EJEMPLOS DE DISEÑO

FILTROS IIR

PROCEDIMIENTO

(6.16) $$\omega = \frac{2}{T}\tan\left(\frac{\Omega}{2}\right)$$

Once the frequency specifications for the analog filter are obtained from those for the digital filter via (6.16), we must determine the order of the chosen analog filter. Of course, we must then determine the transfer function of the analog filter whose order has been just determined. As mentioned earlier, a wealth of analog filter design exists in the literature. We can, therefore, use such information to come up with the transfer function of the required analog filter. Once the analog filter transfer function expressed in the Laplace variable s is found, we replace s in the analog filter transfer function by the term on the right hand side of the bilinear transform of equation (6.9) to obtain the transfer function of the IIR digital filter. Thus,

(6.17) $$H(z) = H_A(s)\Big|_{s=\frac{1(1-z^{-1})}{T(1+z^{-1})}}$$

We enumerate the design procedure used in converting an analog filter into an IIR digital filter using the bilinear transform method as follows: Given the frequency domain specifications of a lowpass IIR digital filter, do the following.

1. Choose an appropriate analog filter type such as, Butterworth, Chebyshev, etc.
2. Prewarp the critical frequencies of the digital filter using equation (6.16).
3. Determine the required order of the analog filter.
4. Determine the corresponding analog filter transfer function.
5. Convert the analog filter transfer function into the digital filter transfer function via equation (6.17).

PROBLEMA 1

Design an IIR lowpass Butterworth digital filter with the following frequency specifications: passband edge $\Omega_p = \frac{\pi}{4}$, stopband edge $\Omega_s = \frac{\pi}{2}$, passband ripple $\alpha_p = 0.5\ dB$, and a minimum stopband attenuation of 20 dB.

ECUACIONES PROBLEMA 1

A Butterworth lowpass filter is also known as the maximally flat filter because its magnitude of the frequency response has the largest number of derivatives with respect to ω equal to zero at zero frequency. In other words, the magnitude of its frequency response is as flat as possible at DC. An Nth-order lowpass analog Butterworth filter with a *cutoff* frequency ω_c has the magnitude squared of frequency response expressed as

(6.18) $$|H_A(\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^{2N}}$$

Note that at $\omega = \omega_c$, the magnitude squared of the frequency response of the Butterworth lowpass analog filter is half or is -3 dB. Therefore, the cutoff frequency here is also called the *half power* frequency or 3 dB frequency. Typical specifications in the frequency domain for a lowpass filter are the passband edge ω_p, stopband edge ω_s, passband ripple, and minimum stopband attenuation, both in dB. The magnitude squared of the Butterworth lowpass filter function in (6.18) drops to

(6.19) $$\left|H_A(\omega_p)\right|^2 = \frac{1}{1+\left(\frac{\omega_p}{\omega_c}\right)^{2N}} = \frac{1}{1+\varepsilon^2}$$

at the passband edge frequency. The passband ripple α in dB is related to ε by

With the filter order and the cutoff frequency known, the corresponding lowpass Butterworth filter transfer function is determined from

(6.26) $$H_A(s) = \frac{1}{D_N\left(\frac{s}{\omega_c}\right)}$$

(6.20) $$-20log_{10}\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right) = \alpha\ dB \Longrightarrow \varepsilon = \sqrt{10^{0.1\alpha} - 1}$$

The actual minimum stopband attenuation is related to the attenuation in dB by

(6.21) $$A = 10^{0.05A_{dB}}$$

The filter order N is related to ε, A, ω_p, and ω_s through the following:

(6.22a) $$\frac{1}{k} \equiv \frac{\omega_s}{\omega_p}$$

(6.22b) $$\frac{1}{k_1} \equiv \frac{\sqrt{A^2-1}}{\varepsilon}$$

(6.23) $$N = \left\lceil \frac{log_{10}\left(\frac{1}{k_1}\right)}{log_{10}\left(\frac{1}{k}\right)} \right\rceil$$

