $E_c = 0.043W^{1.5}\sqrt{f'_c}$ **MPa**
For Normal-Weight Concrete, $W = 2350$ **kg/m³:**

$$E_c = 4730\sqrt{f'_c}\ \text{MPa}$$

f'_c MPa	E_c (kN/mm²)
17.5	20.0
21.0	22.5
28.0	25.0
35.0	29.0
42.0	32.0
49.0	33.5
56.0	36.5

Table B.9(a) Values of ℓ_d/d_b for Various Values of f'_c and f_y (Tension Bars)

f'_c MPa	$f_y = 300$ MPa				$f_y = 400$ MPa			
	≤20 M Bars		≥25 M Bars		≤20 M Bars		≥25 M Bars	
	Conditions Met	Others	Conditions Met	Others	Conditions Met	Others	Conditions Met	Others
20	34.0	50.5	42.0	63.0	45.0	67.0	56.0	84.0
30	27.5	41.5	34.5	51.5	36.5	55.0	46.0	68.5
35	25.5	38.5	32.0	47.5	34.0	51.0	42.5	63.5
40	23.5	35.5	29.5	44.5	31.5	47.5	39.5	59.5

Table B.9(b) Development Length ℓ_d/d_b for Tension Bars and $f_y = 400$ MPa ($\alpha = \beta = \lambda = 1.0$)

		Development Length ℓ_d/d_d (mm) — Tension Bars			
		$f'_c = 20$ MPa		$f'_c = 30$ MPa	
Bar Number	**Bar Diameter (mm)**	**Conditions Met**	**Others**	**Conditions Met**	**Others**
10M	11.3	Ê510	Ê765	Ê415	Ê620
15M	16.0	Ê720	1080	Ê585	Ê875
20M	19.5	Ê880	1320	Ê710	1070
25M	25.2	1410	2120	1160	1740
30M	29.9	1675	2510	1375	2065
35M	35.7	2000	3000	1640	2465

Table B.10 Designations, Areas, and Mass of Bars

	Nominal Dimensions		
Bar No.	**Diameter (mm)**	**Area (mm^2)**	**Mass (kg/m)**
#10	9.5	71	0.560
#13	12.7	129	0.994
#16	15.9	199	1.552
#19	19.1	284	2.235
#22	22.2	387	3.042
#25	25.4	510	3.973
#29	28.7	645	5.060
#32	32.3	819	6.404
#36	35.8	1006	7.907
#43	43.0	1452	11.38
#57	57.3	2581	20.24

ASTM A615M Grade 300 is limited to sizes #10 through #19; otherwise, grades are 400 or 500 MPa. (These bars are soft conversion of #3 to #18 in U.S. customary units.)

Table B.11 ASTM Standard Metric Reinforcing Bars

	Nominal Dimensions		
Bar-size Designation (number)	**Diameter (mm)**	**Area (mm^2)**	**Mass (kg/m)**
10M	11.3	100	0.785
15M	16.0	200	1.570
20M	19.5	300	2.355
25M	25.2	500	3.925
30M	29.9	700	5.495
35M	35.7	1000	7.850
45M	43.7	1500	11.775
55M	56.4	2500	19.625

ASTM A615M grade 300 is limited to size 10M through 20M; otherwise, grades are 400 or 500 MPa.

Table B.12 Areas of Group of Bars (mm²) — Metric

Bar No. Metric	Number of Bars 1	2	3	4	5	6	7	8	9	10
#10	71	142	213	384	355	426	497	568	639	710
#13	129	258	387	516	645	774	903	1032	1161	1290
#16	199	398	597	796	995	1194	1393	1592	1791	1990
#19	284	568	852	1136	1420	1704	1988	2272	2556	2840
#22	387	774	1161	1548	1935	2322	2709	3096	3483	3870
#25	510	1020	1530	2040	2550	3060	3570	4080	4590	5100
#29	645	1290	1935	2580	3225	3870	4515	5160	5805	6450
#32	819	1638	2457	3276	4095	4914	5733	6552	7371	8190
#36	1006	2012	3018	4024	5030	6036	7042	8048	9054	10060

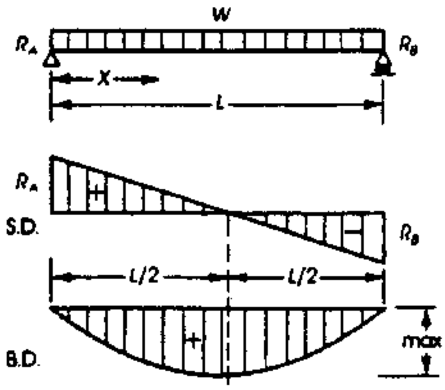
APPENDIX C

STRUCTURAL AIDS

Note: S.S. stands for shearing force diagram. B.D. stands for bending moment diagram.
Bending moments are drawn on the tension sides of beams.
Moments, shearing forces and deflections, for any combination of loadings, are obtained by superposition.

Table C.1 Simple Beams (w = Load/Unit Length)

1. Uniform load



$$W = \text{total load} = wL$$

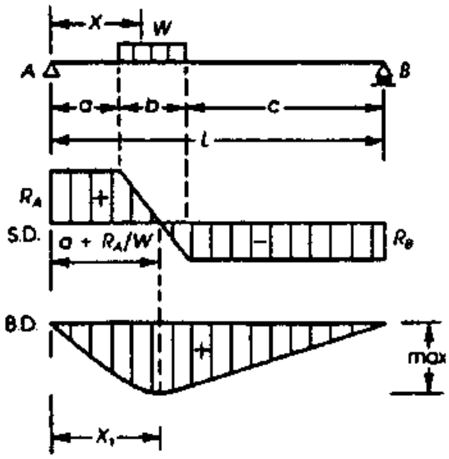
$$R_A = R_B = V_A = V_B = \frac{W}{2}$$

$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right)$$

$$M_{\max} = \frac{WL}{8} \quad \text{(at center)}$$

$$\Delta_{\max} = \frac{5}{384} \times \frac{WL^3}{EI} \quad \text{(at center)}$$

2. Uniform partial load



$$W = \text{total load} = wb$$

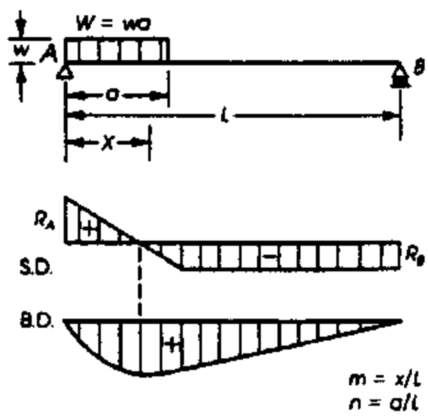
$$R_A = V_A = \frac{W}{L}\left(\frac{b}{2} + c\right)$$

$$R_B = V_B = \frac{W}{L}\left(\frac{b}{2} + a\right)$$

$$M_{\max} = \frac{W}{2b}(x^2 - a^2) \quad \text{when } x = a + \frac{R_A b}{W}$$

$$\Delta_{\max} = \frac{W}{384EI}(8L^3 - 4Lb^2 + b^3) \quad \text{when } a = c$$

3. Uniform partial load at one end



$$W = \text{total load} = wa$$

$$R_A = V_A = W\left(1 - \frac{a}{2L}\right)$$

$$R_B = V_B = \frac{Wa}{2L}$$

$$M_{\max} = \frac{Wa}{2}\left(1 - \frac{a}{2L}\right)^2 \quad \text{when } x = a\left(1 - \frac{a}{2L}\right)$$

