Líneas de transmisión

FACULTAD DE INGENIERIA

Escuela de Electrónica y Telecomunicaciones
Quinto Semestre
Unidad I: Revisión de teoría electromagnética
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Rectangular coordinates:

$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \bar{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \bar{A} = \hat{x}\nabla^2 A_x + \hat{y}\nabla^2 A_y + \hat{z}\nabla^2 A_z$$

Fuente: Boria, V., Martín, C. B., Peñarrocha, V. M. R., Oltra, A. S. B., and Pacheco, P. S. (2002). Líneas de transmisión. Universidad Politecnica de Valencia.

Cylindrical coordinates:

$$\nabla f = \hat{\rho}\frac{\partial f}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial f}{\partial \phi} + \hat{z}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \bar{A} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{z}\frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla \times \nabla \times \bar{A}$$

Fuente: Boria, V., Martín, C. B., Peñarrocha, V. M. R., Oltra, A. S. B., and Pacheco, P. S. (2002). Líneas de transmisión. Universidad Politecnica de Valencia.

Spherical coordinates:

$$\nabla f = \hat{r}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial \theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

$$\nabla \cdot \bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \bar{A} = \frac{\hat{r}}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \frac{\hat{\theta}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(r A_\phi)\right] + \frac{\hat{\phi}}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

$$\nabla^2 \bar{A} = \nabla\nabla\cdot\bar{A} - \nabla\times\nabla\times\bar{A}$$

Fuente: Boria, V., Martín, C. B., Peñarrocha, V. M. R., Oltra, A. S. B., and Pacheco, P. S. (2002). Líneas de transmisión. Universidad Politecnica de Valencia.

$$\bar{A}\cdot\bar{B} = |A||B|\cos\theta, \qquad \text{where } \theta \text{ is the angle between } \bar{A} \text{ and } \bar{B} \tag{B.1}$$
$$|\bar{A}\times\bar{B}| = |A||B|\sin\theta, \qquad \text{where } \theta \text{ is the angle between } \bar{A} \text{ and } \bar{B}. \tag{B.2}$$
$$\bar{A}\cdot\bar{B}\times\bar{C} = \bar{A}\times\bar{B}\cdot\bar{C} = \bar{C}\times\bar{A}\cdot\bar{B} \tag{B.3}$$
$$\bar{A}\times\bar{B} = -\bar{B}\times\bar{A} \tag{B.4}$$
$$\bar{A}\times(\bar{B}\times\bar{C}) = (\bar{A}\cdot\bar{C})\bar{B} - (\bar{A}\cdot\bar{B})\bar{C} \tag{B.5}$$
$$\nabla(fg) = g\nabla f + f\nabla g \tag{B.6}$$
$$\nabla\cdot(f\bar{A}) = \bar{A}\cdot\nabla f + f\nabla\cdot\bar{A} \tag{B.7}$$
$$\nabla\cdot(\bar{A}\times\bar{B}) = (\nabla\times\bar{A})\cdot\bar{B} - (\nabla\times\bar{B})\cdot\bar{A} \tag{B.8}$$
$$\nabla\times(f\bar{A}) = (\nabla f)\times\bar{A} + f\nabla\times\bar{A} \tag{B.9}$$
$$\nabla\times(\bar{A}\times\bar{B}) = \bar{A}\nabla\cdot\bar{B} - \bar{B}\nabla\cdot\bar{A} + (\bar{B}\cdot\nabla)\bar{A} - (\bar{A}\cdot\nabla)\bar{B} \tag{B.10}$$
$$\nabla\cdot(\bar{A}\cdot\bar{B}) = (\bar{A}\cdot\nabla)\bar{B} + (\bar{B}\cdot\nabla)\bar{A} + A\times(\nabla\times\bar{B}) + \bar{B}\times(\nabla\times\bar{A}) \tag{B.11}$$
$$\nabla\cdot\nabla\times\bar{A} = 0 \tag{B.12}$$
$$\nabla\times(\nabla f) = 0 \tag{B.13}$$
$$\nabla\times\nabla\times\bar{A} = \nabla\nabla\cdot\bar{A} - \nabla^2\bar{A} \tag{B.14}$$

