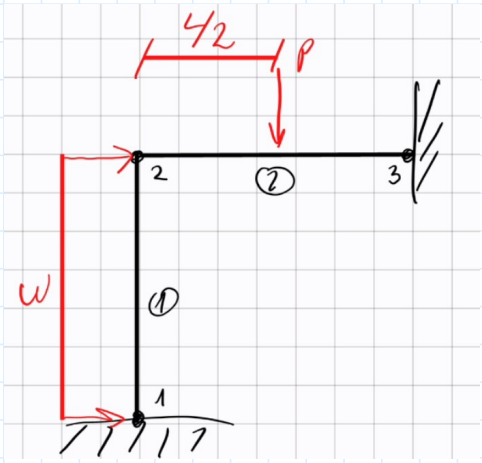
EJERCICIO DE APLICACIÓN - ANÁLISIS MATRICIAL DE PÓRTICOS



$L1 := 300$ V30x50

$L2 := 250$ C50x50

$$XY := \begin{bmatrix} 0 & 0 \\ 0 & 300 \\ 250 & 300 \end{bmatrix}$$

$P := 150$ $E := 3000$

$w := 1$

1------- 1-2
2------- 3-2

Transformación de Coordenada

$$\beta = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Elemento 1

$c1 := 0$ $s1 := \frac{300-0}{300} = 1$

$$\beta 1 := \begin{bmatrix} c1 & s1 & 0 & 0 & 0 & 0 \\ -s1 & c1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c1 & s1 & 0 \\ 0 & 0 & 0 & -s1 & c1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Elemento 2

$$c2 := \frac{-250}{250} = -1 \qquad s2 := \frac{0}{300} = 0$$

$$\beta 2 := \begin{bmatrix} c2 & s2 & 0 & 0 & 0 & 0 \\ -s2 & c2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c2 & s2 & 0 \\ 0 & 0 & 0 & -s2 & c2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matriz de Rigidez

Elemento 1

$$A1 := 50 \cdot 50 = 2500$$

$$I1 := \frac{50^4}{12} = 520833.333$$

$$k1 := \begin{bmatrix} \frac{E \cdot A1}{L1} & 0 & 0 & -\frac{E \cdot A1}{L1} & 0 & 0 \\ 0 & \frac{12 \cdot E \cdot I1}{L1^3} & \frac{6 \cdot E \cdot I1}{L1^2} & 0 & -\frac{12 \cdot E \cdot I1}{L1^3} & \frac{6 \cdot E \cdot I1}{L1^2} \\ 0 & \frac{6 \cdot E \cdot I1}{L1^2} & \frac{4 \cdot E \cdot I1}{L1} & 0 & -\frac{6 \cdot E \cdot I1}{L1^2} & \frac{2 \cdot E \cdot I1}{L1} \\ -\frac{E \cdot A1}{L1} & 0 & 0 & \frac{E \cdot A1}{L1} & 0 & 0 \\ 0 & -\frac{12 \cdot E \cdot I1}{L1^3} & -\frac{6 \cdot E \cdot I1}{L1^2} & 0 & \frac{12 \cdot E \cdot I1}{L1^3} & -\frac{6 \cdot E \cdot I1}{L1^2} \\ 0 & \frac{6 \cdot E \cdot I1}{L1^2} & \frac{2 \cdot E \cdot I1}{L1} & 0 & -\frac{6 \cdot E \cdot I1}{L1^2} & \frac{4 \cdot E \cdot I1}{L1} \end{bmatrix}$$

$$Ki = \beta i^{\mathrm{T}} \cdot ki \cdot \beta i$$

$$K1 := \beta 1^{\mathrm{T}} \cdot k1 \cdot \beta 1$$

$$K1 = \begin{bmatrix} 694.444 & 0 & -104166.667 & -694.444 & 0 & -104166.667 \\ 0 & 25000 & 0 & 0 & -25000 & 0 \\ -104166.667 & 0 & 20833333.333 & 104166.667 & 0 & 10416666.667 \\ -694.444 & 0 & 104166.667 & 694.444 & 0 & 104166.667 \\ 0 & -25000 & 0 & 0 & 25000 & 0 \\ -104166.667 & 0 & 10416666.667 & 104166.667 & 0 & 20833333.333 \end{bmatrix}$$