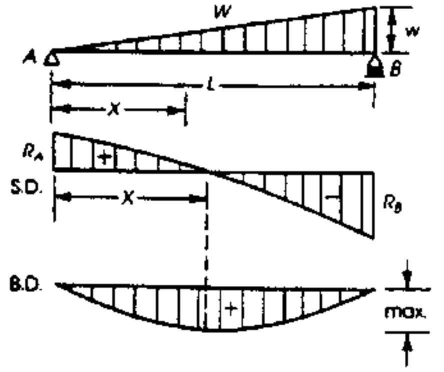
$$\Delta = \frac{WL^4}{24aEI} n^2[2m^3 - 6m^2 + m(4 + n^2) - n^2]$$

when $x \geq a$

$$\Delta = \frac{WL^4 m}{24aEI}[n^2(2 - n)^2 - 2nm^2(2 - n) + m^3] \quad \text{when } x < a$$

Table C.1 *(continued)*

4. Triangular load on span with maximum value at one end



$$W = \text{total load} = \frac{wL}{2}$$

$$R_A = V_A = \frac{W}{3}$$

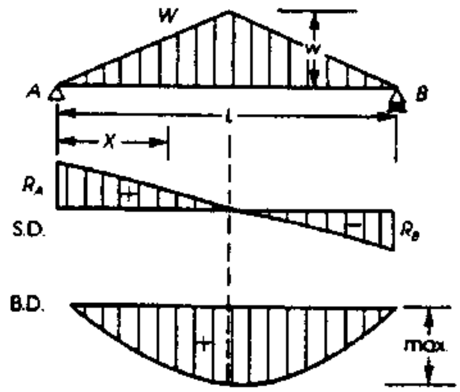
$$R_B = V_B = \frac{2W}{3}$$

$$M_x = \frac{Wx}{3}\left(1 - \frac{x^2}{L^2}\right)$$

$$M_{max} = 0.128WL \quad \text{when } x = 0.5774L$$

$$\Delta_{max} = \frac{0.01304WL^3}{EI} \quad \text{when } x = 0.5193L$$

5. Triangular load with maximum value at midspan



$$W = \text{total load} = \frac{wL}{2}$$

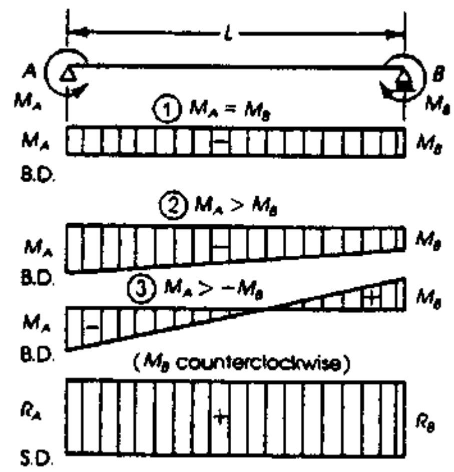
$$R_A = R_B = V_A = V_B = \frac{W}{2}$$

$$M_x = Wx\left(\frac{1}{2} - \frac{2x^2}{3L^2}\right)$$

$$M_{max} = \frac{WL}{6} \quad \text{(at midspan)}$$

$$\Delta_{max} = \frac{WL^3}{60EI} \quad \text{(at midspan)}$$

6. Moments at ends



$$R_A = R_B = V_A = V_B = \frac{M_A - M_B}{L}$$

$$\Delta_{max} \text{ (at midspan)} = \frac{ML^2}{8EI} \quad \text{when } M_A = M_B$$

$$\Delta \text{ (at midspan)} = \frac{M_A L^2}{16EI} \quad \text{when } M_B = 0$$

$$\Delta \text{ (at midspan)} = \frac{M_B L^2}{16EI} \quad \text{when } M_A = 0$$

(continued)

Table C.1 *(continued)*

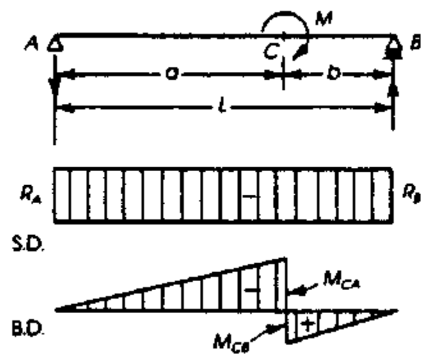
7. External moment at any point

$$R_A = -R_B = V_A = V_B = \frac{M}{L}$$

$$M_{CA} = \frac{Ma}{L}$$

$$M_{CB} = \frac{Mb}{L}$$

$$\Delta_c = \frac{-Mab}{3EIL}(a-b)$$

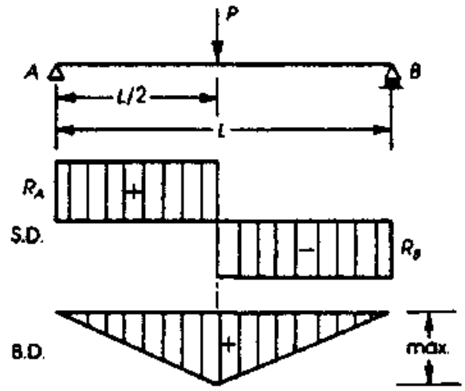


8. Concentrated load at midspan

$$R_A = R_B = V_A = V_B = \frac{P}{2}$$

$$M_{max} = \frac{PL}{4} \quad \text{(at midspan)}$$

$$\Delta_{max} = \frac{PL^3}{48EI} \quad \text{(at midspan)}$$



9. Concentrated load at any point

$$R_A = V_A = \frac{Pb}{L}$$

$$R_B = V_B = \frac{Pa}{L}$$

$$M_{max} = \frac{Pab}{L} \quad \text{(at point load)}$$

$$\Delta_c = \frac{Pa^2b^2}{3EIL} \quad \text{(at point load)}$$

$$\Delta_{max} = \frac{PL^3}{48EI}\left[\frac{3a}{L} - 4\left(\frac{a}{L}\right)^3\right] \quad \text{(when } a \geq b\text{)}$$

at $x = \sqrt{a(b+L)/3}$

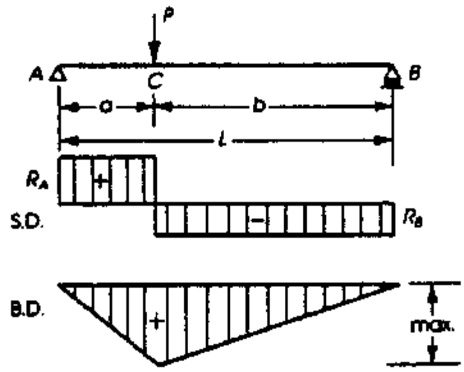


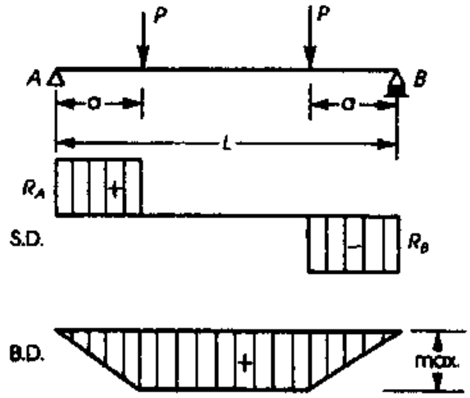
Table C.1 (*continued*)

10. Two symmetrical concentrated loads

$R_A = R_B = V_A = V_B = P$

$M_{\max} = Pa$

$\Delta_{\max} = \frac{PL^3}{6EI}\left[\frac{3a}{4L} - \left(\frac{a}{L}\right)^3\right]$ (at midspan)