Note: the term $\nabla^2\bar{A}$ has meaning only for rectangular components of $\bar{A}$.

$$\int_V \nabla\cdot\bar{A}\,dv = \oint_S \bar{A}\cdot d\bar{s} \qquad \text{(divergence theorem)} \qquad ((B.15))$$

$$\int_S (\nabla\times\bar{A})\cdot d\bar{s} = \oint_C \bar{A}\cdot d\bar{\ell} \qquad \text{(Stokes' theorem)} \qquad ((B.16))$$

Onda plana

- El punto de partida del análisis serán las ecuaciones de Maxwell y las condiciones de contorno, pasando a formular las ecuaciones integrales de campo eléctrico en la superficie PEC, siguiendo con su representación en forma matricial.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

Ecuaciones de Maxwell

- Si se considera una onda electromagnética que viaja e incide en un medio homogéneo, que tenga un valor de permitividad ε y permeabilidad μ, las ecuaciones de Maxwell en el dominio de la frecuencia (el factor de dependencia en el tiempo $e^{j\omega t}$ es suprimido), se puede expresar como Chen y Wang 2015; Balanis 2012:

$$\nabla \times \vec{E} = -\vec{M} - j\omega\mu\vec{H} \tag{1}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E} \tag{2}$$

$$\nabla \cdot \vec{D} = \rho_e \tag{3}$$

$$\nabla \cdot \vec{B} = \rho_m \tag{4}$$

Ecuaciones de Maxwell

- donde $\vec{E}$ es la intensidad de campo eléctrico, $\vec{H}$ la intensidad de campo magnético, $\vec{D} = \varepsilon\vec{E}$ la densidad de flujo eléctrico, $\vec{B} = \mu\vec{H}$ la densidad de flujo magnético, $\vec{J} = \sigma\vec{E}$ la densidad de corriente eléctrica, $\vec{M}$ la densidad de corriente magnética, ρ_e la densidad de carga eléctrica, ρ_m la densidad de carga magnética, ε la permitividad, μ la permeabilidad, y σ la conductividad.
- Las ecuaciones (1), (2), (3) y (4) son independientes y válidas para todos los medios Balanis 2012.

Ecuaciones de Maxwell

- Adicionalmente, se puede escribir dos ecuaciones de continuidad relacionando el cambio de densidad de corriente y la densidad de carga, tal como se expresa en la ecuaciones.

$$\nabla \cdot \vec{J} = -j\omega\rho_e \tag{5}$$

$$\nabla \cdot \vec{M} = -j\omega\rho_m \tag{6}$$

Con estas ecuaciones se puede decribir un problema electromagnético. Esto incluye la corriente inducida en un objeto de una onda entrante, así como la radiación producida a partir de una fuente conocida.

Ecuación de Potencial

- Para analizar las corrientes en un cuerpo arbitrario PEC cuando incide una onda electromagnética, se utiliza las ecuaciones (1), y (2). Además, se considera $\vec{M} = 0$, porque el objeto está compuesto solo por material PEC. Este conjunto de ecuaciones puede ser utilizado para resolver la ecuación de onda compleja Harrington 2001. En una región libre de cargas, la divergencia del campo magnético $\vec{H}$ es cero, como se indica en:

$$\nabla \cdot \vec{H} = 0 \tag{7}$$

Además, al analizar la ecuación (4), se puede deducir que $\vec{H}$ es siempre solenoidal. Por lo tanto, esta ecuación se puede escribir como el rotacional de otro vector arbitrario $\vec{A}$.

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A} \tag{8}$$

Ecuación de Potencial

- Donde $\vec{A}(r) = \frac{\mu}{4\pi}\int_V J(r')\frac{e^{-jkR}}{R}$ es el vector potencial magnético. Este vector depende del vector $\vec{J}$ y la posición R, en el cual $R = \mid r - r' \mid$ indica la distancia entre el punto fuente r' y el punto de observación r. Sustituyendo (8) en (1) se obtiene:

$$\nabla \times \vec{E} = -j\omega\nabla \times \vec{A} \tag{9}$$

$$\nabla \times (\vec{E} + j\omega\vec{A}) = 0 \tag{10}$$

Aplicando la identidad $\nabla \times (-\nabla\Phi) = 0$, el campo eléctrico se puede formular como:

$$\vec{E} = -j\omega\vec{A} - \nabla\Phi \tag{11}$$

donde Φ es un potencial eléctrico escalar arbitrario.

