Líneas de transmisión

FACULTAD DE INGENIERIA

Escuela de Electrónica y Telecomunicaciones
Quinto Semestre
Unidad I: Ecuación de onda
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1. Ecuación de onda
2. Incidencia normal
3. Trabajos propuestos

1 Ecuación de onda

2 Incidencia normal

3 Trabajos propuestos

Ejemplo

Una sonda espacial de 500 kg, situada fuera de la atmósfera, pretende realizar unos movimientos aprovechando la radiación solar. Para ello, despliega una vela cuadrada de 300 m de lado, totalmente reflectora, colocándola perpendicularmente a los rayos solares. Fases a realizar en el problema:

1º.- El radio del Sol es 696000 km con una intensidad de radiación a esa distancia del centro de 63447532,7 $W \cdot m^{-2}$. Demostrar que la intensidad de la radiación a la distancia de la Tierra ($150 \cdot 10^6$ km) es 1366 $W \cdot m^{-2}$.

2º.- Calcular el valor medio del vector de Poynting.

3º.- Imagina una superficie cilíndrica con ondas electromagnéticas en su interior, que incide sobre la vela y le transmite una cantidad de movimiento (o momento lineal). Mediante un balance de cantidad de movimiento, demuestra que la vela recibe el doble de la cantidad de movimiento que transporta la radiación. Calcula la cantidad de movimiento comunicado a la vela.

4º.- Calcular la presión de radiación sobre la vela.

5º.- Calcular la aceleración que adquiere la sonda espacial.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

1º.- Intensidad de la radiación solar a la distancia de la Tierra

El radio del Sol es 696000 km con una intensidad de radiación a esa distancia del centro de 63447532,7 $W \cdot m^{-2}$. Demostrar que la intensidad de la radiación a la distancia de la Tierra ($150 \cdot 10^6$ km) es 1366 $W \cdot m^{-2}$.

Llamemos I_1 a la intensidad de radiación en la superficie del Sol, e I_2 en el exterior de la atmósfera. A partir de la definición de intensidad:

$$I_1 = \frac{P_e}{4\pi R_1^2} \qquad ; \qquad I_2 = \frac{P_e}{4\pi R_2^2}$$

Siendo P_e la potencia emisiva del Sol, R_1 el radio del Sol y R_2 la distancia Sol-Tierra. Dividiendo miembro a miembro ambas ecuaciones, se tiene,

$$\frac{I_1}{I_2} = \frac{R_2^2}{R_1^2}$$

Despejando I_2 se obtiene, $I_2 = 1366 \; W \cdot m^{-2}$ la cual es denominada constante solar.

2º.- Calcular el valor medio del vector de Poynting

$$\langle S \rangle = I_2 = 1366\ \mathrm{W \cdot m^{-2}}$$

3º.- Demuestra que la vela recibe el doble de la cantidad de movimiento que transporta la radiación. Calcula la cantidad de movimiento comunicado a la vela

Imagina una superficie cilíndrica con ondas electromagnéticas en su interior, que incide sobre la vela y le transmite una cantidad de movimiento (o momento lineal). Mediante un balance de cantidad de movimiento, demuestra que la vela recibe el doble de la cantidad de movimiento que transporta la radiación. Calcula la cantidad de movimiento comunicado a la vela.

Balance de cantidad de movimiento,

$$\{p_{\mathrm{cilindro}} + p_{\mathrm{superficie}}\}_{\mathrm{ANTES}} = \{p_{\mathrm{cilindro}} + p_{\mathrm{superficie}}\}_{\mathrm{DESPUÉS}}$$

Siendo reflexión total los valores del momento lineal son,

$$p + 0 = -p + 2\,p$$

lo cual implica que la superficie recibe el doble de cantidad de movimiento que transporta la radiación. El valor correspondiente es, $u = \frac{2\langle S \rangle}{c} = 9{,}11\ \frac{\mu J}{m^3}$

4°.- Calcular la presión de radiación (P) sobre la vela

$$P = u = \frac{2\langle S \rangle}{c} = 9{,}11 \quad \frac{\mu J}{m^3}$$

5°.- Calcular la aceleración que adquiere la sonda espacial

La fuerza que se ejerce sobre la superficie es, $F = M \cdot a = P \cdot A$

Siendo A, el área de la superficie, y "a" la aceleración que adquiere.