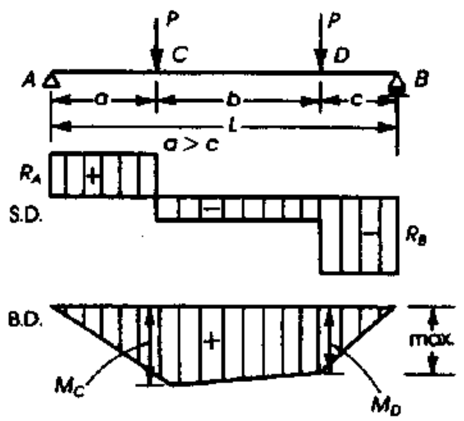
11. Two concentrated loads

$R_A = V_A = \frac{P(b+2c)}{L}$

$R_B = V_B = \frac{P(b+2a)}{L}$

$M_C = \frac{Pa(b+2c)}{L}$

$M_D = \frac{Pc(b+2a)}{L}$

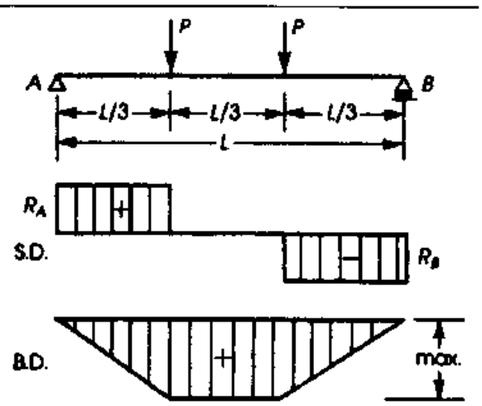


12. Two concentrated loads at one-third points

$R_A = R_B = V_A = V_B = P$

$M_{\max} = \frac{PL}{3}$

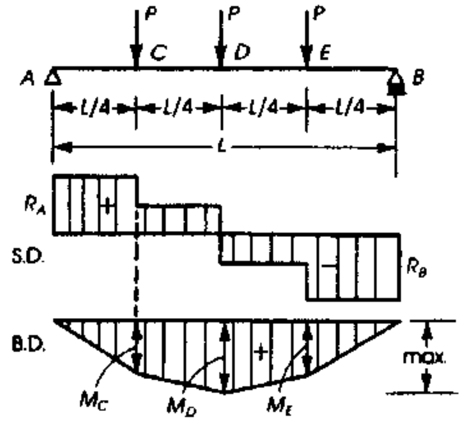
$\Delta_{\max} = \frac{23PL^3}{648EI}$ (at midspan)



(*continued*)

Table C.1 *(continued)*

13. Three concentrated loads at one-fourth points



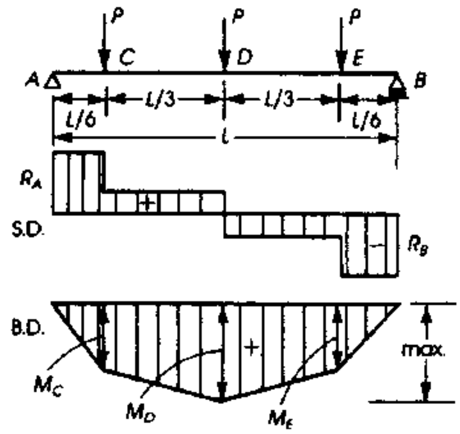
$$R_A = R_B = V_A = V_B = \frac{3P}{2}$$

$$M_C = M_E = \frac{3PL}{8}$$

$$M_D = \frac{PL}{2}$$

$$\Delta_{max} = \frac{19PL^3}{384EI} \quad \text{(at midspan)}$$

14. Three concentrated loads as shown



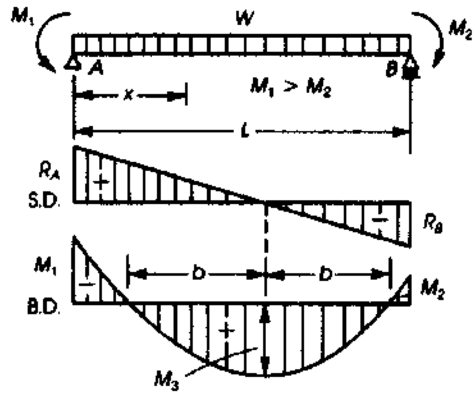
$$R_A = R_B = V_A = V_B = \frac{3P}{2}$$

$$M_C = M_E = \frac{PL}{4}$$

$$M_D = \frac{5PL}{12}$$

$$\Delta_{max} = \frac{53PL^3}{1296EI} \quad \text{(at midspan)}$$

15. Uniformly distributed load and variable end moments



W = total load = wL

$$R_A = V_A = \frac{W}{2} + \frac{M_1 - M_2}{L}$$

$$R_B = V_B = \frac{W}{2} - \frac{M_1 - M_2}{L}$$

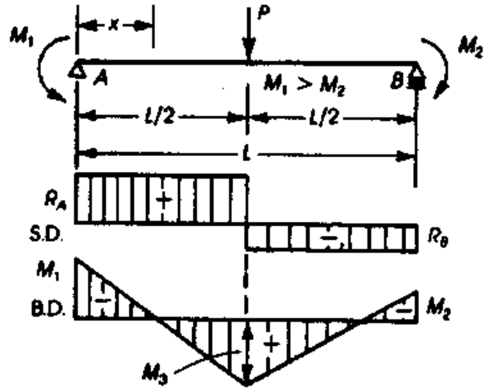
$$M_3 = \frac{WL}{8} - \frac{M_1 + M_2}{2} + \frac{(M_1 - M_2)^2}{2WL}$$

$$\text{at } x = \frac{L}{2} + \frac{M_1 - M_2}{W}$$

$$\Delta_x = \frac{Wx}{24EIL}\left[x^3 - \left(2L + \frac{4M_1}{W} - \frac{4M_2}{W}\right)x^2 + \frac{12M_1L}{W}x + L^3 - \frac{8M_1L^2}{W} - \frac{4M_2L^2}{W}\right]$$

Table C.1 *(continued)*

16. Concentrated load at center and variable end moments



$$R_A = V_A = \frac{P}{2} + \frac{M_1 - M_2}{L}$$

$$R_B = V_B = \frac{P}{2} - \frac{M_1 - M_2}{L}$$

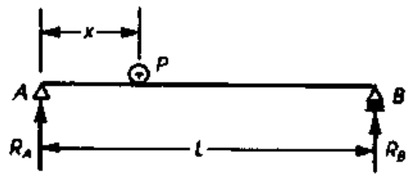
$$M_3 = \frac{PL}{4} - \frac{M_1 + M_2}{2} \quad \text{(at midspan)}$$

$$M_x = \left(\frac{P}{2} + \frac{M_1 - M_2}{L}\right)x - M_1 \quad \text{when } x < \frac{L}{2}$$

$$M_x = \frac{P}{2}\,(L - x) + \frac{(M_1 - M_2)}{L}\,x - M_1 \quad \text{when } x > \frac{L}{2}$$

$$\Delta_x = \frac{Px}{48EI}\left[3L^2 - 4x^2 - \frac{8(L-x)}{PL}\,\{M_1(2L - x) + M_2(L + x)\}\right] \quad \text{when } x < \frac{L}{2}$$

17. One concentrated moving load



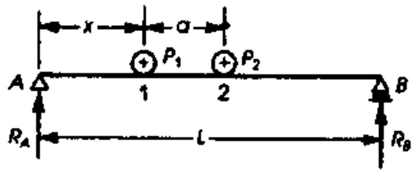
$$R_A \max = V_A \max = P \quad \text{at } x = 0$$

$$R_B \max = V_A \max = P \quad \text{at } x = L$$

$$M_{max} = \frac{PL}{4} \quad \text{at } x = \frac{L}{2}$$

$$M_x = \frac{P}{L}\,(L - x)x$$

18. Two equal concentrated moving loads



$$R_A \max = V_A \max = P\left(2 - \frac{a}{L}\right) \quad \text{at } x = 0$$

$$M_{max} = \frac{P}{2L}\left(L - \frac{a}{2}\right)^2$$

when $a < 0.586L$ under load 1 at $x = \frac{1}{2}\left(L - \frac{a}{2}\right)$

$$M_{max} = \frac{PL}{4} \quad \text{when } a > 0.5L \text{ with one load at midspan}$$