Ecuación de Potencial

- Tomando el rotacional a ambos lados de la ecuación (8) y usando la identidad vectorial $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$ se llega a:

$$\mu \nabla \times \vec{H} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \tag{12}$$

Reemplazando la ecuación en (2), se obtiene:

$$\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J} + j\omega\mu\varepsilon \vec{E} \tag{13}$$

Combinando con (11) se llega a la ecuación indicada:

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu \vec{J} + \nabla(\nabla \cdot \vec{A} + j\omega\varepsilon\mu\nabla\Phi) \tag{14}$$

donde k representa el número de onda y se define por la relación $k = \omega\sqrt{\varepsilon\mu}$.

Ecuación de Potencial

- Considerando el lado derecho de (14) y teniendo en cuenta una región libre de cargas, el vector potencial $\vec{A}$ está sujeto a la condición de Lorentz Deeley 1996:

$$\nabla(\nabla \cdot \vec{A}) = -j\omega\varepsilon\mu\Phi \tag{15}$$

El teorema de Lorentz (15) se puede usar para simplificar (14), obteniendo la ecuación inhomogénea de Helmholtz.

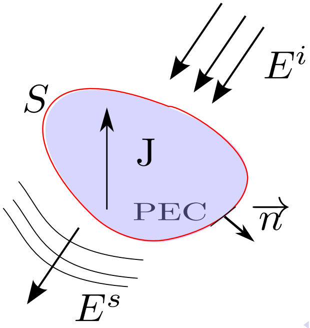
$$\nabla^2\vec{A} + k^2\vec{A} = -\mu\vec{J} \tag{16}$$

En una región libre de cargas, el campo eléctrico enunciado en (11) se puede expresar en función del vector potencial eléctrico como sigue:

$$\vec{E} = -j\omega\vec{A} - \nabla\Phi = -j\omega\vec{A} - \frac{j}{\omega\varepsilon\mu}\nabla(\nabla \cdot \vec{A}) \tag{17}$$

Análisis de problemas de difracción.

- Si se considera una onda plana incidente E^i que ilumina una estructura arbitraria PEC de superficie S, se inducirán un conjunto de corrientes superficiales $\vec{J}$ en la superficie del cuerpo conductor, que irradiarán energía electromagnética en el espacio libre, de forma que el flujo de potencia total en la superficie sea igual a cero Miers 2016.



Análisis de problemas de difracción.

- El análisis se simplifica si la superficie de la estructura tiene una conducción eléctrica perfecta (PEC) como se muestra en la Figura, cumpliéndose la ecuación (18)

$$(\vec{E}^i + \vec{E}^s)_{tan} = 0. \tag{18}$$

donde el subíndice *tan* representa la componente tangencial del campo eléctrico Chen y Wang 2015.
Con la ecuación (18) es posible caracterizar el campo eléctrico dispersado por la estructura $\vec{E}^s$ a través del conocimiento del campo eléctrico incidente $\vec{E}^i$.
El campo dispersado $\vec{E}^s$ se puede expresar en términos de la corriente superficial inducida Thornton y Huang 2013. Seguidamente es necesario considerar las condiciones de contorno, para lo cual se emplea la función de Green, que matemáticamente se expresa como:

Análisis de problemas de difracción.

- $$G(r, r') = \frac{e^{jkR}}{4\pi R} \tag{19}$$

  donde $R = \mid r - r' \mid$ indica la distancia entre el punto fuente r' y el punto de observación r.
  La función de Green incorpora las condiciones de contorno en el dominio en el que se encuentra la fuente, y permite encontrar la solución mediante un operador de convolución. Además, relacionando (19) con (17), el campo eléctrico dispersado se puede expresar como:

  $$\vec{E}^s = -j\omega\vec{A}(r) - \nabla\Phi(r) = -\frac{j\omega\mu_0}{4\pi}\int_s G(r, r')\vec{J}(r')dS' - \frac{j}{4\pi\varepsilon_0\omega}\nabla\int_s G(r, r')\nabla' \tag{20}$$

  donde ε_0 y μ_0, son la permitividad y permeabilidad en el espacio libre, respectivamente.