Entonces,

$$a = \frac{2 \cdot I \cdot A}{M \cdot c} = \frac{2(1366)(300^2)}{500(3 \cdot 10^8)} = 0{,}00164 \quad m \cdot s^{-2}$$

$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0.$$

The two independent solutions to this equation are easily seen, by substitution, to be of the form

$$E_x(z) = E^+ e^{-jkz} + E^- e^{jkz},$$

where E^+ and E^- are arbitrary amplitude constants.

The above solution is for the time harmonic case at frequency ω. In the time domain, this result is written as

$$\mathcal{E}_x(z, t) = E^+ \cos(\omega t - kz) + E^- \cos(\omega t + kz),$$

where we have assumed that E^+ and E^- are real constants. Consider the first term

This term represents a wave traveling in the $+z$ direction because, to maintain a fixed point on the wave ($\omega t - kz =$ constant), one must move in the $+z$ direction as time increases. Similarly, the second term represents a wave traveling in the negative z direction—hence the notation E^+ and E^- for these wave amplitudes.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

The velocity of the wave in this sense is called the *phase velocity* because it is the velocity at which a fixed phase point on the wave travels, and it is given by

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - \text{constant}}{k}\right) = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

In free-space, we have $v_p = 1/\sqrt{\mu_0\epsilon_0} = c = 2.998 \times 10^8$ m/sec, which is the speed of light.

The *wavelength*, λ, is defined as the distance between two successive maxima (or minima, or any other reference points) on the wave at a fixed instant of time. Thus,

$$(\omega t - kz) - [\omega t - k(z + \lambda)] = 2\pi,$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f}.$$

A complete specification of the plane wave electromagnetic field should include the magnetic field. In general, whenever $\bar{E}$ or $\bar{H}$ is known, the other field vector can be readily found by using one of Maxwell's curl equations. Thus, applying to the electric field gives $H_x = H_z = 0$, and

$$H_y = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} = \frac{1}{\eta}(E^+ e^{-jkz} - E^- e^{jkz}),$$

where $\eta = \omega\mu/k = \sqrt{\mu/\epsilon}$ is known as the *intrinsic impedance* of the medium. The ratio

$$\nabla^2 \bar{E} + \omega^2 \mu\epsilon \left(1 - j\frac{\sigma}{\omega\epsilon}\right) \bar{E} = 0,$$

where we see a similarity with the wave equation for $\bar{E}$ in the lossless case. The difference is that the quantity $k^2 = \omega^2\mu\epsilon$ is replaced by $\omega^2\mu\epsilon[1 - j(\sigma/\omega\epsilon)]$ We then define a *complex propagation constant* for the medium as

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$$

where α is the *attenuation constant* and β is the *phase constant*. If we again assume an electric field with only an $\hat{x}$ component and uniform in x and y, the wave equation of reduces to

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0,$$

which has solutions

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z}.$$

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z}.$$

The positive traveling wave then has a propagation factor of the form

$$e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z},$$

which in the time domain is of the form

$$e^{-\alpha z} \cos(\omega t - \beta z).$$

We see that this represents a wave traveling in the $+z$ direction with a phase velocity $v_p = \omega/\beta$, a wavelength $\lambda = 2\pi/\beta$, and an exponential damping factor. The rate of decay with distance is given by the attenuation constant, α. The negative traveling wave term is similarly damped along the $-z$ axis. If the loss is removed, $\sigma = 0$, and we have $\gamma = jk$ and $\alpha = 0$, $\beta = k$.

As discussed loss can also be treated through the use of a complex permittivity. with $\sigma = 0$ but $\epsilon = \epsilon' - j\epsilon''$ complex, we have that

$$\gamma = j\omega\sqrt{\mu\epsilon} = jk = j\omega\sqrt{\mu\epsilon'(1 - j\tan\delta)},$$

where $\tan\delta = \epsilon''/\epsilon'$ is the loss tangent of the material.