(continued)

Table C.1 *(continued)*

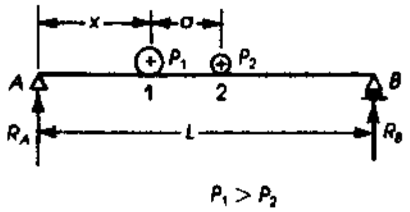
19. Two unequal concentrated moving loads

$$R_A\text{max} = V_A\text{max} = P_1 + P_2\left(\frac{L-a}{L}\right) \quad \text{at } x = 0$$

$$M_{\text{max}} = (P_1 + P_2)\frac{x^2}{L}$$

$$\text{under load } P_1 \text{ at } x = \frac{1}{2}\left(L - \frac{P_2 a}{P_1 + P_2}\right)$$

$$M_{\text{max}} = \frac{P_1 L}{4}$$ may occur with larger load at center of span and other load off span



20. General rules for simple beams carrying moving concentrated loads V_{max} occurs at one support and other loads on span (trial method) For M_{max}:place center line of beam midway between center of gravity of loads and nearest concentrated load. M_{max} occurs under this load (here P_1)

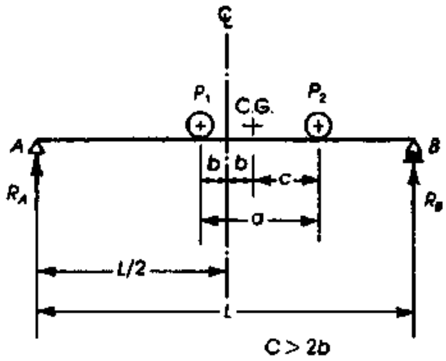


Table C.2 Cantilever Beams

21. Uniform load

W = total load = wL

$R_A = V_A = W$

$M_A = \dfrac{WL}{2}$ (at support A)

$$M_x = \frac{Wx^2}{2L}$$

$$\Delta_B \text{ max} = \frac{WL^3}{8EI}$$

$$\Delta_x = \frac{W}{24EIL}(x^4 - 4L^3x + 3L^4)$$

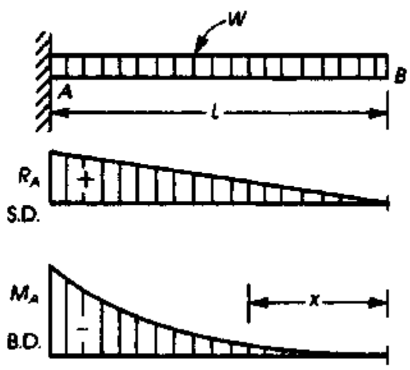


Table C.2 *(continued)*

22. Partial uniform load starting from support

W = total load = wa

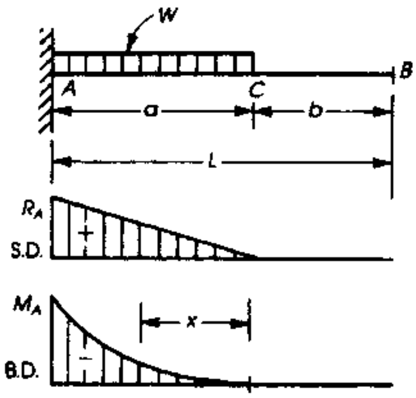
$R_A = V_A = W$

$M_A = \dfrac{Wa}{2}$ (at support A)

$M_x = \dfrac{Wx^2}{2a}$

$\Delta_C = \dfrac{Wa^3}{8EI}$

$\Delta_B \max = \dfrac{Wa^3}{8EI}\left(1 + \dfrac{4b}{3a}\right)$



23. Concentrated load

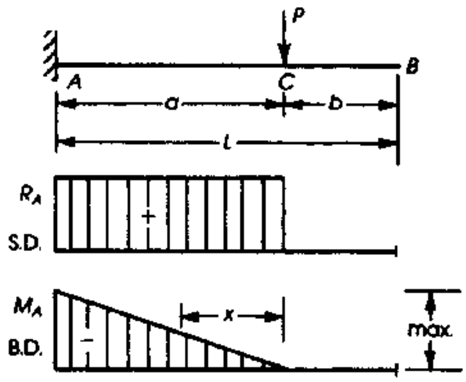
$R_A = V_A = P$

$M_{\max} = Pa$ (at support A)

$M_x = Px$

$\Delta_C = Pa^3/3EI$

$\Delta_B \max = \dfrac{Pa^3}{3EI}\left(1 + \dfrac{3b}{2a}\right)$ (at free end)



24. Concentrated load at free end

$R_A = V_A = P$

$M_{\max} = PL$ (at A)

$M_x = Px$

$\Delta_B \max = \dfrac{PL^3}{3EI}$

$\Delta_x = \dfrac{P}{6EI}(2L^3 - 3L^2x + x^3)$

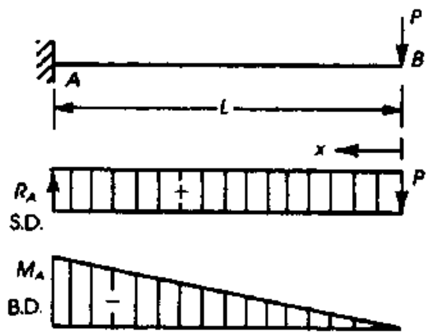


Table C.3 Propped Beams

25. Uniform load

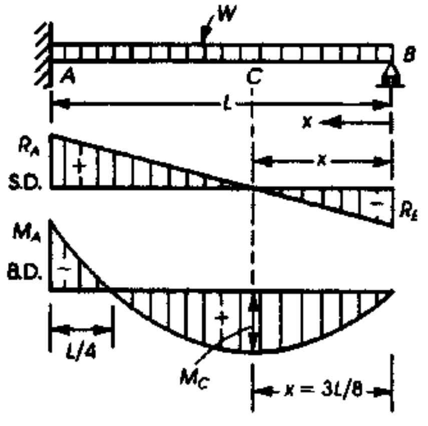
$W = \text{total load} = wL$

$R_A = V_A = \frac{5W}{8} \quad R_B = V_B = \frac{3W}{8}$

$M_A = -\frac{WL}{8} \quad M_C = \frac{9WL}{128}\left(\text{at} x = \frac{3}{8}L\right)$

$\Delta_x = \frac{WL^3}{48EI}(m - 3m^3 + 2m^4) \text{where} m = \frac{x}{L}$

$\Delta_{\max} = \frac{WL^3}{185EI} \text{at a distance} x = 0.4215L \text{(from support B)}$



26. Partial uniform load starting from hinged support

$W = wb \quad n = \frac{b}{L}$

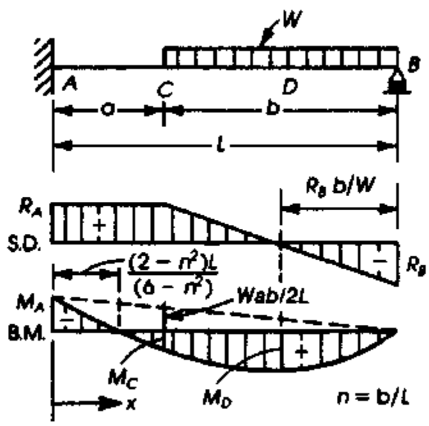
$R_A = V_A = \frac{Wn}{8}(6 - n^2)$

$R_B = V_B = \frac{W}{8}(n^3 - 6n + 8)$

$M_A = -\frac{Wb}{8}(2 - n^2) \quad M_C = \frac{Wb}{8}(6n - n^3 - 4)$

$\Delta_x = \frac{WbL^2}{48EI}[(n^2 - 6)m^3 - (3n^2 - 6)m^2] \text{ when } x \le a$