Elemento 2

$$A2 := 30 \cdot 50 = 1500$$

$$I2 := \frac{30 \cdot 50^3}{12} = 312500$$

$$k2 := \begin{bmatrix} \frac{E \cdot A2}{L2} & 0 & 0 & -\frac{E \cdot A2}{L2} & 0 & 0 \\ 0 & \frac{12 \cdot E \cdot I2}{L2^3} & \frac{6 \cdot E \cdot I2}{L2^2} & 0 & -\frac{12 \cdot E \cdot I2}{L2^3} & \frac{6 \cdot E \cdot I2}{L2^2} \\ 0 & \frac{6 \cdot E \cdot I2}{L2^2} & \frac{4 \cdot E \cdot I2}{L2} & 0 & -\frac{6 \cdot E \cdot I2}{L2^2} & \frac{2 \cdot E \cdot I2}{L2} \\ -\frac{E \cdot A2}{L2} & 0 & 0 & \frac{E \cdot A2}{L2} & 0 & 0 \\ 0 & -\frac{12 \cdot E \cdot I2}{L2^3} & -\frac{6 \cdot E \cdot I2}{L2^2} & 0 & \frac{12 \cdot E \cdot I2}{L2^3} & -\frac{6 \cdot E \cdot I2}{L2^2} \\ 0 & \frac{6 \cdot E \cdot I2}{L2^2} & \frac{2 \cdot E \cdot I2}{L2} & 0 & -\frac{6 \cdot E \cdot I2}{L2^2} & \frac{4 \cdot E \cdot I2}{L2} \end{bmatrix}$$

$$Ki = \beta i^{\mathrm{T}} \cdot ki \cdot \beta i$$

$$K2 := \beta 2^{\mathrm{T}} \cdot k2 \cdot \beta 2$$

$$K2 = \begin{bmatrix} 18000 & 0 & 0 & -18000 & 0 & 0 \\ 0 & 720 & -90000 & 0 & -720 & -90000 \\ 0 & -90000 & 15000000 & 0 & 90000 & 7500000 \\ -18000 & 0 & 0 & 18000 & 0 & 0 \\ 0 & -720 & 90000 & 0 & 720 & 90000 \\ 0 & -90000 & 7500000 & 0 & 90000 & 15000000 \end{bmatrix}$$

Matriz de Colocación

Elemento 1

$$L\_1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Elemento 2

$$L\_2 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Matriz de Rigidez de la Estructura

$$KE = \sum KE\_i$$

$$KE\_i = L\_i^{T} \cdot Ki \cdot L\_i$$

$$KE\_1 := L\_1^{T} \cdot K1 \cdot L\_1 \qquad KE\_2 := L\_2^{T} \cdot K2 \cdot L\_2$$

$$KE := KE\_1 + KE\_2$$

$$KE = \begin{bmatrix} 694.444 & 0 & -104166.667 & -694.444 & 0 & -104166.667 & 0 & 0 & 0 \\ 0 & 25000 & 0 & 0 & -25000 & 0 & 0 & 0 & 0 \\ -104166.667 & 0 & 20833333.333 & 104166.667 & 0 & 10416666.667 & 0 & 0 & 0 \\ -694.444 & 0 & 104166.667 & 18694.444 & 0 & 104166.667 & -18000 & 0 & 0 \\ 0 & -25000 & 0 & 0 & 25720 & 90000 & 0 & -720 & 90000 \\ -104166.667 & 0 & 10416666.667 & 104166.667 & 90000 & 35833333.333 & 0 & -90000 & 7500000 \\ 0 & 0 & 0 & -18000 & 0 & 0 & 18000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -720 & -90000 & 0 & 720 & -90000 \\ 0 & 0 & 0 & 0 & 90000 & 7500000 & 0 & -90000 & 15000000 \end{bmatrix}$$

Fuerzas de empotramiento

Elemento 1

$$\begin{bmatrix} femp11 \\ femp21 \\ femp31 \\ femp41 \\ femp51 \\ femp61 \end{bmatrix} := \begin{bmatrix} 0 \\ \frac{w \cdot L1}{2} \\ \frac{w \cdot L1^{2}}{12} \\ 0 \\ \frac{w \cdot L1}{2} \\ -\frac{w \cdot L1^{2}}{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 150 \\ 7500 \\ 0 \\ 150 \\ -7500 \end{bmatrix}$$