The associated magnetic field can be calculated as

The associated magnetic field can be calculated as

$$H_y = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} = \frac{-j\gamma}{\omega\mu}(E^+e^{-\gamma z} - E^-e^{\gamma z}).$$

The intrinsic impedance of the conducting medium is now complex,

$$\eta = \frac{j\omega\mu}{\gamma},$$

but is still identified as the wave impedance, which expresses the ratio of electric to magnetic field components. This allows to be rewritten as

$$H_y = \frac{1}{\eta}(E^+e^{-\gamma z} - E^-e^{\gamma z}).$$

Note that although η of (1.57) is, in general, complex, it reduces to the lossless case of $\eta = \sqrt{\mu/\epsilon}$ when $\gamma = jk = j\omega\sqrt{\mu\epsilon}$.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

Many problems of practical interest involve loss or attenuation due to good (but not perfect) conductors. A good conductor is a special case of the preceding analysis, where the conductive current is much greater than the displacement current, which means that $\sigma \gg \omega\epsilon$. Most metals can be categorized as good conductors. In terms of a complex ϵ, rather than conductivity, this condition is equivalent to $\epsilon'' \gg \epsilon'$. The propagation constant of (1.52) can then be adequately approximated by ignoring the displacement current term, to give

$$\gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}.$$

The *skin depth*, or characteristic depth of penetration, is defined as

$$\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}.$$

Thus the amplitude of the fields in the conductor will decay by an amount $1/e$, or 36.8%, after traveling a distance of one skin depth, because $e^{-\alpha z} = e^{-\alpha\delta_s} = e^{-1}$. At microwave frequencies, for a good conductor, this distance is very small. The practical importance of this result is that only a thin plating of a good conductor (e.g., silver or gold) is necessary for low-loss microwave components.

	Type of Medium		
Quantity	Lossless ($\epsilon'' = \sigma = 0$)	General Lossy	Good Conductor ($\epsilon'' \gg \epsilon'$ or $\sigma \gg \omega\epsilon'$)
Complex propagation constant	$\gamma = j\omega\sqrt{\mu\epsilon}$	$\gamma = j\omega\sqrt{\mu\epsilon}$ $= j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\frac{\sigma}{\omega\epsilon'}}$	$\gamma = (1+j)\sqrt{\omega\mu\sigma/2}$
Phase constant (wave number)	$\beta = k = \omega\sqrt{\mu\epsilon}$	$\beta = \mathrm{Im}\{\gamma\}$	$\beta = \mathrm{Im}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Attenuation constant	$\alpha = 0$	$\alpha = \mathrm{Re}\{\gamma\}$	$\alpha = \mathrm{Re}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Impedance	$\eta = \sqrt{\mu/\epsilon} = \omega\mu/k$	$\eta = j\omega\mu/\gamma$	$\eta = (1+j)\sqrt{\omega\mu/2\sigma}$
Skin depth	$\delta_s = \infty$	$\delta_s = 1/\alpha$	$\delta_s = \sqrt{2/\omega\mu\sigma}$
Wavelength	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$
Phase velocity	$v_p = \omega/\beta$	$v_p = \omega/\beta$	$v_p = \omega/\beta$

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

In the case of time-harmonic excitation we use phasor representation and replace the partial derivative with respect to time by the factor $j\omega$

$$\Delta \vec{E} = j\omega\mu_0\sigma \vec{E}$$

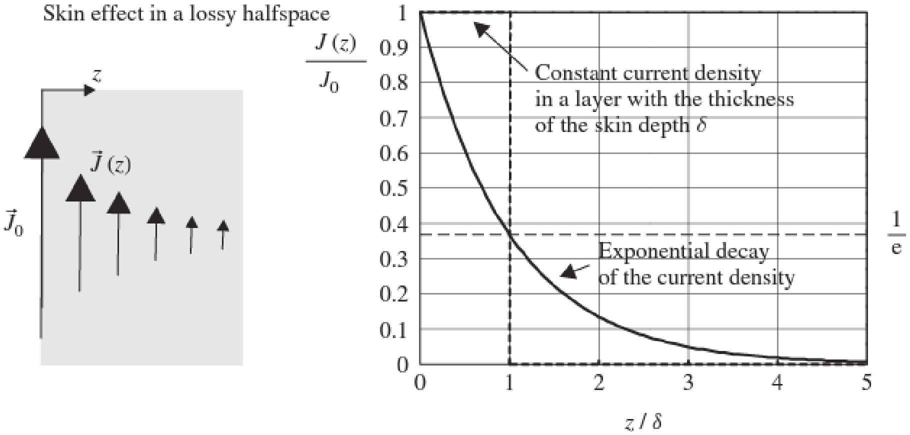
The electric field strength in the conductive halfspace is given by the following solution

$$\vec{E}(z) = E_0 e^{-z/\delta} e^{-jz/\delta} \vec{e}_x = E_0 e^{-(1+j)z/\delta} \vec{e}_x$$