$\Delta_x = \frac{WL^4}{48bEI}[2P^4 - p^3n(n^3 - 6n + 8) + Pn^2(3n^2 - 8n + 6)] \text{ when } x \ge a \text{ and } P = \frac{L - x}{L}$



27. Partial uniform load starting from fixed end

$W = wa \quad n = \frac{a}{L}$

$R_A = V_A = \frac{W}{8}[8 - n^2(4 - n)]$

$R_B = V_B = \frac{Wn^2}{8}(4 - n) \quad Y = b + an^2(4 - n)$

$M_A = -\frac{W_a}{8}(2 - n)^2$

$M_{\max} = \frac{Wa}{8}\left\{-\frac{[8 - n^2(4 - n)]^2}{16} + 4 - n(4 - n)\right\}$

$\Delta_C = \frac{Wa^3}{48EI}(6 - 12n + 7n^2 - n^3)$

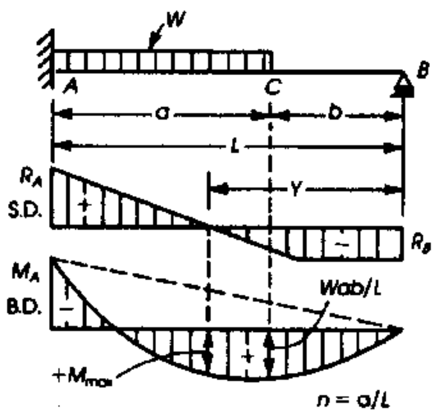


Table C.3 (*continued*)

28. Triangular load on all span L

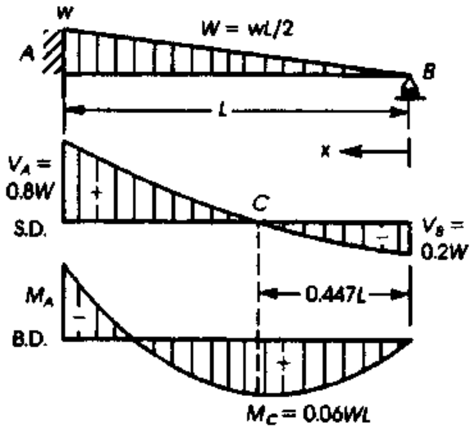
$$W = \text{total load} = \frac{wL}{2}$$

$$R_A = V_A = \frac{4}{5}W \quad R_B = \frac{W}{5} = V_B$$

$$M_A = -\frac{2}{15}WL$$

$$M_C = +\frac{3}{50}WL$$

$$\Delta_{max} = \frac{WL^3}{212EI} \text{ (at } x = 0.447L)$$



29. Triangular load on part of the span

$$W = \frac{wa}{2}$$

$$R_B = V_B = \frac{Wa^2}{20L^3}(5L - a)$$

$$R_A = W - R_B$$

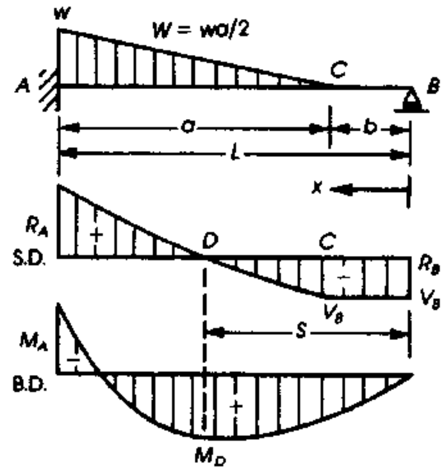
$$M_A = \frac{Wa}{60L^2}(3a^2 - 15aL + 20L^2)$$

Maximum positive moment at

$$S = b + \frac{a^2}{2L}\sqrt{1 - \frac{a}{5L}}$$

M_{max} (positive at) D:

$$M_D = R_B S - \frac{WL}{3a^3}(-b + S)^3$$



30. Concentrated load at midspan

$$R_A = V_A = \frac{11P}{16}$$

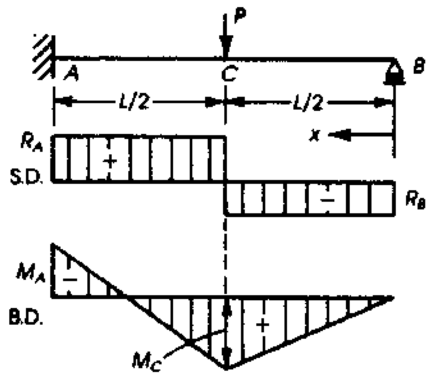
$$R_B = V_B = \frac{5P}{16}$$

$$M_A = -\frac{3PL}{16}$$

$$M_C = \frac{5PL}{32}$$

$$\Delta_C = \frac{7PL^3}{768EI}$$

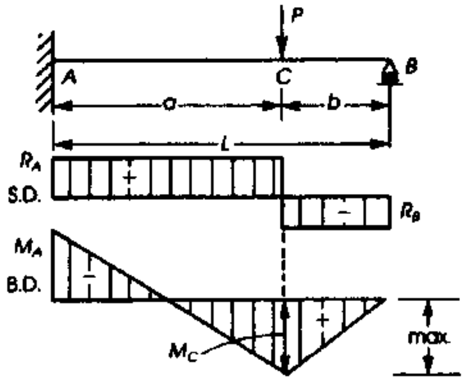
$$\Delta_{max} = \frac{PL^3}{107EI} \text{ (at } x = 0.447L \text{ from B)}$$



(*continued*)

Table C.3 *(continued)*

31. Concentrated load at any point



$$R_A = V_A = P - R_B \quad R_B = V_B = \frac{Pa^2}{2L^3}(b + 2L)$$

$$M_A = -\frac{Pb(L^2 - b^2)}{2L^2}$$

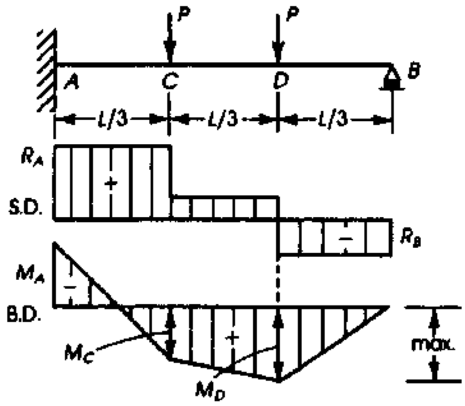
M_A max $= 0.193PL$ when $b = 0.577L$

$$M_C = \frac{Pb}{2}\left(2 - \frac{3b}{L} + \frac{b^3}{L^3}\right)$$

M_C max $= 0.174PL$ when $b = 0.366L$

$$\Delta_C = \frac{Pa^3b^2}{12EIL^3}(4L - a)$$

32. Two concentrated loads at one-third points



$$R_A = V_A = \frac{4P}{3}$$

$$R_B = V_B = \frac{2P}{3}$$

$$M_A = -\frac{PL}{3}$$

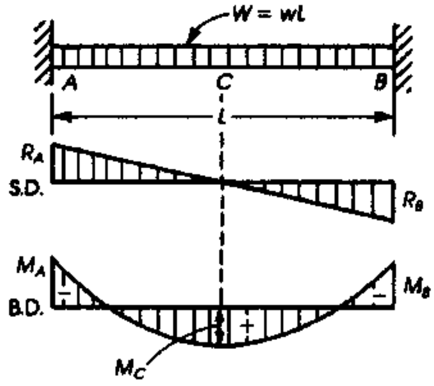
$$M_C = \frac{PL}{9} \quad M_D = \frac{2PL}{9}$$

