Resolución Prueba

① Ecuación de Bernoulli

$$3xy' - 2y = \frac{x^3}{y^2}$$

$$y' - \frac{2}{3x}y = \frac{x^2}{3y^2}$$

$$n = -2$$

$$u = y^{1-(-2)} = y^3$$

$$u = y^3$$

$$\frac{du}{dx} = 3y^2\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3y^2}\frac{du}{dx}$$

$$\frac{1}{3y^2}\frac{du}{dx} - \frac{2}{3x}y = \frac{x^2}{3y^2}$$

$$(1-n)y^{-n}$$

$$[1-(-2)]y^{-(-2)}$$

$$3y^2$$

$$3y^2\left[\frac{1}{3y^2}\frac{du}{dx} - \frac{2}{3x}y = \frac{x^2}{3y^2}\right]$$

$$\frac{du}{dx} - \frac{2}{x}y^3 = x^2$$

$$\frac{du}{dx} - \frac{2}{x}u = x^2$$

$$\mu(x) = e^{-\int\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = \frac{1}{x^2}$$

$$u = \frac{1}{\frac{1}{x^2}}\left(\int\frac{1}{x^2}\cdot x^2\,dx + c\right)$$

$$u = x^2\left(\int dx + c\right)$$

$$u = x^2(x + c)$$

$$u = x^3 + x^2c$$

$$y^3 = x^3 + x^2c$$

② Ecuación de Ricatti

$$y' = y^2 + y - e^{2x} \qquad y_1 = e^x$$

$$y = y_1 + u \qquad u = y - e^x$$

$$y = e^x + u$$

$$\frac{dy}{dx} = e^x + \frac{du}{dx}$$

$$e^x + \frac{du}{dx} = (e^x + u)^2 + (e^x + u) - e^{2x}$$

$$\cancel{e^x} + \frac{du}{dx} = \cancel{e^{2x}} + 2e^x u + u^2 + \cancel{e^x} + u - \cancel{e^{2x}}$$

$$\frac{du}{dx} = 2e^x u + u + u^2$$

$$\frac{du}{dx} - (2e^x + 1) u = u^2$$

$$s = u^{1-n} = u^{1-2} = u^{-1}$$

s

$$s = u^{-1}$$

$$\frac{ds}{dx} = -u^{-2} \frac{du}{dx}$$

$$\frac{du}{dx} = -u^2 \frac{ds}{dx}$$

$$-u^2 \frac{ds}{dx} - (2e^x + 1) u = u^2 \qquad (1-n) u^{-n}$$

$$(1-2) u^{-2} \Rightarrow -u^{-2}$$

$$-u^{-2} \left(-u^2 \frac{ds}{dx} - (2e^x + 1) u = u^2 \right)$$

$$\frac{ds}{dx} + (2e^x + 1) u^{-1} = -1$$

$$\frac{ds}{dx} + (2e^x + 1) s = -1$$

$$u(s) = e^{\int (2e^x + 1) dx} = e^{2\int e^x dx + \int dx} = e^{2e^x + x}$$

$$s = \frac{1}{e^{2e^x + x}} \cdot \left(\int e^{2e^x} \cdot e^x \, (-1) \, dx + c \right)$$

$$s = \frac{1}{e^{2e^x} \cdot e^x} \cdot \left(- \int e^{2e^x} \cdot e^x dx + c \right)$$

$u = e^x$
$du = e^x dx$

$$s = \frac{1}{e^{2e^x} \cdot e^x} \left(- \int e^{2u} \cdot du + c \right)$$

$$s = \frac{1}{e^{2e^x} \cdot e^x} \left(- \frac{e^{2u}}{2} + c \right)$$

$$s = \frac{1}{\cancel{e^{2e^x}} \cdot e^x} \left(- \frac{\cancel{e^{2e^x}}}{2} + c \right)$$

