Resolución Taller # 2

① Resolver las ecuaciones

$$2\cos x\, dy = (y\sin x - y^3)\, dx$$

$$2\cos x \frac{dy}{dx} = y\sin x - y^3$$

$$\frac{dy}{dx} = \frac{y\sin x - y^3}{2\cos x}$$

$$\frac{dy}{dx} = \frac{y\sin x}{2\cos x} - \frac{y^3}{2\cos x}$$

$$\frac{dy}{dx} - \frac{y\sin x}{2\cos x} = -\frac{y^3}{2\cos x}$$

$$u = y^{1-3} = y^{-2}$$

$$\frac{du}{dx} = -2y^{-3}\frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{y^3}{2}\frac{du}{dx}$$

$$-\frac{y^3}{2}\frac{du}{dx} - \frac{\sin x}{2\cos x}y = -\frac{y^3}{2\cos x}$$

$$(1-n)y^{-n} = (1-3)y^{-3} = -2y^{-3}$$

$$(-2y^{-3})\left(-\frac{y^3}{2}\frac{du}{dx} - \frac{\sin x}{2\cos x}y = -\frac{y^3}{2\cos x}\right)$$

$$\frac{du}{dx} + \frac{\sin x}{\cos x}y^{-2} = \frac{1}{\cos x}$$

$$\frac{du}{dx} + \frac{\sin x}{\cos x}u = \frac{1}{\cos x}$$

$$\mu(x) = e^{\int \frac{\sin x}{\cos x}dx}$$

$u = \cos x$
$du = -\operatorname{Sen} x\, dx$
$-du = \operatorname{Sen} x\, dx$

$$\mu(x) = e^{\int -\frac{du}{u}} = e^{-\ln u} = u^{-1} = \frac{1}{\cos x}$$

$$u = \frac{1}{\frac{1}{\cos x}}\left(\int \frac{1}{\cos x}\cdot\frac{1}{\cos x}\,dx + C\right)$$

$$u = \cos x \left(\int \frac{1}{\cos^2 x} dx + c \right)$$

$$u = \cos x \left(\int \csc^2 x + c \right)$$

$$u = \cos x \left(-\operatorname{ctg} x + c \right)$$

$$y^{-2} = \cos x \left(-\frac{\cos x}{\operatorname{sen} x} + c \right)$$

$$y^2 = \frac{1}{-\frac{\cos^2 x}{\operatorname{sen} x} + c\cos x}$$

$$y^2 = \frac{1}{\frac{-\cos^2 x + c\operatorname{sen} x \cos x}{\operatorname{sen} x}}$$

$$y^2 = \frac{\operatorname{sen} x}{-\cos^2 x + c\operatorname{sen} x \cos x}$$

(2) Resuelve las ecuaciones

$$y' = y^2 \ln(x) - 3y + \frac{1}{x} \quad , \quad y_1 = \frac{1}{x}$$

$$y = \frac{1}{x} + u$$

$$y' = -\frac{1}{x^2} + \frac{du}{dx}$$

$$-\frac{1}{x^2} + \frac{du}{dx} = \left(\frac{1}{x} + u \right)^2 \ln x - 3\left(\frac{1}{x} + u \right) + \frac{1}{x}$$

$$-\frac{1}{x^2} + \frac{du}{dx} = \left(\frac{1}{x^2} + \frac{2u}{x} + u^2 \right) \ln x - \frac{3}{x} - 3u + \frac{1}{x}$$

$$-\frac{1}{x^2} + \frac{du}{dx} = \frac{1}{x^2}\ln x + \frac{2}{x} u \ln x + u^2 \ln x - \cancel{\frac{3}{x}} - 3u + \cancel{\frac{1}{x}}$$

$$\frac{du}{dx} - \frac{2}{x} u \ln x + 3u = u^2 \ln x + \frac{1}{x^2} + \frac{1}{x^2}\ln x$$