$$\Delta_{max} = \frac{PL^3}{65.8EI}$$

occurs at point $= 0.423L$ from support B

Table C.4 Fixed End Beams

33. Uniform load



W = total load = wL

$$R_A = V_A = R_B = V_B = \frac{W}{2}$$

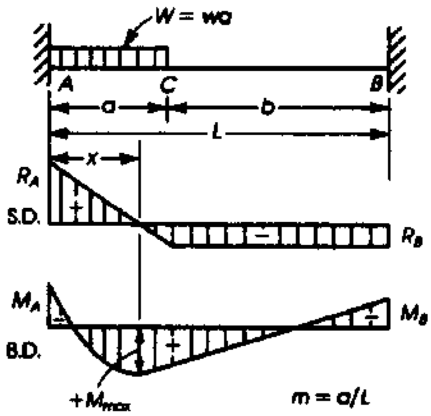
$$M_A = M_B = -\frac{WL}{12} \quad \text{(at support)}$$

$$M_C \text{ max} = \frac{WL}{24} \quad \text{(at midspan)}$$

$$\Delta_{max} = \frac{WL^3}{384EI} \quad \text{(at midspan)}$$

$$\Delta_x = \frac{Wx^3}{24EIL}(L-x)^2 \quad \text{(from } A \text{ or } B\text{)}$$

34. Uniform partial load at one end



W = total load = $wa \quad m = \dfrac{a}{L}$

$$R_A = V_A = \frac{W(m^3 - 2m^2 + 2)}{2}$$

$$R_B = V_B = \frac{Wm^2(2-m)}{2} = W - R_A$$

$$M_A = \frac{WLm}{12}(3m^2 - 8m + 6)$$

$$M_B = \frac{WLm^2}{12}(4 - 3m)$$

$$M_{max} = \frac{WLm^2}{12}\left(-\frac{3}{2}m^5 + 6m^4 - 6m^3 - 6m^2 + 15m - 8\right)$$

when $x = \dfrac{a}{2}(m^3 - 2m^2 + 2)$

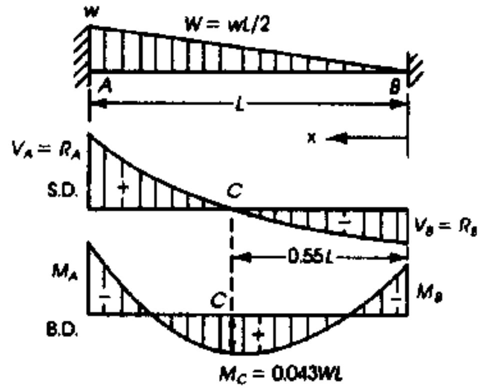
$$\Delta_{max} = \frac{WL^3}{333EI}$$

$$\Delta_C = \frac{WL^3}{384EI}$$

(*continued*)

Table C.4 (*continued*)

35. Triangular load



$$W = \frac{wL}{2}$$

$$R_A = V_A = 0.7W$$

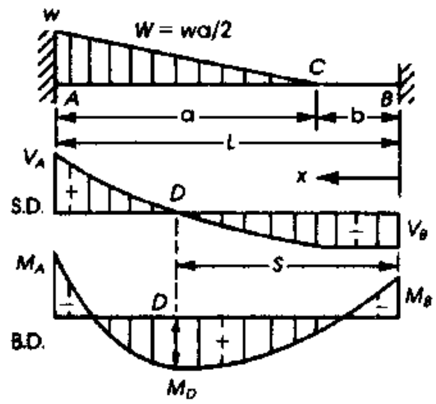
$$R_B = V_B = 0.3W$$

$$M_A = \frac{WL}{10} \quad M_B = \frac{WL}{15}$$

$$\Delta_{max} = \frac{WL^3}{382EI}(\text{at } x = 0.55L \text{ from B})$$

$$M_C(\text{maximum positive moment}) = +\frac{WL}{23.3}(\text{at}0.55L \text{ from B})$$

36. Triangular load on part of the span



$$W = \frac{wa}{2}$$

$$R_B = V_B = \frac{Wa^2}{10L^3}(5L - 2a)$$

$$R_A = W - R_B$$

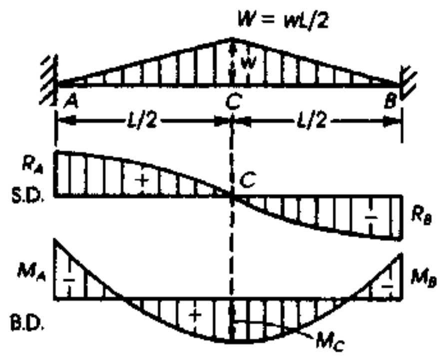
$$M_A = \frac{Wa}{30L^2}(3a^2 + 10bL)$$

$$M_B = \frac{Wa}{30L^2}(-3a^2 + 5aL)$$

Maximum positive moment at $S = b + \frac{a^2}{3.16L}\sqrt{5 - \frac{2a}{L}}$

$$M_D = R_B S - \frac{WL}{3a^3}(a + S - L)^3 - M_B$$

37. Triangular load, maximum intensity at midspan



$$W = \text{total load} = \frac{wL}{2}$$

$$R_A = R_B = \frac{W}{2}$$

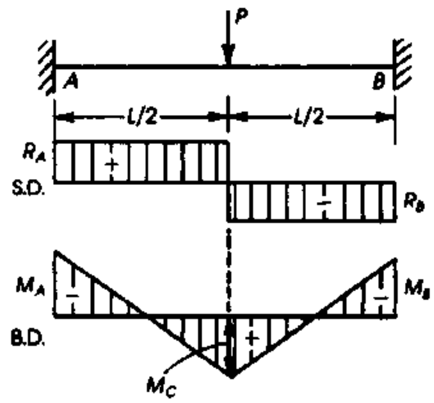
$$M_A = M_B = -\frac{5}{48}WL$$

$$M_C(\text{maximum positive}) = \frac{WL}{16}$$

$$\Delta_{max} = \frac{1.4WL^3}{384EI}(\text{at midspan})$$

Table C.4 *(continued)*

38. Concentrated load at midspan



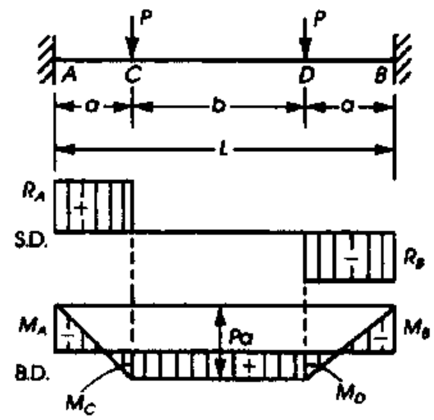
$$R_A = V_A = R_B = V_B = \frac{P}{2}$$

$$M_A = M_B = M_C = -\frac{PL}{8}$$

$$\Delta_{\max} = \frac{PL^3}{192EI}\text{(at midspan)}$$

$$\Delta_x = \frac{Px^2}{48EI}(3L - 4x)\left(x < \frac{L}{2}\right)$$

39. Two symmetrical concentrated loads



$$R_A = V_A = R_B = V_B = P$$

$$M_A = M_B = -\frac{Pa(L-a)}{L}$$

$$M_C = M_D = \frac{Pa^2}{L}$$

$$\Delta_{\max} = \frac{PL^3}{6EI}\left[\frac{3a^2}{4L^2} - \left(\frac{a}{L}\right)^3\right]\text{(at midspan)}$$