For a dielectric material, an applied electric field $\bar{E}$ causes the polarization of the atoms or molecules of the material to create electric dipole moments that augment the total displacement flux, $\bar{D}$. This additional polarization vector is called $\bar{P}_e$, the *electric polarization*, where

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}_e.$$

In a linear medium the electric polarization is linearly related to the applied electric field as

$$\bar{P}_e = \epsilon_0 \chi_e \bar{E},$$

where χ_e, which may be complex, is called the *electric susceptibility*. Then,

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}_e = \epsilon_0 (1 + \chi_e) \bar{E} = \epsilon \bar{E},$$

where

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon_0 (1 + \chi_e)$$

is the complex permittivity of the medium. The imaginary part of ϵ accounts for loss in the medium (heat) due to damping of the vibrating dipole moments.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

Material	Frequency	ϵ_r	$\tan\delta$ (25°C)
Alumina (99.5%)	10 GHz	9.5–10.	0.0003
Barium tetratitanate	6 GHz	$37 \pm 5\%$	0.0005
Beeswax	10 GHz	2.35	0.005
Beryllia	10 GHz	6.4	0.0003
Ceramic (A-35)	3 GHz	5.60	0.0041
Fused quartz	10 GHz	3.78	0.0001
Gallium arsenide	10 GHz	13.0	0.006
Glass (pyrex)	3 GHz	4.82	0.0054
Glazed ceramic	10 GHz	7.2	0.008
Lucite	10 GHz	2.56	0.005
Nylon (610)	3 GHz	2.84	0.012
Parafin	10 GHz	2.24	0.0002
Plexiglass	3 GHz	2.60	0.0057
Polyethylene	10 GHz	2.25	0.0004
Polystyrene	10 GHz	2.54	0.00033
Porcelain (dry process)	100 MHz	5.04	0.0078
Rexolite (1422)	3 GHz	2.54	0.00048
Silicon	10 GHz	11.9	0.004
Styrofoam (103.7)	3 GHz	1.03	0.0001

Linear material: A material is *linear* if the material properties (relative permittivity ε_r, relative permeability μ_r and electric conductivity σ) do not depend on the field strengths in the material. The material parameters are not functions of the magnitude of the electric and magnetic field, that is $\varepsilon_r \neq \varepsilon_r(\vec{E}, \vec{H})$, $\mu_r \neq \mu_r(\vec{E}, \vec{H})$ and $\sigma \neq \sigma(\vec{E}, \vec{H})$. If the material is not linear it is called *non-linear*.

Time-invariant material: A material is *time-invariant* if the material parameters do not change with time. The material parameters are not functions of time, that is $\varepsilon_r \neq \varepsilon_r(t)$, $\mu_r \neq \mu_r(t)$ and $\sigma \neq \sigma(t)$. If the material properties depend on time the material is called *time-variant*.

Isotropic material: A material is *isotropic* if the material parameters are independent of the direction in space. The material parameters are then defined by a scalar value. If a material shows different material properties when oriented in different directions it is called *anisotropic*.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

Non-dispersive material: A material is *non-dispersive* if the material parameters do not vary with frequency. The material parameters are not functions of frequency, that is $\varepsilon_r \neq \varepsilon_r(f)$, $\mu_r \neq \mu_r(f)$ and $\sigma \neq \sigma(f)$. If the material properties depend on frequency the material is called *dispersive*.

Homogeneous material: A material is homogeneous if the material parameters do not vary in space. The material parameters are not functions of the spatial coordinates, that is $\varepsilon_r \neq \varepsilon_r(\vec{r})$, $\mu_r \neq \mu_r(\vec{r})$ and $\sigma \neq \sigma(\vec{r})$. If the material properties are not constant in space the material is called *inhomogeneous*.

Fortunately, most RF engineering materials are *simple materials* which are linear, time-invariant and isotropic. Furthermore, at least in narrow frequency bands, they can be regarded as non-dispersive[11] and in confined regions they are homogeneous.