No es lineal

③ Resuelva la ecuación

$$3xy' - 2y = \frac{x^3}{y^2}$$

$$y' - \frac{2y}{3x} = \frac{x^2}{3y^2}$$

$$u = y^{1-(-2)}$$

$$u = y^{1+2} = y^3$$

$$\frac{du}{dx} = 3y^2\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{y^{-2}}{3}\frac{du}{dx}$$

$$\frac{y^{-2}}{3}\frac{du}{dx} - \frac{2}{3x}y = \frac{x^2}{3y^2}$$

$$(1-n)y^{-n} = [1-(-2)]\,y^{-(-2)} = (3y^2)$$

$$\cancel{3y^2}\left(\frac{\cancel{y^{-2}}}{\cancel{3}}\frac{du}{dx} - \frac{2}{3x}y = \frac{x^2}{3y^2}\right)$$

$$\frac{du}{dx} - \frac{2}{x}y^3 = x^2$$

$$\frac{du}{dx} - \frac{2}{x}u = x^2$$

$$\mu(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$$

$$u = \frac{1}{\frac{1}{x^2}}\left(\int \frac{1}{x^2}\cdot x^2\,dx + c\right)$$

$$u = x^2\left(\int dx + c\right)$$

$$u = x^2(x+c)$$

$$y^3 = x^2(x+c)$$

(4)

$$y = x\sqrt{y'} + \frac{1}{2}(y')^2 - \frac{2}{3}(y')^{3/2}$$

$$y = x\sqrt{p} + \frac{1}{2}p^2 - \frac{2}{3}p^{\frac{3}{2}}$$

$$y' = \sqrt{p} + x\left(\frac{1}{2}\right)p^{-\frac{1}{2}}\frac{dp}{dx} + \frac{1}{2}(2p)\frac{dp}{dx} - \frac{2}{3}\left(\frac{3}{2}\right)p^{\frac{1}{2}}\frac{dp}{dx}$$

$$p = \sqrt{p} + \frac{x}{2\sqrt{p}}\frac{dp}{dx} + p\frac{dp}{dx} - \sqrt{p}\frac{dp}{dx}$$

$$p - \sqrt{p} = \frac{dp}{dx}\left(\frac{x p^{-1/2}}{2} + p - \sqrt{p}\right)$$

$$p^{1/2}(p^{1/2} - 1) = \frac{dp}{dx}\left[p^{1/2}\left(\frac{x}{2}p^{-1} + p^{\frac{1}{2}} - 1\right)\right]$$

$$\frac{dp}{dx} = \frac{p^{1/2}(p^{1/2} - 1)}{p^{\frac{1}{2}}\left(\frac{x}{2}p^{-1} + p^{1/2} - 1\right)}$$

$$\frac{dx}{dp} = \frac{\left(\frac{x}{2}p^{-1} + p^{\frac{1}{2}} - 1\right)}{p^{\frac{1}{2}} - 1}$$

$$\frac{dx}{dp} = \frac{\frac{x}{2}p^{-1}}{p^{1/2} - 1} + \frac{\cancel{p^{\frac{1}{2}} - 1}}{\cancel{p^{1/2} - 1}}$$

$$\frac{dx}{dp} = \frac{\frac{x}{2p}}{p^{\frac{1}{2}} - 1} + 1$$

$$\frac{dx}{dp} = \frac{x}{2p(p^{1/2} - 1)} + 1$$

$$\frac{dx}{dp} - \frac{x}{2p(\sqrt{p} - 1)} = 1$$

$$e^{-\int \frac{1}{2p(\sqrt{p} - 1)}dp}$$

$u = \sqrt{p}$

$u^2 = p$

$dp = 2u\,du$

$$e^{-\int \frac{\cancel{2u}}{2\cancel{u^2}(u-1)}du}$$

$$e^{-\int \frac{du}{u(u-1)}}$$

$$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1} = \frac{A(u-1)+Bu}{u(u-1)}$$

$u=0$ $\qquad$ $u=1$

$A(-1)=1$ $\qquad$ $B(1)=1$

$A=-1$ $\qquad$ $B=1$

$$-\int \frac{1}{u}\,du + \int \frac{1}{u-1}\,du$$

$$-\ln u + \ln(u-1)$$

$$e$$

$$e^{\ln u^{-1} + \ln(u-1)}$$

$$\mu(p) = \frac{u-1}{u} = \frac{\sqrt{p}-1}{\sqrt{p}}$$

$$x(p) = \frac{1}{\frac{\sqrt{p}-1}{\sqrt{p}}}\left(\int \frac{\sqrt{p}-1}{\sqrt{p}}\,dp + C\right)$$

$$x(p) = \frac{p}{\sqrt{p}-1}\left(\int \frac{\sqrt{p}}{\sqrt{p}}\,dp - \int \frac{1}{\sqrt{p}}\,dp + C\right)$$