If $a = \dfrac{L}{3}$,

$$M_A = M_B = \frac{2}{9}PL$$

$$\Delta_{\max} = \frac{5PL^3}{648EI}\text{(at centerline)}$$

If $a = \dfrac{L}{4}$,

$$M_A = M_B = \frac{3}{16}PL$$

$$\Delta_{\max} = \frac{PL^3}{192EI}\text{(at centerline)}$$

(continued)

Table C.4 *(continued)*

40. Concentrated load at any point

$$R_A = V_A = P\left(\frac{b}{L}\right)^2\left(1+\frac{2a}{L}\right)$$

$$R_B = V_B = P\left(\frac{a}{L}\right)^2\left(1+\frac{2b}{L}\right)$$

$$M_A = -\frac{Pab^2}{L^2} \quad M_B = -\frac{Pba^2}{L^2} \quad M_C = \frac{2Pa^2b^2}{L^3}$$

$$\Delta_C = \frac{Pa^3b^3}{3EIL^3} \quad \text{(at point C)}$$

$$\Delta_{max} = \frac{2Pa^2b^3}{3EI(3L-2a)^2} \quad \text{when } x = \frac{2bL}{3L-2a} \quad \text{and } b > a$$

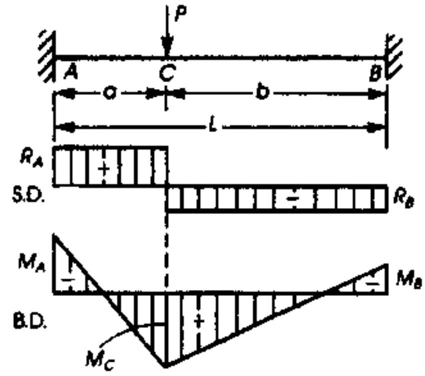
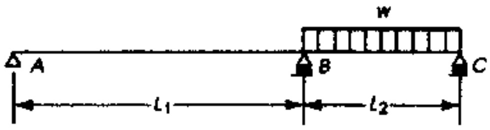


Table C.5 Moments in Two Unequal Spans and Values of the Coefficient K (w = Unit Load/Unit Length)

1. Load on short span

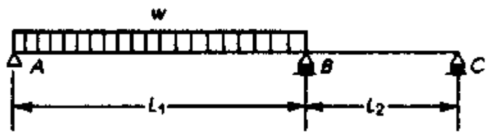
$$M_B = \frac{wL_2^3}{8(L_1+L_2)} = \frac{wL_2^2}{K}$$



L_2/L_1	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
K	46.0	40.0	34.7	28.0	24.0	21.4	19.5	18.0	16.9	15.9

2. Load on long span

$$M_B = \frac{wL_1^3}{8(L_1+L_2)} = \frac{wL_1^2}{K}$$



L_2/L_1	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
K	9.6	10.0	10.4	11.2	12.0	12.8	13.6	14.4	15.2	15.9

3. Both spans loaded with w_1 on L_1 and w_2 on L_2

$$M_B = \frac{w_1L_1^3 + w_2L_2^3}{8(L_1+L_2)}$$

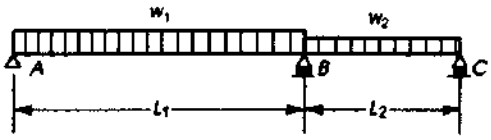
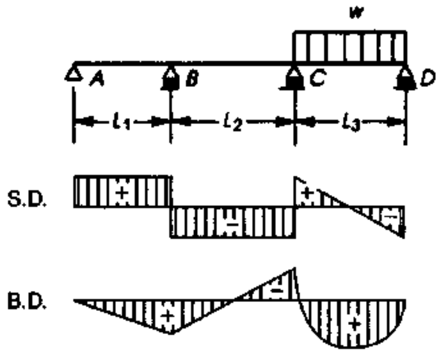


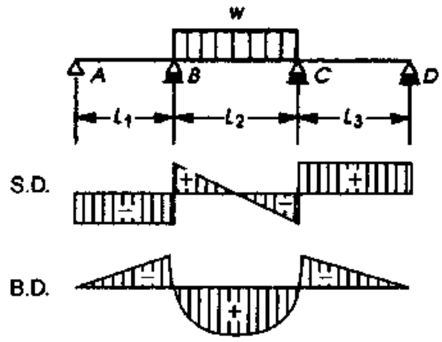
Table C.6 Moments in Three Unequal Spans and Values of the Coefficient K (w = Load/Unit Length)

4. Load on span *CD*



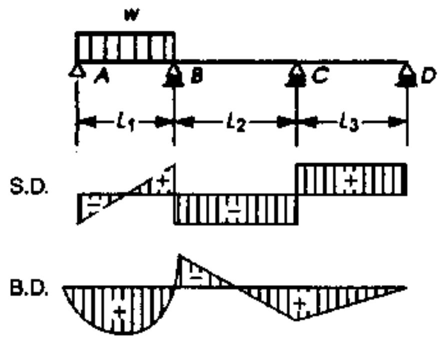
L_2/L_3	$M_B = \frac{wL_3^2}{K}$ (positive)	$M_C = \frac{wL_3^2}{K}$ (negative)
0.25	100.0	9.9
0.30	90.9	10.3
0.40	76.3	11.0
0.50	70.4	11.7
0.60	65.8	12.3
0.70	62.9	13.0
0.80	61.7	13.7
1.00	59.9	14.9

5. Load on middle span



L_2/L_1	$M_B = M_C = \frac{wL_2^2}{K}$ (negative)
0.25	43.5
0.30	38.5
0.40	32.3
0.50	27.8
0.60	25.6
0.70	23.3
0.80	22.2
1.00	20.0

6. Load on span *AB*

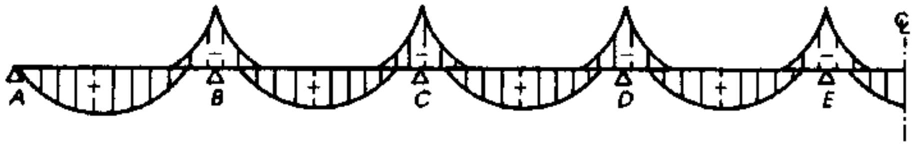


L_2/L_1	$M_B = \frac{wL_1^2}{K}$ (negative)	$M_C = \frac{wL_1^2}{K}$ (positive)
0.25	9.9	100.0
0.30	10.3	90.9
0.40	11.0	76.3
0.50	11.7	70.4
0.60	12.3	65.8
0.70	13.0	62.9
0.80	13.7	61.7
1.00	14.9	59.9

Table C.7 Maximum and Minimum Moments in Equal Spans Continuous Beams

7. Uniform loads

$$M = \frac{wL^2}{K}$$



where w = (D.L. + L.L.) per unit length D.L. = Uniform dead load L.L. = Uniform live load

Values of coefficient K

Ratio D.L./w	First span AB (positive moment)				Second support B (negative moment)			
	Number of spans				Number of spans			
	2	3	4	5	2	3	4	5
0.0	10.5	10.0	10.2	10.1	8.0	8.6	8.3	8.3
0.1	10.8	10.2	10.4	10.3	8.0	8.7	8.4	8.5
0.2	11.1	10.4	10.6	10.6	8.0	8.8	8.5	8.6
0.3	11.4	10.6	10.9	10.8	8.0	9.0	8.6	8.7
0.4	11.8	10.9	11.1	11.0	8.0	9.1	8.6	8.8
0.5	12.1	11.1	11.4	11.3	8.0	9.2	8.8	8.9
0.6	12.5	11.4	11.7	11.6	8.0	9.4	8.9	9.0
0.7	12.9	11.6	12.0	11.9	8.0	9.5	9.0	9.1
0.8	13.3	11.9	12.3	12.2	8.0	9.7	9.1	9.2
0.9	12.8	12.2	12.6	12.5	8.0	9.8	9.2	9.4
1.0	14.3	12.5	13.0	12.8	8.0	9.9	9.3	9.5