Finally, in order to find the Nth-order transfer function of the analog lowpass Butterworth filter, we need to know the cutoff frequency. This can be found from equation (6.19) as

(6.24) $$\omega_c = \varepsilon^{-\frac{1}{N}}\omega_p$$

Alternatively, the cutoff frequency can also be found from

(6.25) $$|H_A(\omega_s)|^2 = \frac{1}{1+\left(\frac{\omega_s}{\omega_c}\right)^{2N}} = \frac{1}{A^2}$$

(6.27) $$H_A(s) = \frac{{\omega_c}^N}{\prod_{m=1}^{N}(s-p_m)}$$

where the poles are described by

(6.28) $$p_m = \omega_c e^{j\left(\frac{(N+2m-1)\pi}{2N}\right)}, 1 \le m \le N$$

SOLUCION PROBLEMA 1

$$\omega_p = \tan\left(\frac{\Omega_p}{2}\right) = \tan\left(\frac{\pi}{8}\right) \approx 0.414214$$

$$\omega_s = \tan\left(\frac{\Omega_s}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

onding to the passband ripple and minimum stopband attenuation, we have

$$\varepsilon^2 = 10^{0.1\alpha_p} - 1 = 10^{0.05} - 1 = 0.122018$$

$$20log_{10}A = 20 \Longrightarrow A = 10$$

e find the filter order as follows:

$$\frac{1}{k} = \frac{\omega_s}{\omega_p} = \frac{1}{0.414214} = 2.41421$$

(6.31b) $$\frac{1}{k_1} = \frac{\sqrt{A^2-1}}{\varepsilon} = 28.4843$$

(6.31c) $$N = \left\lceil\frac{log_{10}(28.4843)}{log_{10}(2.41421)}\right\rceil = \lceil 3.8002\rceil = 4$$

Note that in equation (6.31c), we need to round the number to the integer that is greater than or equal to the number within the ceiling operator. Having determined the required order of the Butterworth lowpass filter, we need to determine the cutoff frequency of the lowpass filter, which is obtained from (6.24):

(6.32) $$\omega_c = \varepsilon^{-\frac{1}{N}}\omega_p = 0.538793$$

The poles of the 4th-order Butterworth lowpass analog filter are found from equation (6.28):

(6.33) $$p_m = \omega_c e^{j\pi\frac{(4+2m-1)}{8}}, 1 \leq m \leq 4$$

SOLUCION PROBLEMA 1 CODIGO 1

(6.33) $$p_m = \omega_c e^{j\pi\frac{(4+2m-1)}{8}}, 1 \le m \le 4$$

We notice from (6.33) that p_1 and p_4 are complex conjugates of each other and similarly, p_2 and p_3 are complex conjugates of each other. Using (6.27) with algebraic simplification, we obtain the analog transfer function of the lowpass Butterworth filter as

(6.34) $$H_A(s) = \frac{H_0}{(s^2+a_1 s+a_2)(s^2+b_1 s+b_2)}$$

where, the constants are: $H_0 = 0.0842722$, $a_1 = 0.412374$, $a_2 = 0.290297$, $b_1 = 0.995558$, and $b_2 = 0.290297$. We finally obtain the transfer function of the desired IIR digital Butterworth lowpass filter using equation (6.17), which is given in (6.35) after algebraic manipulation:

(6.35) $$H(z) = \frac{0.0217(1+4z^{-1}+6z^{-2}+4z^{-3}+z^{-4})}{(1-0.8336z^{-1}+0.5156z^{-2})(1-0.6209z^{-1}+0.12189z^{-2})}$$

DISEÑO CHEBYS

A Chebyshev type I filter has an equiripple characteristic in the passband and falls off monotonically in the stopband. More specifically, an Nth-order Chebyshev type I analog lowpass filter with a passband edge ω_p has the transfer function whose magnitude square is described by