$$x(p) = \frac{p}{\sqrt{p}-1}\left(p - \int p^{-\frac{1}{2}}\,dp + C\right)$$

$$x(p) = \frac{p}{\sqrt{p}-1}\left(p - \frac{p^{\frac{1}{2}}}{\frac{1}{2}} + C\right)$$

$$x(p) = \frac{p}{\sqrt{p}-1}\left(p - 2\sqrt{p} + C\right)$$

$$y(p) = \left[\frac{p}{\sqrt{p}-1}\left(p - 2\sqrt{p} + C\right)\right]\sqrt{p} + \frac{1}{2}p^2 - \frac{2}{3}p^{\frac{3}{2}}$$

Escaneado con CamScanner

(5) Resuelva

$$y = e^{y'}x + \ln(1+y'^2)$$

$$y = e^{p}x + \ln(1+p^2)$$

$$y' = xe^{p}\frac{dp}{dx} + e^{p} + \frac{1}{1+p^2}\,2p\,\frac{dp}{dx}$$

$$p = xe^{p}\frac{dp}{dx} + e^{p} + \frac{2p}{1+p^2}\frac{dp}{dx}$$

$$p - e^{p} = \frac{dp}{dx}\left(xe^{p} + \frac{2p}{1+p^2}\right)$$

$$\frac{dp}{dx} = \frac{p-e^{p}}{xe^{p} + \frac{2p}{1+p^2}}$$

$$\frac{dx}{dp} = \frac{xe^{p} + \frac{2p}{1+p^2}}{p-e^{p}}$$

$$\frac{dx}{dp} = \frac{xe^{p}}{p-e^{p}} + \frac{\frac{2p}{1+p^2}}{p-e^{p}}$$

$$\frac{d}{dp} - \frac{e^{p}}{p-e^{p}} = \frac{2p}{(p-e^{p})(1+p^2)}$$

$$u(p) = e^{-\int\frac{e^{p}}{p-e^{p}}} = e^{-[-\ln(p-e^{p})]} = e^{\ln(p-e^{p})}$$

$$u(p) = p - e^{p}$$

$$x(p) = \frac{1}{p-e^{p}}\int\left(\cancel{(p-e^{p})}\cdot\frac{2p}{\cancel{(p-e^{p})}(1+p^2)}\,dp + c\right)$$

$$x(p) = \frac{1}{p-e^{p}}\left(\int\frac{2p}{1+p^2}\,dp + c\right)$$

$$u = 1+p^2$$

$$du = 2p\,dp$$

$$x(p) = \frac{1}{p-e^{p}}\left(\int\frac{du}{u} + c\right)$$

$$x(p) = \frac{1}{p-e^p}\left(\ln u + c \right)$$

$$x(p) = \frac{1}{p-e^p}\left[\ln(1+p^2) + c \right]$$

$$y(p) = e^p\left\{ \frac{1}{p-e^p}\left[\ln(1+p^2) + c \right] \right\} + \ln(1+p^2)$$

⑥ Resuelva las ecuaciones

$$y = (y')^2 x + \sin(y') + 3$$

$$y = p^2 x + \sin p + 3$$

$$y' = x(2p)\frac{dp}{dx} + p^2 + \cos p \frac{dp}{dx}$$

$$p = 2xp\frac{dp}{dx} + p^2 + \cos p \frac{dp}{dx}$$

$$p - p^2 = \frac{dp}{dx}\left(2xp + \cos p \right)$$

$$\frac{dp}{dx} = \frac{p-p^2}{(2xp+\cos p)}$$

$$\frac{dx}{dp} = \frac{2xp+\cos p}{p-p^2}$$

$$\frac{dx}{dp} = \frac{2xp}{p(1-p)} + \frac{\cos p}{p(1-p)}$$

$$\frac{dx}{dp} - \frac{2x}{1-p} = \frac{\cos p}{p(1-p)}$$

$$\mu(p) = e^{-\int \frac{2}{1-p} dp} = e^{-2\int \frac{1}{1-p} dp}$$

$u = 1-p$
$du = -dp$
$-du = dp$

$$\mu(p) = e^{-2\left[-\ln(1-p)\right]} = e^{\ln(1-p)^2}$$

$$\mu(p) = (1-p)^2$$

$$x(p) = \frac{1}{(1-p)^2}\left(\int (1-p)^2 \cdot \frac{\cos p}{p(1-p)} dp + c \right)$$