Elemento 2

$$\begin{bmatrix} femp72 \\ femp82 \\ femp92 \\ femp42 \\ femp52 \\ femp62 \end{bmatrix} := \begin{bmatrix} 0 \\ -\frac{P}{2} \\ -\frac{P \cdot L2}{8} \\ 0 \\ -\frac{P}{2} \\ \frac{P \cdot L2}{8} \end{bmatrix} = \begin{bmatrix} 0 \\ -75 \\ -4687.5 \\ 0 \\ -75 \\ 4687.5 \end{bmatrix}$$

$$Femp\_i = \beta i^{T} \cdot femp\_i$$

$$\begin{bmatrix} Femp11 \\ Femp21 \\ Femp31 \\ Femp41 \\ Femp51 \\ Femp61 \end{bmatrix} := \beta 1^{T} \cdot \begin{bmatrix} femp11 \\ femp21 \\ femp31 \\ femp41 \\ femp51 \\ femp61 \end{bmatrix} = \begin{bmatrix} -150 \\ 0 \\ 7500 \\ -150 \\ 0 \\ -7500 \end{bmatrix} \qquad \begin{bmatrix} Femp72 \\ Femp82 \\ Femp92 \\ Femp42 \\ Femp52 \\ Femp62 \end{bmatrix} := \beta 2^{T} \cdot \begin{bmatrix} femp72 \\ femp82 \\ femp92 \\ femp42 \\ femp52 \\ femp62 \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \\ -4687.5 \\ 0 \\ 75 \\ 4687.5 \end{bmatrix}$$

$$Femp\_1 := L\_1^{T} \cdot \begin{bmatrix} Femp11 \\ Femp21 \\ Femp31 \\ Femp41 \\ Femp51 \\ Femp61 \end{bmatrix} = \begin{bmatrix} -150 \\ 0 \\ 7500 \\ -150 \\ 0 \\ -7500 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad Femp\_2 := L\_2^{T} \cdot \begin{bmatrix} Femp72 \\ Femp82 \\ Femp92 \\ Femp42 \\ Femp52 \\ Femp62 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 75 \\ 4687.5 \\ 0 \\ 75 \\ -4687.5 \end{bmatrix}$$

$$Femp\_E := Femp\_1 + Femp\_2 = \begin{bmatrix} -150 \\ 0 \\ 7500 \\ -150 \\ 75 \\ -2812.5 \\ 0 \\ 75 \\ -4687.5 \end{bmatrix}$$

Fuerzas y desplazamientos en grados de libertad

$$F = \begin{bmatrix} R1 \\ R2 \\ R3 \\ 0 \\ 0 \\ 0 \\ R7 \\ R8 \\ R9 \end{bmatrix} \qquad U = \begin{bmatrix} u1 \\ u2 \\ u3 \\ u4 \\ u5 \\ u6 \\ u7 \\ u8 \\ u9 \end{bmatrix}$$

$u1 := 0$ $u7 := 0$

$u2 := 0$ $u8 := 0$

$u3 := 0$ $u9 := 0$

Partición de Matrices

$$Up = Kpp^{-1} \cdot (Fp - Kps \cdot Us - Femp\_p)$$

$$Fs = Ksp \cdot Up + Kss \cdot Us + Femp\_s$$

p ----- 4,5,6
s ----- 1,2,3,7,8,9

$$Kpp := \begin{bmatrix} 18694.444 & 0 & 104166.667 \\ 0 & 25720 & 90000 \\ 104166.667 & 90000 & 35833333.333 \end{bmatrix}$$

$$Kps := \begin{bmatrix} -694.444 & 0 & 104166.667 & -18000 & 0 & 0 \\ 0 & -25000 & 0 & 0 & -720 & 90000 \\ -104166.667 & 0 & 10416666.667 & 0 & -90000 & 7500000 \end{bmatrix}$$

$$Ksp := \begin{bmatrix} -694.444 & 0 & -104166.667 \\ 0 & -25000 & 0 \\ 104166.667 & 0 & 10416666.667 \\ -18000 & 0 & 0 \\ 0 & -720 & -90000 \\ 0 & 90000 & 7500000 \end{bmatrix}$$

$$Kss := \begin{bmatrix} 694.444 & 0 & -104166.667 & 0 & 0 & 0 \\ 0 & 25000 & 0 & 0 & 0 & 0 \\ -104166.667 & 0 & 20833333.333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 18000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 720 & -90000 \\ 0 & 0 & 0 & 0 & -90000 & 15000000 \end{bmatrix}$$

$$Femp\_p := \begin{bmatrix} -150 \\ 75 \\ -2812.5 \end{bmatrix} \qquad Femp\_s := \begin{bmatrix} -150 \\ 0 \\ 7500 \\ 0 \\ 75 \\ -4687.5 \end{bmatrix} \qquad Fp := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Fs = \begin{bmatrix} R1 \\ R2 \\ R3 \\ R7 \\ R8 \\ R9 \end{bmatrix} \qquad Up = \begin{bmatrix} u4 \\ u5 \\ u6 \end{bmatrix} \qquad Us := \begin{bmatrix} u1 \\ u2 \\ u3 \\ u7 \\ u8 \\ u9 \end{bmatrix}$$