(6.36) $$|H_A(\omega)|^2 = \frac{1}{1+\varepsilon^2 T_N^2\left(\frac{\omega}{\omega_p}\right)}$$

where the Nth-order Chebyshev polynomial is defined as

(6.37) $$T_N(x) = \begin{cases} \cos(N\cos^{-1}(x)), |x| \leq 1 \\ \cosh(N\cosh^{-1}(x)), |x| > 1 \end{cases}$$

From equation (6.37), we observe that the Chebyshev polynomial alternates between +1 and -1 in the interval between -1 and +1 N times, and for $|x| > 1$, it increases monotonically. Therefore, the lowpass Chebyshev filter's frequency response has ripples in the passband. The filter order can be determined from the following equations.

(6.38) $$|H_A(\omega_s)|^2 = \frac{1}{1+\varepsilon^2 T_N^2\left(\frac{\omega_s}{\omega_p}\right)} = \frac{1}{A^2} \Longrightarrow T_N^2\left(\frac{\omega_s}{\omega_p}\right) = \frac{A^2-1}{\varepsilon^2}$$

where ω_s is the stopband edge and A is the minimum stopband attenuation. Since $\omega_s > \omega_p$, we have from (6.38)

(6.39) $$T_N\left(\frac{1}{k}\right) = \cosh\left(N\cosh^{-1}\left(\frac{1}{k}\right)\right) = \frac{\sqrt{A^2-1}}{\varepsilon} \equiv \frac{1}{k_1}$$

where we have used the fact that

(6.40) $$\frac{1}{k} = \frac{\omega_s}{\omega_p}$$

From equations (6.39) and (6.40), we determine the value of N as

(6.41) $$N = \frac{\cosh^{-1}\left(\frac{1}{k_1}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)} = \frac{ln\left(\frac{1}{k_1}+\sqrt{\left(\frac{1}{k_1}\right)^2-1}\right)}{ln\left(\frac{1}{k}+\sqrt{\left(\frac{1}{k}\right)^2-1}\right)}$$

Once the filter order is found, the transfer function of the lowpass Chebyshev type I analog filter in terms of its poles are obtained from

(6.42) $$H_A(s) = \frac{H_0}{\prod_{n=1}^{N}(s-p_n)}$$

where H_0 is a normalization factor, and the poles are

(6.43a) $$p_n = \sigma_n + j\omega_n, 1 \leq n \leq N$$

(6.43b) $$\sigma_n = -\omega_p a_1 \sin\left(\frac{(2n-1)\pi}{2N}\right), 1 \leq n \leq N$$

(6.43c) $$\omega_n = \omega_p a_2 \cos\left(\frac{(2n-1)\pi}{2N}\right), 1 \leq n \leq N$$

(6.43d) $$a_1 = \frac{\gamma^2-1}{2\gamma}$$

(6.43e) $$a_2 = \frac{\gamma^2+1}{2\gamma}$$

(6.43f) $$\gamma = \left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{\frac{1}{N}}$$

PROBLEMA 2

- Design a lowpass IIR Chebyshev type I digital filter using bilinear transformation. Use the same frequency specifications as in Example 1

SOLUCION PROBLEMA 2

$$\omega_p = \tan\left(\frac{\Omega_p}{2}\right) = \tan\frac{\pi}{8} = 0.414214$$

$$\omega_s = \tan\left(\frac{\Omega_s}{2}\right) = \tan\frac{\pi}{4} = 1$$

Corresponding to 0.5 dB passband ripple, we have

$$\varepsilon^2 = 10^{0.1*0.5} - 1 = 0.122018$$

We already know that the minimum stopband attenuation A is 20 dB or 10 in actual value. From (6.39) and (6.40), we determine $\frac{1}{k_1} = 28.4843$ and $\frac{1}{k} = 2.41421$. Using these values in (6.41), we find the filter order $N = \lceil 2.6444 \rceil = 3$. Note that the order of the lowpass Chebyshev type I analog filter is less than that of the Butterworth filter for the same frequency specifications. We now determine the poles of the Chebyshev type I analog filter using equation (6.43) and are given by