$$s = - \frac{1}{2e^x} + \frac{c}{e^{2e^x} \cdot e^x}$$

$$u^{-1} = - \frac{1}{2e^x} + \frac{c}{e^{2e^x} \cdot e^x}$$

$$(y - e^x)^{-1} = - \frac{1}{2e^x} + \frac{c}{e^{2e^x} \cdot e^x}$$

$$\frac{1}{y - e^x} = - \frac{1}{2e^x} + \frac{c}{e^{2e^x} \cdot e^x}$$

③ Ecuación de Lagrange

$$y = x(y'+3) - 2(y')^2 \qquad y' = p$$

$$y = x(p+3) - 2p^2$$

$$\frac{dy}{dx} = p+3 + x\frac{dp}{dx} - 4p\frac{dp}{dx}$$

$$p = p+3 + x\frac{dp}{dx} - 4p\frac{dp}{dx}$$

$$p - p - 3 = \frac{dp}{dx}(x - 4p)$$

$$\frac{dp}{dx} = \frac{-3}{x-4p}$$

$$\frac{dx}{dp} = \frac{x-4p}{-3}$$

$$\frac{dx}{dp} = -\frac{x}{3} + \frac{4}{3}p$$

$$\frac{dx}{dp} + \frac{x}{3} = \frac{4}{3}p$$

$$\mu(p) = e^{\int \frac{1}{3}p} = e^{\frac{1}{3}p}$$

$$x(p) = \frac{1}{e^{\frac{1}{3}p}}\left(\int e^{\frac{1}{3}p} \cdot \frac{4}{3}p \, dp + c\right)$$

$$x(p) = \frac{1}{e^{\frac{1}{3}p}}\left(\frac{4}{3}\int e^{\frac{1}{3}p} \cdot p \, dp + c\right)$$

$$u = p \qquad dv = e^{\frac{1}{3}p}$$

$$du = dp \qquad v = \frac{e^{\frac{1}{3}p}}{\frac{1}{3}} = 3e^{\frac{1}{3}p}$$

$$x(p) = \frac{1}{e^{\frac{1}{3}p}}\left[\frac{4}{3}\left(3pe^{\frac{1}{3}p} - \int 3e^{\frac{1}{3}p}\,dp + c\right]\right.$$

$$x(p) = \frac{1}{e^{\frac{1}{3}p}}\left[4pe^{\frac{1}{3}p} - 4\frac{e^{\frac{1}{3}p}}{\frac{1}{3}} + c\right]$$

$$x(p) = \frac{1}{e^{\frac{1}{3}p}}\left(4pe^{\frac{1}{3}p} - 12e^{\frac{1}{3}p} + C\right)$$

$$\begin{cases} x(p) = 4p - 12 + \dfrac{C}{e^{\frac{1}{3}p}} \\ y(p) = \left(4p - 12 + \dfrac{C}{e^{\frac{1}{3}p}}\right)(p+3) - 2p^2 \end{cases}$$

Escaneado con CamScanner

Resolución Prueba

① Ecuación de Lagrange

$$y = x + y' - 3(y')^2 \qquad y' = p$$

$$y = x + p - 3p^2$$

$$y' = 1 + \frac{dp}{dx} - 6p\frac{dp}{dx}$$

$$p = 1 + \frac{dp}{dx} - 6p\frac{dp}{dx}$$

$$p - 1 = \frac{dp}{dx}(1 - 6p)$$

$$\frac{dp}{dx} = \frac{p-1}{1-6p}$$

$$\int \frac{1-6p}{p-1}\,dp = \int dx$$

$$\int \frac{1}{p-1}\,dp - 6\int \frac{p}{p-1}\,dp = x$$

$$u = p-1 \qquad u = p-1 \qquad u+1 = p$$
$$du = dp \qquad du =$$

$$\int \frac{du}{u} - 6\int \frac{u+1}{u}\,du = x + c$$

$$\ln u - 6\int \frac{u}{u}\,du - 6\int \frac{1}{u}\,du = x + c$$

$$\ln(p-1) - 6u - 6\ln u = x + c$$

$$\ln(p-1) - 6(p-1) - 6\ln(p-1) = x + c$$

$$-5\ln(p-1) - 6(p-1) = x + c$$

$$\begin{cases} x = -5\ln(p-1) - 6(p-1) - c \\ y = [-5\ln(p-1) - 6(p-1) - c] + p - 3p^2 \end{cases}$$