Cálculo de desplazamientos y reacciones

$$\begin{bmatrix} u4 \\ u5 \\ u6 \end{bmatrix} := Kpp^{-1} \cdot \left(Fp - Kps \cdot Us - Femp\_p \right) = \begin{bmatrix} 0.00766667 \\ -0.00314028 \\ 0.00006409 \end{bmatrix}$$

$$\begin{bmatrix} R1 \\ R2 \\ R3 \\ R7 \\ R8 \\ R9 \end{bmatrix} := Ksp \cdot \begin{bmatrix} u4 \\ u5 \\ u6 \end{bmatrix} + Kss \cdot Us + Femp\_s = \begin{bmatrix} -162 \\ 78.507 \\ 8966.202 \\ -138 \\ 71.493 \\ -4489.459 \end{bmatrix}$$

Fuerzas internas en los elementos

Ejes Globales

$F = K \cdot U + Femp$

K : Matriz de Rigidez del elemento en ejes globales y tamaño de elemento (6x6)

Femp: Fuerzas de empotramiento en cada elemento en ejes globales y tamaño del elemento (6x1)

U: Vector de desplazamientos del elemento (6x1)

Ejes locales

$f = \beta \cdot F$

βi: Matriz de transformación de coordenadas de cada elemento

F: Fuerzas internas en ejes globales

Elemento 1

$$\begin{bmatrix} F11 \\ F21 \\ F31 \\ F41 \\ F51 \\ F61 \end{bmatrix} := K1 \cdot \begin{bmatrix} u1 \\ u2 \\ u3 \\ u4 \\ u5 \\ u6 \end{bmatrix} + \begin{bmatrix} Femp11 \\ Femp21 \\ Femp31 \\ Femp41 \\ Femp51 \\ Femp61 \end{bmatrix} = \begin{bmatrix} -162 \\ 78.507 \\ 8966.202 \\ -138 \\ -78.507 \\ -5366.206 \end{bmatrix}$$

$$\begin{bmatrix} f11 \\ f21 \\ f31 \\ f41 \\ f51 \\ f61 \end{bmatrix} := \beta 1 \cdot \begin{bmatrix} F11 \\ F21 \\ F31 \\ F41 \\ F51 \\ F61 \end{bmatrix} = \begin{bmatrix} 78.507 \\ 162 \\ 8966.202 \\ -78.507 \\ 138 \\ -5366.206 \end{bmatrix}$$

Elemento 2

$$\begin{bmatrix} F72 \\ F82 \\ F92 \\ F42 \\ F52 \\ F62 \end{bmatrix} := K2 \cdot \begin{bmatrix} u7 \\ u8 \\ u9 \\ u4 \\ u5 \\ u6 \end{bmatrix} + \begin{bmatrix} Femp72 \\ Femp82 \\ Femp92 \\ Femp42 \\ Femp52 \\ Femp62 \end{bmatrix} = \begin{bmatrix} -138 \\ 71.493 \\ -4489.459 \\ 138 \\ 78.507 \\ 5366.206 \end{bmatrix}$$

$$\begin{bmatrix} f72 \\ f82 \\ f92 \\ f42 \\ f55 \\ f62 \end{bmatrix} := \beta 2 \cdot \begin{bmatrix} F72 \\ F82 \\ F92 \\ F42 \\ F55 \\ F62 \end{bmatrix} = ?$$

Comprobación: Sumatoria de Fuerzas Internas igual a Fuerzas Externas

$R1 = -162$ $\quad$ $F11 = -162$

$R2 = 78.507$ $\quad$ $F21 = 78.507$

$R3 = 8966.202$ $\quad$ $F31 = 8966.202$

$P4 := 0$ $\quad$ $F41 + F42 = 0$

$P5 := 0$ $\quad$ $F51 + F52 = 0$

$P6 := 0$ $\quad$ $F61 + F62 = 0$

$R7 = -138$ $\quad$ $F72 = -138$

$R8 = 71.493$ $\quad$ $F82 = 71.493$

$R9 = -4489.459$ $\quad$ $F92 = -4489.459$