(6.44a) $p_1 = -0.1297 + j0.4233$

(6.44b) $p_2 = -0.2595$

(6.44c) $p_3 = p_1^* = -0.1297 - j0.4233$

From these poles we obtain the analog transfer function of the lowpass Chebyshev type I filter as

(6.45) $H_A(s) = \frac{H_0}{(s+0.2595)(s^2+0.2594s+0.196)}$

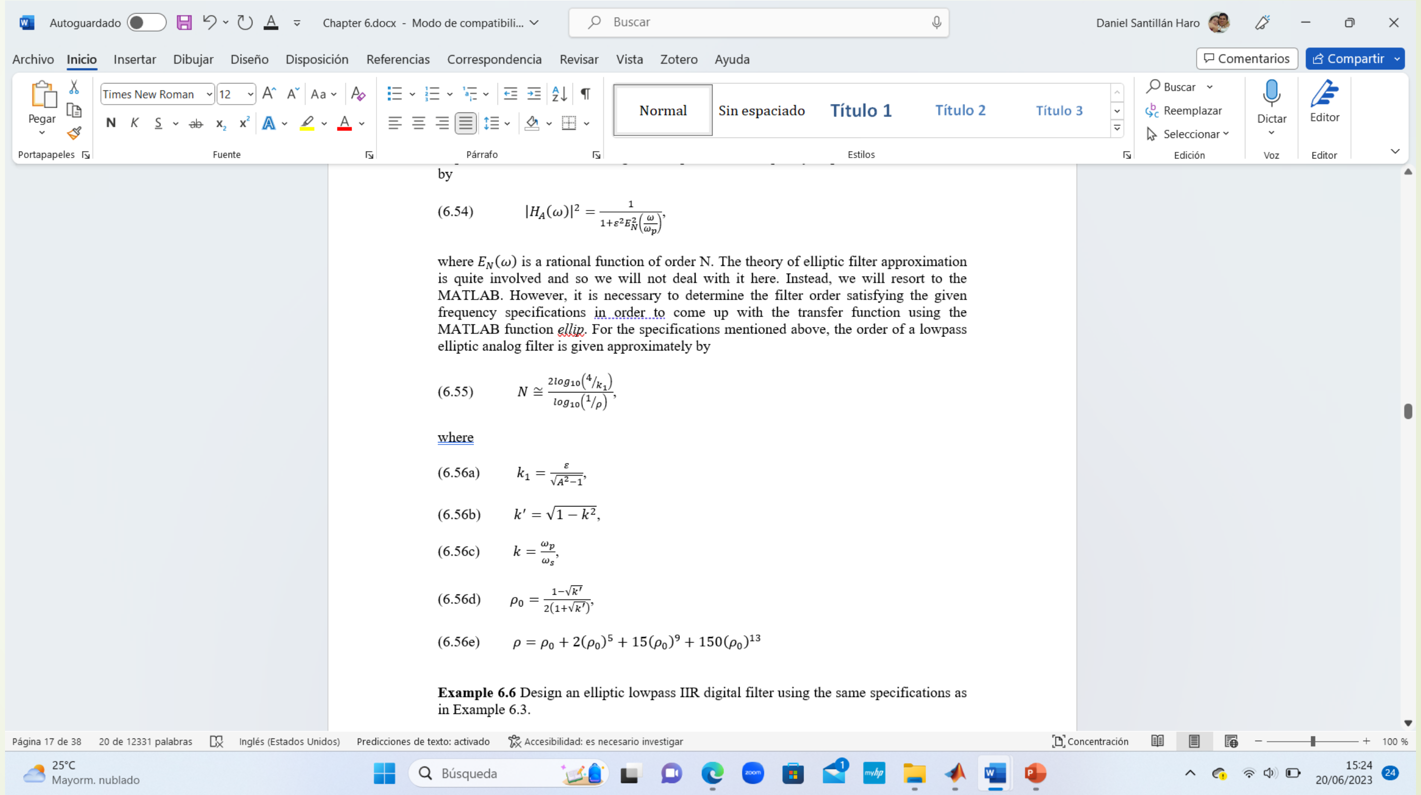
SOLUCION PROBLEMA 2 CODIGO 2 MATLAB

After applying the bilinear transformation with $\frac{2}{T} = 1$ to the analog filter function in (6.45) and with some algebraic manipulation, we get