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

In the preceding discussion it was assumed that P_e was a vector in the same direction as $\bar{E}$. Such materials are called *isotropic* materials, but not all materials have this property. Some materials are *anisotropic* and are characterized by a more complicated relation between $\bar{P}_e$ and $\bar{E}$, or $\bar{D}$ and $\bar{E}$. The most general linear relation between these vectors takes the form of a tensor of rank two (a dyad), which can be written in matrix form as

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = [\epsilon] \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}.$$

It is thus seen that a given vector component of $\bar{E}$ gives rise, in general, to three components of $\bar{D}$. Crystal structures and ionized gases are examples of anisotropic dielectrics. For a linear isotropic material, the matrix reduces to a diagonal matrix with elements ϵ.

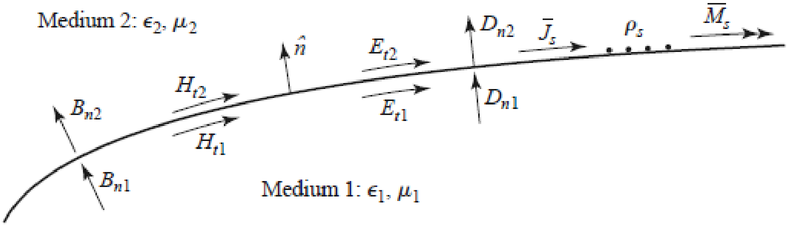
An analogous situation occurs for magnetic materials. An applied magnetic field may align magnetic dipole moments in a magnetic material to produce a *magnetic polarization* (or magnetization) $\bar{P}_m$. Then,

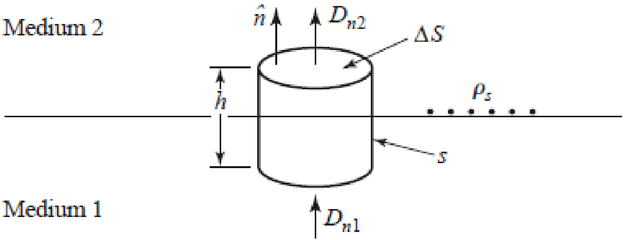
$$\bar{B} = \mu_0(\bar{H} + \bar{P}_m).$$

For a linear magnetic material, $\bar{P}_m$ is linearly related to $\bar{H}$ as

$$\bar{P}_m = \chi_m \bar{H},$$

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.


Medium 2: ϵ_2, μ_2
$\hat{n}$
D_{n2}
$\bar{J}_s$
ρ_s
$\overline{M}_s$
E_{t2}
H_{t2}
B_{n2}
E_{t1}
D_{n1}
H_{t1}
B_{n1}
Medium 1: ϵ_1, μ_1


Medium 2
$\hat{n}$
D_{n2}
ΔS
h
ρ_s
S
Medium 1
D_{n1}

For the tangential components of the electric field we use the phasor

$$\oint_C \bar{E} \cdot d\bar{l} = -j\omega \int_S \bar{B} \cdot d\bar{s} - \int_S \bar{M} \cdot d\bar{s},$$

in connection with the closed contour C shown in Figure In the limit as $h \to 0$, the surface integral of $\bar{B}$ vanishes (because $S = h\Delta\ell$ vanishes). The contribution from the surface integral of $\bar{M}$, however, may be nonzero if a magnetic surface current density $\bar{M}_s$ exists on the surface. The Dirac delta function can then be used to write

$$\bar{M} = \bar{M}_s \delta(h),$$

where h is a coordinate measured normal from the interface.

$$\Delta\ell E_{t1} - \Delta\ell E_{t2} = -\Delta\ell M_s,$$

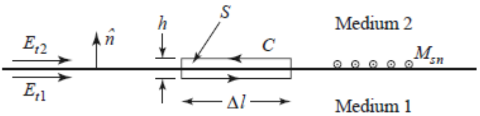


FIGURE Closed contour C

Fuente: POZAR, David M. Microwave engineering. John wiley and sons, 2009.