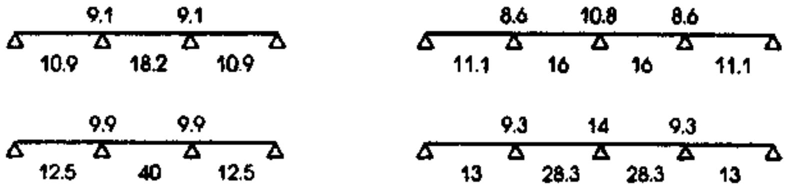
Ratio D.L./w	Second span BC (positive moment)			Third support C (negative moment)		Third span CD (positive moment)	Interior span (positive moment)	Interior support (negative moment)
	Number of spans			Spans		Span		
	3	4	5	4	5	5		
0.0	13.4	12.4	12.7	9.3	9.0	11.7	12.0	8.8
0.1	14.3	13.2	13.5	9.7	9.3	12.3	12.6	9.1
0.2	15.4	14.0	14.3	10.0	9.6	12.9	13.3	9.8
0.3	16.7	14.9	15.3	10.4	9.9	13.6	14.1	9.5
0.4	18.2	16.0	16.5	10.8	10.2	14.3	15.0	9.9
0.5	20.0	17.2	17.9	11.5	10.5	15.2	16.0	10.1
0.6	22.2	18.7	19.5	11.7	10.9	16.2	17.2	10.5
0.7	25.0	20.4	21.4	12.2	11.3	17.3	18.4	10.8
0.8	28.6	22.4	23.8	12.7	11.7	18.5	20.0	11.2
0.9	33.3	24.9	26.6	13.3	12.2	20.0	21.8	11.6
1.0	40.0	28.3	30.0	14.0	12.7	21.7	24.0	12.0

Table C.7 *(continued)*

Example: K values

1. $\dfrac{\text{D.L.}}{w} = 0.4$

2. $\dfrac{\text{D.L.}}{w} = 1.0$

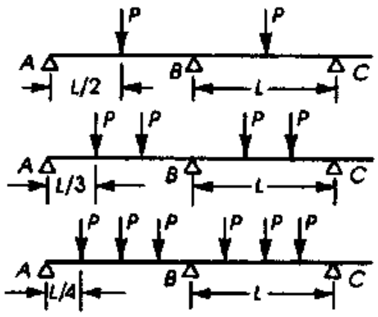


8. Concentrated loads

P' = concentrated dead load

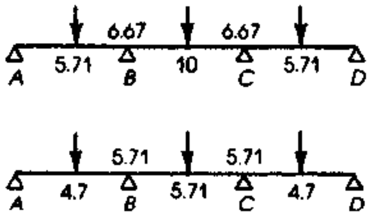
P'' = concentrated live load

$$M = \left(\frac{P'}{K_1} + \frac{P''}{K_2}\right) L$$



Number of Spans	*First Span AB*						*Second Support B*					
	K_1 (D.L.)			K_2 (L.L.)			K_1 (D.L.)			K_2 (L.L.)		
	2	3	4	2	3	4	2	3	4	2	3	4
Central load	6.40	5.71	5.89	4.92	4.70	4.76	5.35	6.67	6.22	5.33	5.71	5.53
One-third-point loads	4.50	4.09	4.20	3.60	3.46	3.50	3.00	3.75	3.50	3.00	3.21	3.11
One-fourth-point loads	3.67	3.20	3.34	2.61	2.46	2.50	2.13	2.67	2.49	2.13	2.28	2.21

Number of Spans	Second span BC				Third Support C	
	K_1		K_2		K_1	K_2
	3	4	3	4	4	4
Central load	10.00	8.61	5.71	5.46	9.33	6.22
One-third-point loads	15.00	9.00	5.00	4.50	5.25	3.50
One-fourth-point loads	8.00	6.05	3.20	3.01	3.72	2.49



Example: K values

K_1(dead load) $\quad M_{AB}(\text{max}) = \left(\dfrac{P'}{5.71} + \dfrac{P''}{4.7}\right) L$

$$-M_B(\text{max}) = \left(\frac{P'}{6.67} + \frac{P''}{5.71}\right) L$$

K_2(live load) $\quad M_{BC}(\text{max}) = \left(\dfrac{P'}{10} + \dfrac{P''}{5.71}\right) L$

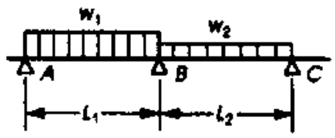
Table C.8 Moments in Unequal Spans Continuous Beams Subjected to Unequal Loads

9. Unequal spans and unequal loads. For approximate bending moments in continuous beams, use

$L' = 0.8L$ for spans continuous at both ends

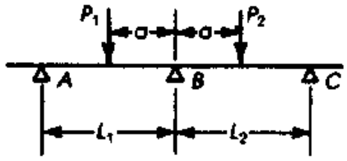
$L' = L$ for spans continuousatonly one end

1. Uniform loads (load on two adjacent spans):



$$M_B = \frac{w_1 L_1'^3 + w_2 L_2'^3}{8.5(L_1' + L_2')}$$

2. Concentrated loads:



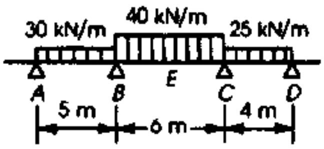
$$M_B = \frac{K P_1 L_1'^2}{L_1' + L_2'} \text{ due to load } P_1$$

$$M_B = \frac{K P_2 L_2'^2}{L_1' + L_2'} \text{ due to load } P_1$$

a/L	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K	0.080	0.136	0.168	0.182	0.176	0.158	0.128	0.090	0.050	0.000

3. Moments within span:
   - Maximum positive moment is obtained by superposing B.M. due to D.L. + L.L. and the negative moments at supports due to D.L. only.
   - Maximum negative moment is obtained by superposing B.M. due to D.L. only and the negative moments at supports due to D.L. + L.L.

Example:



$$M_B = -\frac{30(0.8 \times 5)^3 + 40(0.8 \times 6)^3}{8.5(0.8 \times 5 + 0.8 \times 6)} = -84.8\text{kN m}$$

$$M_C = -\frac{40(0.8 \times 6)^3 + 25(0.8 \times 6)^3}{8.5(0.8 \times 4 + 0.8 \times 6)} = -77.1\text{kN m}$$

$$M_E = \text{(at centerline of BC)} = +\frac{wL^2}{8} + \frac{1}{2}(M_B + M_C) = \frac{40 \times 36}{8} + \frac{1}{2}(-84.8 - 77.1) = +99\text{kN·m}$$

INDEX

CIVIL ENGINEERING/STRUCTURES

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THE FOURTH EDITION OF *STRUCTURAL CONCRETE: THEORY AND DESIGN* brings this text fully up to date while maintaining its acclaimed easy-to-follow, logical approach. Working with the text's numerous step-by-step examples, students quickly grasp the principles and techniques of analyzing and designing reinforced and prestressed concrete elements. Moreover, the authors' emphasis on a top quality, economical approach helps students design concrete structures and members with confidence.

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COVER DESIGN: HOLLY WITTENBERG
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ISBN 978-0-470-17094-6
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