② Ecuación de Bernoulli

$$\frac{dy}{dx} = e^{x} y^{7} + 2y$$

$$\frac{dy}{dx} - 2y = e^{x} y^{7}$$

$$u = y^{1-n} = y^{1-7} = y^{-6}$$

$$u = y^{-6}$$

$$\frac{du}{dx} = -6y^{-7} \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{y^{7}}{6} \frac{du}{dx}$$

$$-\frac{y^{7}}{6} \frac{du}{dx} - 2y = e^{x} y^{7}$$

$(1-n)y^{-7}$

$(1-7)y^{-7}$

$-6y^{-7}$

$$(-6y^{-7})\left(-\frac{y^{7}}{6} \frac{du}{dx} - 2y = e^{x} y^{7}\right)$$

$$\frac{du}{dx} + 12y^{-6} = -6e^{x}$$

$$\frac{du}{dx} + 12u = -6e^{x}$$

$$u(x) = e^{\int 12 dx} = e^{12x}$$

$$u = \frac{1}{e^{12x}}\left(\int e^{12x} \cdot -6e^{x} \, dx + c\right)$$

$$u = \frac{1}{e^{12x}}\left(-6\int e^{13x} dx + c\right)$$

$$u = \frac{1}{e^{12x}}\left(-6\frac{e^{13x}}{13} + c\right)$$

$$u = -\frac{6}{13}e^{x} + \frac{c}{e^{12x}}$$

$$y^{-6} = -\frac{6}{13}e^{x} + \frac{c}{e^{12x}}$$

(3) Ecuación de Ricatti

$$y' - 2x^2 - \frac{y}{x} = -2y^2 \qquad y_1 = x$$

$$y = y_1 + u \qquad u = y - x$$

$$y = x + u$$

$$y' = 1 + \frac{du}{dx}$$

$$y' = 2x^2 + \frac{y}{x} - 2y^2$$

$$1 + \frac{du}{dx} = 2x^2 + \frac{(x+u)}{x} - 2(x+u)^2$$

$$1 + \frac{du}{dx} = 2x^2 + \frac{x}{x} + \frac{u}{x} - 2(x^2 + 2xu + u^2)$$

$$\cancel{1} + \frac{du}{dx} = \cancel{2x^2} + \cancel{1} + \frac{u}{x} - \cancel{2x^2} - 4xu - 2u^2$$

$$\frac{du}{dx} + \left(4x - \frac{1}{x}\right)u = -2u^2$$

$$s = u^{1-2} = u^{-1}$$

$$s = u^{-1}$$

$$\frac{ds}{dx} = -u^{-2}\frac{du}{dx}$$

$$\frac{du}{dx} = -u^2\frac{ds}{dx}$$

$$-u^2\frac{ds}{dx} + \left(4x - \frac{1}{x}\right)u = -2u^2 \qquad (1-n)u^{-n}$$

$$-u^{-2}\left[-u^2\frac{ds}{dx} + \left(4x - \frac{1}{x}\right)u = -2u^2\right] \qquad (1-2)u^{-2}$$

$$-u^{-2}$$

$$\frac{ds}{dx} - \left(4x - \frac{1}{x}\right)u^{-1} = 2$$

$$\frac{ds}{dx} - \left(4x - \frac{1}{x}\right)s = 2$$

$$\mu(x) = e^{-\int 4x\,dx + \int \frac{1}{x}dx} = e^{-\frac{4x^2}{2} + \ln x} = e^{-2x^2 + \ln x} = xe^{-2x^2}$$