Escaneado con CamScanner

$$x(p) = \frac{1}{(1-p)^2}\left(\int \frac{(1-p)\cos p}{p}\,dp + C\right)$$

$$x(p) = \frac{1}{(1-p)^2}\left(\int \frac{\cos p\,dp}{p} - \int \frac{p\cos p}{p}\,dp + C\right)$$

$$x(p) = \frac{1}{1-p^2}\left(Gp - \sin p + C\right)$$

$$y(p) = p^2\left[\frac{1}{1-p^2}\left(Gp - \sin p + C\right)\right] + \sin p + 3$$

7) $y' = \cos(x) - 2y\sin x + y^2 \qquad y_1 = tg\,x$

$y = tg\,x + u$

$y' = \sec^2 x + \frac{du}{dx}$

$$\sec^2 x + \frac{du}{dx} = \cos x - 2(tg\,x + u)\sin x + (Tg\,x + u)^2$$

$$\sec^2 x + \frac{du}{dx} = \cos x - 2tg\,x\sin x - 2u\sin x + Tg^2 x + 2Tg\,x\,u + u^2$$

$$\frac{1}{\cos^2 x} + \frac{du}{dx} = \cos x - \frac{2\sin x}{\cos x}\cdot\sin x - 2u\sin x + \frac{\sin^2 x}{\cos^2 x} + 2u\frac{\sin x}{\cos x} + u^2$$

$$\frac{du}{dx} + 2u\sin x + 2u\frac{\sin x}{\cos x} = u^2 - \frac{1}{\cos^2 x} - \frac{2\sin^2 x}{\cos x} + \cos x + \frac{\sin^2 x}{\cos^2 x}$$

No es lineal

③ Resuelva la ecuación

$$x^2y' + 2x^3y = y^2(1+2x^2)$$

$$y' + 2xy = \frac{y^2(1+2x^2)}{x^2}$$

$$u = y^{1-2}$$

$$u = y^{-1}$$

$$\frac{du}{dx} = -y^{-2}\frac{dy}{dx}$$

$$\frac{dy}{dx} = -y^2\frac{du}{dx}$$

$$-y^2\frac{du}{dx} + 2xy = \frac{y^2(1+2x^2)}{x^2}$$

$$(1-n)y^{-n}$$

$$-y^{-2}$$

$$-y^{-2}\left[-y^2\frac{du}{dx} + 2xy = \frac{y^2(1+2x^2)}{x^2}\right]$$

$$\frac{du}{dx} - 2xy^{-1} = -\frac{(1+2x^2)}{x^2}$$

$$\frac{du}{dx} - 2xu = -\frac{(1+2x^2)}{x^2}$$

$$u(x) = e^{\int -2x\,dx} = e^{-2\frac{x^2}{2}} = e^{-x^2}$$

$$u = \frac{1}{e^{-x^2}}\left(\int e^{-x^2}\cdot\frac{-(1+2x^2)}{x^2}dx + c\right)$$

$$y^{-1} = e^{x^2}\left(-\int e^{-x^2}\cdot\frac{1+2x^2}{x^2}dx + c\right)$$

PAPELIKA

9. Resuelva la ecuación

$$y' = x\ln x - 3\ln(x)y + y^2 \quad , \quad y_1 = \ln(x)$$

$$y = u + \ln x$$

$$y' = \frac{1}{x} + \frac{du}{dx}$$

$$\frac{1}{x} + \frac{du}{dx} = x\ln x - 3\ln x\left(\ln x + u\right) + \left(u + \ln x\right)^2$$

$$\frac{1}{x} + \frac{du}{dx} = x\ln x - 3\ln^2 x - 3u\ln x + u^2 + 2u\ln x + \ln^2 x$$

$$\frac{du}{dx} = x\ln x - \frac{1}{x} - 2\ln^2 x - 3u\ln x + 2u\ln x + u^2$$

$$\frac{du}{dx} = x\ln x - \frac{1}{x} - 2\ln^2 x - u\ln x + u^2$$

$$\frac{du}{dx} + u\ln x = u^2 + \left(\frac{x^2\ln x - 1 - 2x\ln^2 x}{x}\right)$$

No es lineal