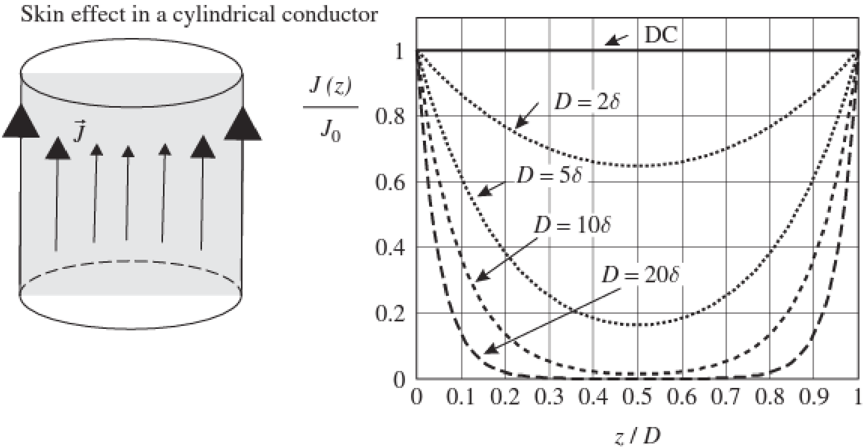
where δ is the so called *skin depth*. The skin depth is an important measure when describing the penetration of the electromagnetic field in conductive regions. It is given by

$$\boxed{\delta = \sqrt{\frac{2}{\omega\sigma\mu_0}}} \qquad \text{(Skin depth)}$$

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.



Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.



Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

Onda plana

- En el espacio libre, la ecuación de Helmholtz para el campo eléctrico $\vec{E}$ se puede escribir como:
  $\nabla^2\vec{E} + k_0^2\vec{E} = \frac{\partial^2\vec{E}}{\partial x^2} + \frac{\partial^2\vec{E}}{\partial y^2} + \frac{\partial^2\vec{E}}{\partial z^2} + k_0^2\vec{E} = 0$
  Esta ecuación es válida para cada componente rectangular de la forma:
  $\frac{\partial^2\vec{E}_i}{\partial x^2} + \frac{\partial^2\vec{E}_i}{\partial y^2} + \frac{\partial^2\vec{E}_i}{\partial z^2} + k_0^2\vec{E}_i = 0$
  donde el subíndice $i = x, y, o\ z$

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

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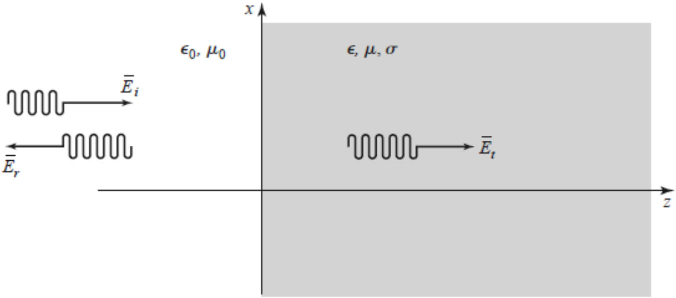

Plane wave reflection from an arbitrary medium; normal incidence.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

for $z < 0$, as

$$\bar{E}_i = \hat{x} E_0 e^{-jk_0 z},$$

$$\bar{H}_i = \hat{y} \frac{1}{\eta_0} E_0 e^{-jk_0 z},$$

where η_0 is the impedance of free-space and E_0 is an arbitrary amplitude. Also in the region $z < 0$, a reflected wave may exist with the form

$$\bar{E}_r = \hat{x} \Gamma E_0 e^{+jk_0 z},$$

$$\bar{H}_r = -\hat{y} \frac{\Gamma}{\eta_0} E_0 e^{+jk_0 z},$$

where Γ is the unknown *reflection coefficient* of the reflected electric field. Note that the sign in the exponential terms has been chosen as positive, to represent waves traveling in the $-\hat{z}$ direction of propagation. This is also consistent with the Poynting vector $\bar{S}_r = \bar{E}_r \times \bar{H}_r^* = -|\Gamma|^2 |E_0|^2 \hat{z}/\eta_0$, which shows power to be traveling in the $-\hat{z}$ direction for the reflected wave.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