$$s = \frac{1}{xe^{-2x^2}}\left(\int xe^{-2x^2} \cdot 2\,dx + c\right)$$

$$s = \frac{1}{xe^{-2x^2}}\left(2\int xe^{-2x^2}dx + c\right)$$

$$u = -2x^2$$

$$du = -4x\,dx$$

$$-\frac{du}{4} = x\,dx$$

$$s = \frac{1}{xe^{-2x^2}}\left(2\int e^{u} \cdot -\frac{du}{4} + c\right)$$

$$s = \frac{1}{xe^{-2x^2}}\left(-\frac{2}{4}\int e^{u}du + c\right)$$

$$s = \frac{1}{xe^{-2x^2}}\left(-\frac{1}{2}e^{u} + c\right)$$

$$s = \frac{1}{xe^{-2x^2}}\left(-\frac{1}{2}e^{-2x^2} + c\right)$$

$$s = -\frac{1}{2x} + \frac{c}{xe^{-2x^2}}$$

$$u^{-1} = -\frac{1}{2x} + \frac{c}{xe^{-2x^2}}$$

$$(y-x)^{-1} = -\frac{1}{2x} + \frac{c}{xe^{-2x^2}}$$

$$\frac{1}{y-x} = -\frac{1}{2x} + \frac{c}{xe^{-2x^2}}$$

Resolución Prueba

① Ecuación de Bernoulli

$$y' + \frac{y}{x} = \frac{\ln x}{x} y^3$$

$$u = y^{1-n} = y^{1-3} = y^{-2}$$

$$\frac{du}{dx} = -2y^{-3} \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{y^3}{2} \frac{du}{dx}$$

$$-\frac{y^3}{2}\frac{du}{dx} + \frac{y}{x} = \frac{\ln x}{x} \cdot y^3$$

$(1-n)y^{-n}$

$(1-3)y^{-3}$

$-2y^{-3}$

$$-2y^{-3}\left(-\frac{y^3}{2}\frac{du}{dx} + \frac{y}{x} = \frac{\ln x}{x} y^3\right)$$

$$\frac{du}{dx} + 2\frac{y^{-2}}{x} = \frac{-2\ln x}{x}$$

$$\frac{du}{dx} - \frac{2u}{x} = -2\frac{\ln x}{x}$$

$$u(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = \cancel{e}^{\cancel{\ln} x^{-2}} = x^{-2} = \frac{1}{x^2}$$

$$u = \frac{1}{\frac{1}{x^2}}\left(\int \frac{1}{x^2} \cdot -\frac{2\ln x}{x}dx + c\right)$$

$$u = x^2\left(-2\int \frac{\ln x}{x^3} + c\right)$$

$u = \ln x \qquad dv = \frac{1}{x^3}$

$du = \frac{1}{x}dx \qquad v = \frac{-1}{2x^2}$

$$u = x^2\left(-2\ln x\left(-\frac{1}{2x^2}\right) + 2\int -\frac{1}{2x^2}\cdot\frac{1}{x}dx + c\right)$$

$$u = x^2\left[\frac{\ln x}{x^2} - \int \frac{1}{x^3}dx + c\right]$$

$$u = x^2\left(\frac{\ln x}{x^2} + \frac{x^{-2}}{2} + c\right)$$

$$u = x^2\left(\frac{\ln x}{x^2} + \frac{1}{2x^2} + c\right)$$

$$u = \ln x + \frac{1}{2} + cx^2$$

$$y^{-2} = \ln x + \frac{1}{2} + cx^2$$

(2) Ecuación de Riccati

$$y' = \csc^2 x + y\,\text{ctg}\,x + y^2 \qquad y_1 = -\text{ctg}\,x$$

$$y = y_1 + u \qquad y - y_1 = u$$

$$y = -\text{ctg}\,x + u \qquad y + \text{ctg}\,x = u$$

$$y' = \csc^2 x + \frac{du}{dx}$$

$$\csc^2 x + \frac{du}{dx} = \csc^2 x + (-\text{ctg}\,x + u)\,\text{ctg}\,x + (-\text{ctg}\,x + u)^2$$