At an interface between two lossless dielectric materials, no charge or surface current densities will ordinarily exist. Equations

$$\hat{n} \cdot \bar{D}_1 = \hat{n} \cdot \bar{D}_2,$$

$$\hat{n} \cdot \bar{B}_1 = \hat{n} \cdot \bar{B}_2,$$

$$\hat{n} \times \bar{E}_1 = \hat{n} \times \bar{E}_2,$$

$$\hat{n} \times \bar{H}_1 = \hat{n} \times \bar{H}_2.$$

In words, these equations state that the normal components of $\bar{D}$ and $\bar{B}$ are continuous across the interface, and the tangential components of $\bar{E}$ and $\bar{H}$ are continuous across the interface. Because Maxwell's equations are not all linearly independent, the six boundary conditions contained in the above equations are not all linearly independent.

Many problems in microwave engineering involve boundaries with good conductors (e.g., metals), which can often be assumed as lossless ($\sigma \to \infty$). In this case of a perfect conductor, all field components must be zero inside the conducting region. This result can be seen by considering a conductor with finite conductivity ($\sigma < \infty$) and noting that the skin depth (the depth to which most of the microwave power penetrates) goes to zero as $\sigma \to \infty$. If we also assume here that $\bar{M}_s = 0$, which would be the case if the perfect conductor filled all the space on one side of the boundary, the following:

$$\hat{n} \cdot \bar{D} = \rho_s,$$
$$\hat{n} \cdot \bar{B} = 0,$$
$$\hat{n} \times \bar{E} = 0,$$
$$\hat{n} \times \bar{H} = \bar{J}_s,$$

where ρ_s and $\bar{J}_s$ are the electric surface charge density and current density, respectively, on the interface, and $\hat{n}$ is the normal unit vector pointing out of the perfect conductor.

Dual to the preceding boundary condition is the *magnetic wall* boundary condition, where the tangential components of $\bar{H}$ must vanish. Such a boundary does not really exist in practice but may be approximated by a corrugated surface or in certain planar transmission line problems. In addition, the idealization that $\hat{n} \times \bar{H} = 0$ at an interface is often a convenient simplification, as we will see in later chapters. We will also see that the magnetic wall boundary condition is analogous to the relations between the voltage and current at the end of an open-circuited transmission line, while the electric wall boundary condition is analogous to the voltage and current at the end of a short-circuited transmission line. The magnetic wall condition, then, provides a degree of completeness in our formulation of boundary conditions and is a useful approximation in several cases of practical interest.

The fields at a magnetic wall satisfy the following conditions:

$$\hat{n} \cdot \bar{D} = 0,$$
$$\hat{n} \cdot \bar{B} = 0,$$
$$\hat{n} \times \bar{E} = -\bar{M}_s,$$
$$\hat{n} \times \bar{H} = 0,$$

where $\hat{n}$ is the normal unit vector pointing out of the magnetic wall region.

Condiciones de radiación

When dealing with problems that have one or more infinite boundaries, such as plane waves in an infinite medium, or infinitely long transmission lines, a condition on the fields at infinity must be enforced. This boundary condition is known as the *radiation condition* and is essentially a statement of energy conservation. It states that, at an infinite distance from a source, the fields must either be vanishingly small (i.e., zero) or propagating in an outward direction. This result can easily be seen by allowing the infinite medium to contain a small loss factor (as any physical medium would have). Incoming waves (from infinity) of finite amplitude would then require an infinite source at infinity and so are disallowed.

Fuente: Boria, V., Martín, C. B., Peñarrocha, V. M. R., Oltra, A. S. B., and Pacheco, P. S. (2002). Líneas de transmisión. Universidad Politecnica de Valencia.

1. Ecuaciones de Maxwell
2. Condiciones de frontera
3. Trabajos propuestos

Problema 1

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A plane wave propagating in a lossless dielectric medium has an electric field given as $\mathcal{E}_x = E_0 \cos(\omega t - \beta z)$ with a frequency of 5.0 GHz and a wavelength in the material of 3.0 cm. Determine the propagation constant, the phase velocity, the relative permittivity of the medium, and the wave impedance.

Compute the skin depth of aluminum, copper, gold, and silver at a frequency of 10 GHz.

An artificial anisotropic dielectric material has the tensor permittivity $[\epsilon]$ given as follows:

$$[\epsilon] = \epsilon_0 \begin{bmatrix} 1 & 3j & 0 \\ -3j & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

At a certain point in the material the electric field is known to be $\bar{E} = 3\hat{x} - 2\hat{y} + 5\hat{z}$. What is $\bar{D}$ at this point?

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