$z > 0$ in the lossy medium can be written as

$$\bar{E}_t = \hat{x} T E_0 e^{-\gamma z},$$

$$\bar{H}_t = \frac{\hat{y} T E_0}{\eta} e^{-\gamma z},$$

where T is the *transmission coefficient* of the transmitted electric field and η is the intrinsic impedance (complex) of the lossy medium in the region $z > 0$. the intrinsic impedance is

$$\eta = \frac{j\omega\mu}{\gamma},$$

and the propagation constant is

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\sigma/\omega\epsilon}.$$

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

constants Γ and T are found by applying boundary conditions for E_x and H_y at $z = 0$. Since these tangential field components must be continuous at $z = 0$, we arrive at the following two equations:

$$1 + \Gamma = T,$$
$$\frac{1 - \Gamma}{\eta_0} = \frac{T}{\eta}.$$

Solving these equations for the reflection and transmission coefficients gives

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0},$$
$$T = 1 + \Gamma = \frac{2\eta}{\eta + \eta_0}.$$

This is a general solution for reflection and transmission of a normally incident wave at the interface of an arbitrary material, where η is the intrinsic impedance of the material. We now consider three special cases of this result.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

Lossless Medium

If the region for $z > 0$ is a lossless dielectric, then $\sigma = 0$, and μ and ϵ are real quantities. The propagation constant in this case is purely imaginary and can be written as

$$\gamma = j\beta = j\omega\sqrt{\mu\epsilon} = jk_0\sqrt{\mu_r\epsilon_r},$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the propagation constant (wave number) of a plane wave in free-space. The wavelength in the dielectric is

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{\lambda_0}{\sqrt{\mu_r\epsilon_r}},$$

the phase velocity is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}},$$

(slower than the speed of light in free-space) and the intrinsic impedance of the dielectric is

$$\eta = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{\mu}{\epsilon}} = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}}.$$

For this lossless case, η is real, so both Γ and T from (1.105) are real, and $\bar{E}$ and $\bar{H}$ are in phase with each other in both regions.

strated by computing the Poynting vectors in the two regions. Thus, for $z < 0$, the complex Poynting vector is found from the total fields in this region as

$$\begin{aligned}\bar{S}^- &= \bar{E} \times \bar{H}^* = (\bar{E}_i + \bar{E}_r) \times (\bar{H}_i + \bar{H}_r)^* \\ &= \hat{z}|E_0|^2 \frac{1}{\eta_0}(e^{-jk_0z} + \Gamma e^{jk_0z})(e^{-jk_0z} - \Gamma e^{jk_0z})^* \\ &= \hat{z}|E_0|^2 \frac{1}{\eta_0}(1 - |\Gamma|^2 + \Gamma e^{2jk_0z} - \Gamma^* e^{-2jk_0z}) \\ &= \hat{z}|E_0|^2 \frac{1}{\eta_0}(1 - |\Gamma|^2 + 2j\Gamma \sin 2k_0z),\end{aligned}$$

since Γ is real. For $z > 0$ the complex Poynting vector is

$$\bar{S}^+ = \bar{E}_t \times \bar{H}_t^* = \hat{z}\frac{|E_0|^2|T|^2}{\eta},$$

which can be rewritten,

$$\bar{S}^+ = \hat{z}|E_0|^2 \frac{4\eta}{(\eta + \eta_0)^2} = \hat{z}|E_0|^2 \frac{1}{\eta_0}(1 - |\Gamma|^2).$$

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

Good Conductor

If the region for $z > 0$ is a good (but not perfect) conductor, the propagation constant can be

$$\gamma = \alpha + j\beta = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} = (1+j)\frac{1}{\delta_s}.$$