$$\cancel{\csc^2 x} + \frac{du}{dx} = \cancel{\csc^2 x} - \cancel{\text{ctg}^2 x} + \cancel{u\,\text{ctg}\,x} + \cancel{\text{ctg}^2 x} - 2\cancel{u}\,\text{ctg}\,x + u^2$$

$$\frac{du}{dx} = -u\,\text{ctg}\,x + u^2$$

$$\frac{du}{dx} + u\,\text{ctg}\,x = u^2$$

$$s = u^{1-n} = u^{1-2} = u^{-1}$$

$$\frac{ds}{dx} = -u^{-2}\frac{du}{dx}$$

$$\frac{du}{dx} = -u^2\frac{ds}{dx}$$

$$-u^2\frac{ds}{dx} + u\,\text{ctg}\,x = u^2 \qquad (1-n)u^{-n}$$

$$(-u^{-2})\left(-u^2\frac{ds}{dx} + u\,\text{ctg}\,x = u^2\right) \qquad (1-2)u^{-2}$$

$$-u^{-2}$$

$$\frac{ds}{dx} - u^{-1}\,\text{ctg}\,x = -1$$

$$\frac{ds}{dx} - s\,\text{ctg}\,x = -1$$

$$u(x) = e^{\int -\operatorname{ctg} x \, dx} = e^{-\ln(\operatorname{Sen} x)} = e^{\cancel{\ln} \operatorname{sen}^{-1} x} = \frac{1}{\operatorname{Sen} x}$$

$$s = \frac{1}{\frac{1}{\operatorname{Sen} x}} \cdot \left(\int \frac{1}{\operatorname{Sen} x} \cdot -1 \, dx + c \right)$$

$$s = \operatorname{Sen} x \left(- \int \csc x \, dx + c \right)$$

$$s = \operatorname{Sen} x \left[- \left(-\ln(\csc x + \operatorname{ctg} x) \right] + c \right]$$

$$s = \operatorname{Sen} x \left(\ln(\csc x + \operatorname{ctg} x) + c \right]$$

$$u^{-1} = \operatorname{Sen} x \left(\ln(\csc x + \operatorname{ctg} x) + c \right]$$

$$(y + \operatorname{ctg} x)^{-1} = \operatorname{Sen} x \left[\ln(\csc x + \operatorname{ctg} x) + c \right]$$

3) Ecuación de Lagrange.

$$y = 2xy' + \ln y' \qquad y' = p$$

$$y = 2xp + \ln p$$

$$y' = 2x \frac{dp}{dx} + 2p + \frac{1}{p} \frac{dp}{dx}$$

$$p = 2x \frac{dp}{dx} + 2p + \frac{1}{p} \frac{dp}{dx}$$

$$p - 2p = \frac{dp}{dx} \left(2x + \frac{1}{p} \right)$$

$$-p = \frac{dp}{dx} \left(2x + \frac{1}{p} \right)$$

$$\frac{dp}{dx} = \frac{-p}{2x + \frac{1}{p}}$$

$$\frac{dx}{dp} = \frac{2x + \frac{1}{p}}{-p}$$

$$\frac{dx}{dp} = -\frac{2x}{p} - \frac{\frac{1}{p}}{p}$$

$$\frac{dx}{dp} + \frac{2x}{p} = -\frac{1}{p^2}$$

$$u(p) = e^{\int +\frac{2}{p}\,dp} = e^{2\ln p} = e^{\ln p^2} = p^2$$

$$x(p) = \frac{1}{p^2}\left(\int p^2 \cdot -\frac{1}{p^2}\,dp + c\right)$$

$$x(p) = \frac{1}{p^2}\left(\int -\,dp + c\right)$$

$$x(p) = \frac{1}{p^2}\left(-p + c\right)$$

$$\begin{cases} x(p) = -\dfrac{1}{p} + \dfrac{c}{p^2} \\[2ex] y(p) = 2\left(-\dfrac{1}{p} + \dfrac{c}{p^2}\right)p + \ln p \end{cases}$$