$$(6.46) \qquad H(z) = H_A(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.5455(1+3z^{-1}+3z^{2}+z^{-3})}{(1-0.5879z^{-1})(1-1.1049z^{-1}+0.6435z^{-2})}$$

The magnitude of the frequency response of the lowpass Chebyshev type I IIR digital filter is shown in Figure 6.6. As mentioned above, the magnitude has ripples in the passband and decreases monotonically in the stopband. It is found that the minimum stopband attenuation for the 3rd-order lowpass Chebyshev type I filter is 24.7 dB. Compare this with that for the Butterworth filter, which is 3 dB less! We can also design the Chebyshev type I digital filter using MATLAB. To design an Nth-order lowpass analog Chebyshev type I filter, we use the function *cheby1*. The actual function call is *[z,p,k]=cheby1(N,ap,wp,'s')*, where the arguments are the filter order, passband ripple in dB, passband edge, and the letter s in single quote to imply analog filter. The function

DISEÑO FILTRO ELÍPTICO



by

(6.54) $$|H_A(\omega)|^2 = \frac{1}{1+\varepsilon^2 E_N^2\left(\frac{\omega}{\omega_p}\right)},$$

where $E_N(\omega)$ is a rational function of order N. The theory of elliptic filter approximation is quite involved and so we will not deal with it here. Instead, we will resort to the MATLAB. However, it is necessary to determine the filter order satisfying the given frequency specifications in order to come up with the transfer function using the MATLAB function *ellip*. For the specifications mentioned above, the order of a lowpass elliptic analog filter is given approximately by

(6.55) $$N \cong \frac{2log_{10}\left(4/k_1\right)}{log_{10}\left(1/\rho\right)},$$

where

(6.56a) $$k_1 = \frac{\varepsilon}{\sqrt{A^2-1}},$$

(6.56b) $$k' = \sqrt{1-k^2},$$

(6.56c) $$k = \frac{\omega_p}{\omega_s},$$

(6.56d) $$\rho_0 = \frac{1-\sqrt{k'}}{2(1+\sqrt{k'})},$$

(6.56e) $$\rho = \rho_0 + 2(\rho_0)^5 + 15(\rho_0)^9 + 150(\rho_0)^{13}$$

Example 6.6 Design an elliptic lowpass IIR digital filter using the same specifications as in Example 6.3.

PROBLEMA 3

- Design an elliptic lowpass IIR digital filter using the same specifications as in Example 1

SOLUCIÓN PROBLEMA 3

Solution: Corresponding to the passband ripple of 0.5 dB, we find

$$-20log_{10}\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right) = -0.5 \Rightarrow \varepsilon \cong 0.34931$$

Therefore, $k_1 \cong 0.035107$. Using the passband and stopband edges, we have $k = \tan\frac{\pi}{8}$. Next, we find $k' \cong 0.91018$, $\rho_0 = 0.011762$, and $\rho \cong \rho_0 = 0.011762$. Using the values of k_1 and ρ in (6.55), we get $N = \lceil 2.1318 \rceil = 3$. So, a 3rd-order elliptic filter will satisfy the given specifications. We use the MATLAB function *ellip*, which accepts the filter order, passband ripple in dB, minimum stopband attenuation in dB, passband edge, and the letter s in single quotes. We can use either the statement *[z,p,k]* = *ellip(N,rp,rs,wp,'s')*, which calculates the zeros, poles, and the gain or the statement