Similarly, the intrinsic impedance of the conductor simplifies to

$$\eta = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\frac{1}{\sigma\delta_s}.$$

Now the impedance is complex, with a phase angle of 45°, so $\bar{E}$ and $\bar{H}$ will be 45° out of phase, and Γ and T will be complex. $\delta_s = 1/\alpha$ is the skin depth, as defined

For $z < 0$ the complex Poynting vector can be evaluated at $z = 0$ to give

$$\bar{S}^{-}(z=0) = \hat{z}|E_0|^2\frac{1}{\eta_0}(1 - |\Gamma|^2 + \Gamma - \Gamma^*).$$

For $z > 0$ the complex Poynting vector is

$$\bar{S}^{+} = \bar{E}_t \times \bar{H}_t^* = \hat{z}|E_0|^2|T|^2\frac{1}{\eta^*}e^{-2\alpha z},$$

Observe that if we were to compute the separate incident and reflected Poynting vectors for $z < 0$ as

$$\bar{S}_i = \bar{E}_i \times \bar{H}_i^* = \hat{z}\frac{|E_0|^2}{\eta_0},$$

$$\bar{S}_r = \bar{E}_r \times \bar{H}_r^* = -\hat{z}\frac{|E_0|^2|\Gamma|^2}{\eta_0},$$

we would not obtain $\bar{S}_i + \bar{S}_r = \bar{S}^-$ of (1.115a), even for $z = 0$. It is possible, however, to consider real power flow in terms of the individual traveling wave components. Thus, the time-average power flows through a 1 m^2 cross section are

$$P^- = \frac{1}{2}\mathrm{Re}(\bar{S}^- \cdot \hat{z}) = \frac{1}{2}|E_0|^2\frac{1}{\eta_0}(1 - |\Gamma|^2),$$

$$P^+ = \frac{1}{2}\mathrm{Re}(\bar{S}^- \cdot \hat{z}) = \frac{1}{2}|E_0|^2\frac{1}{\eta_0}(1 - |\Gamma|^2)e^{-2\alpha z},$$

which shows power balance at $z = 0$. In addition, $P_i = |E_0|^2/2\eta_0$ and $P_r = -|E_0|^2\,|\Gamma|^2\,/\,2\eta_0$, so that $P_i + P_r = P^-$, showing that the real power flow for $z < 0$ can be decomposed into incident and reflected wave components.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

Perfect Conductor

Now assume that the region $z > 0$ contains a perfect conductor. The above results can be specialized to this case by allowing $\sigma \to \infty$. Then, from $\alpha \to \infty$; $\eta \to 0$; $\delta_s \to 0$; $T \to 0$ and $\Gamma \to -1$. The fields for $z > 0$ thus decay infinitely fast and are identically zero in the perfect conductor. The perfect conductor can be thought of as "shorting out" the incident electric field. For $z < 0$, from the total $\bar{E}$ and $\bar{H}$ fields are, since $\Gamma = -1$,

$$\bar{E} = \bar{E}_i + \bar{E}_r = \hat{x} E_0 (e^{-jk_0 z} - e^{jk_0 z}) = -\hat{x} 2 j E_0 \sin k_0 z,$$

$$\bar{H} = \bar{H}_i + \bar{H}_r = \hat{y} \frac{1}{\eta_0} E_0 (e^{-jk_0 z} + e^{jk_0 z}) = \hat{y} \frac{2}{\eta_0} E_0 \cos k_0 z.$$

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

La onda plana uniforme:

$$\vec{E}_i = 10(2j\hat{y})e^{j15\pi z} \;\; \text{V/m}$$

se propaga en el vacío y en $z = 0$ incide normalmente sobre un medio no magnético de permitividad $\varepsilon = 36\varepsilon_0$. Calcular:

a) Longitud de onda, frecuencia, dirección de propagación, polarización y signo de polarización de la onda incidente.

b) Expresión del campo eléctrico y del campo magnético de la onda reflejada.

c) Expresión del campo eléctrico instantáneo transmitido.

d) Densidad de potencia media transportada por la onda reflejada.

e) Densidad de potencia media transportada por la onda transmitida.

f) Densidad de potencia media en cada medio.

Sea una onda plana que se propaga en la dirección del eje Z en un medio caracterizado por $\varepsilon_r = 4$ y que incide normalmente sobre una discontinuidad caracterizada por $\varepsilon_r = 36$. Si el campo eléctrico incidente es:

$$\vec{E} = 10 \cdot (2\hat{x} + j\hat{y}) \cdot e^{-j\vec{k}_r \cdot \vec{r}}$$

Calcular:

a) Los coeficientes de reflexión y de transmisión.

b) La expresión del campo eléctrico reflejado.

c) La expresión del campo eléctrico transmitido.

d) La expresión del campo magnético transmitido.

e) La densidad de potencia transmitida al medio.

Una onda plana uniforme se propaga por un medio 1 no magnético ($\mu = \mu_0$; $\varepsilon = 4\varepsilon_0$; $\sigma = 0$) a una frecuencia de 600 MHz. La expresión fasorial de la intensidad de campo eléctrico es:

$$\vec{E}(x, y, z) = E_0 \left(\hat{x} j + A \left(\sqrt{3} \hat{y} - \hat{z} \right) \right) e^{-jg\left(y + \sqrt{3} z\right)} \ \mathrm{V/m}$$

Sabiendo que la onda transporta una densidad de potencia media de 15π W/m^2 y que la constante E_0 vale 30π V/m, calcular:

a) El valor de la constante g.

b) Dirección de propagación de la onda.

c) El valor de la constante A.

d) Si la onda incide perpendicularmente en una superficie que separa el medio 1 de un medio 2, no magnético ($\mu = \mu_0$; $\varepsilon = 9\varepsilon_0$; $\sigma = 0$), obtener el valor de densidad de potencia media que se transmite al segundo medio.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

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Problema 1

- Problema 1.

La amplitud compleja de un campo eléctrico de una onda plana uniforme tiene por expresión:

$$\vec{E}_0 = 100\hat{x} + 20e^{j30^o}\hat{y} \ \text{V/m}$$

Calcular:

a) La expresión fasorial de dicho campo si se sabe que la onda se propaga hacia delante en la dirección de z, en espacio libre y con frecuencia 10 MHz.

b) La expresión del campo eléctrico instantáneo.

Consider a plane wave propagating in a lossy dielectric medium for $z < 0$, with a perfectly conducting plate at $z = 0$. Assume that the lossy medium is characterized by $\epsilon = (5 - j2)\epsilon_0$, $\mu = \mu_0$, and that the frequency of the plane wave is 1.0 GHz, and let the amplitude of the incident electric field be 4 V/m at $z = 0$. Find the reflected electric field for $z < 0$ and plot the magnitude of the total electric field for $-0.5 \leq z \leq 0$.

A plane wave at 1 GHz is normally incident on a thin copper sheet of thickness t. (a) Compute the transmission losses, in dB, of the wave at the air–copper and the copper–air interfaces. (b) If the sheet is to be used as a shield to reduce the level of the transmitted wave by 150 dB, what is the minimum sheet thickness?

A uniform lossy medium with $\epsilon_r = 3.0$, $\tan\delta = 0.1$, and $\mu = \mu_0$ fills the region between $z = 0$ and $z = 20$ cm, with a ground plane at $z = 20$ cm, as shown in the accompanying figure. An incident plane wave with an electric field

$$\bar{E}_i = \hat{x}100e^{-\gamma z} \text{ V/m}$$

is present at $z = 0$ and propagates in the $+z$ direction. The frequency is 3.0 GHz.

(a) Compute S_i, the power density of the incident wave, and S_r, the power density of the reflected wave, at $z = 0$.

(b) Compute the input power density, S_{in}, at $z = 0$ from the total fields at $z = 0$. Does $S_{in} = S_i - S_r$?

Problemas propuestos

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(a) Compute S_i, the power density of the incident wave, and S_r, the power density of the reflected wave, at $z = 0$.

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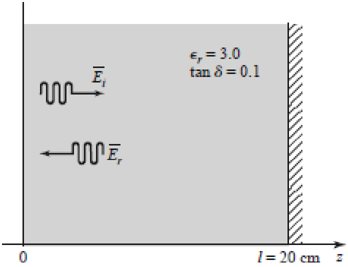

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