Resolución Prueba

① Demuestre que es solución implícita o explícita.

$$(\cos x \operatorname{Sen} x - xy^2)dx + [y(1-x^2)]\,dy = 0$$

$$(\cos x \operatorname{Sen} x - xy^2) + y(1-x^2)\frac{dy}{dx} = 0$$

$$y^2(1-x^2) - \cos^2 x = c$$

$$2y\frac{dy}{dx}(1-x^2) + y^2(-2x) + 2\cos x \operatorname{Sen} x = 0$$

$$2y(1-x^2)\frac{dy}{dx} - 2xy^2 + 2\cos x \operatorname{Sen} x = 0$$

$$\frac{dy}{dx} = \frac{2xy^2 - 2\cos x \operatorname{Sen} x}{2y(1-x^2)}$$

$$\frac{dy}{dx} = \frac{2(xy^2 - \cos x \operatorname{Sen} x)}{2y(1-x^2)}$$

$$\frac{dy}{dx} = \frac{xy^2 - \cos x \operatorname{Sen} x}{y(1-x^2)}$$

$$\cos x \operatorname{Sen} x - xy^2 + y(1-x^2)\left(\frac{xy^2 - \cos x \operatorname{Sen} x}{y(1-x^2)}\right) = 0$$

$$\cos x \operatorname{Sen} x - xy^2 + xy^2 - \cos x \operatorname{Sen} x = 0$$

$$0 = 0$$

② Variables separables

$$\frac{dy}{dx} = \frac{xy + 2x - y - 2}{x^2}$$

$$(xy - y) + (2x - 2)$$

$$y(x-1) + 2(x-1)$$

$$\frac{dy}{dx} = \frac{(x-1)(y+2)}{x^2}$$

$$\int \frac{dy}{y+2} = \int \frac{x-1}{x^2}\,dx$$

$u = y+2$

$du = dy$

$$\int \frac{du}{u} = \int \frac{x}{x^2}dx - \int \frac{1}{x^2}dx$$

$$\ln u = \int \frac{1}{x}\,dx - \int x^{-2}\,dx$$

$$\ln(y+2) = \ln x - \frac{x^{-1}}{-1} + C$$

$$\ln(y+2) = \ln x + \frac{1}{x} + C$$

③ Ecuación diferencial lineal

$$y' - \frac{2x}{2-x^2}y = x^3$$

$$\mu(x) = e^{\int -\frac{2x}{2-x^2}dx}$$

$$e^{-\int \frac{2x}{2-x^2}dx}$$

$u = 2-x^2$

$du = -2x\,dx$

$-du = 2x\,dx$

$$e^{-\int -\frac{du}{u}}$$

$$e^{\ln u} = u$$

$$\mu(x) = 2-x^2$$

$$y(x) = \frac{1}{2-x^2}\left(\int (2-x^2)\,x^3\,dx + C\right)$$

$$y(x) = \frac{1}{2-x^2}\left(\int 2x^3\,dx - \int x^5\,dx + C\right)$$

$$y(x) = \frac{1}{2-x^2}\left(2\int x^3\,dx - \frac{x^6}{6} + C\right)$$

$$y(x) = \frac{1}{2-x^2}\left(\frac{2x^4}{4} - \frac{x^6}{6} + C\right)$$

$$y(x) = \frac{1}{2-x^2}\left(\frac{1}{2}x^4 - \frac{x^6}{6} + C\right)$$

④ Ecuación diferencial exacta.

$$(3x^2y + e^x \operatorname{Sen} y) + (x^3 + e^x \operatorname{Cos} y)y' = 0$$

$$(3x^2y + e^x \operatorname{Sen} y)dx + (x^3 + e^x \operatorname{Cos} y)dy = 0$$

$$\frac{\partial m}{\partial y} = 3x^2 + e^x \operatorname{Cos} y \qquad \frac{\partial n}{\partial x} = 3x^2 + e^x \operatorname{Cos} y$$

$$F(x,y) = \int (3x^2y + e^x \operatorname{Sen} y)\,dx$$

$$F(x,y) = \int 3x^2y\,dx + \int e^x \operatorname{Sen} y\,dx$$

$$F(x,y) = 3y \int x^2 dx + \operatorname{Sen} y \int e^x dx$$

$$F(x,y) = \frac{3x^3}{3}y + \operatorname{Sen} y\, e^x + g(y)$$

$$\frac{\partial F}{\partial y} = x^3 + e^x \operatorname{Cos} y + g'(y)$$

$$x^3 + e^x \operatorname{Cos} y = x^3 + e^x \operatorname{Cos} y + g'(y)$$

$$g'(y) = 0$$

$$g(y) = 0$$

$$k = x^3y + e^x \operatorname{Sen} y$$

⑤ Ecuación homogenea.

$$-(x^2 - y^2)dx + 2xy\,dy = 0$$

Si es homogénea

$$y = ux$$

$$\frac{dy}{dx} = x\frac{du}{dx} + u$$

$$2xy\,dy = (x^2 - y^2)dx$$

$$2xy\frac{dy}{dx} = x^2 - y^2$$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

$$x\frac{du}{dx} + u = \frac{x^2 - (ux)^2}{2x(ux)}$$

$$x\frac{du}{dx} + u = \frac{x^2 - u^2x^2}{2ux^2}$$

Escaneado con CamScanner

$$x\frac{du}{dx}+u=\frac{x^2(1-u^2)}{2ux^2}$$

$$x\frac{du}{dx}=\frac{1-u^2}{2u}-u$$

$$x\frac{du}{dx}=\frac{1-u^2-2u^2}{2u}$$

$$x\frac{du}{dx}=\frac{1-3u^2}{2u}$$

$$\int\frac{2u}{1-3u^2}\,du=\int\frac{dx}{x}$$

$u=1-3u^2$
$du=-6u\,du$
$-\frac{1}{3}du=2u\,du$

$$\int\frac{-\frac{du}{3}}{u}$$

$$-\frac{1}{3}\int\frac{du}{u}=\ln x+c$$

$$-\frac{1}{3}\ln u=\ln x+c$$

$$-\frac{1}{3}\ln(1-3u^2)=\ln x+c$$

$$-\frac{1}{3}\ln\left(1-3\left(\frac{y}{x}\right)^2\right)=\ln x+c$$

$$-\frac{1}{3}\ln\left[1-\frac{3y^2}{x^2}\right]=\ln x+c$$

$$-\frac{1}{3}\ln\left[\frac{x^2-3y^2}{x^2}\right]=\ln x+c$$

$$\ln\left(\frac{x^2-3y^2}{x^2}\right)^{-\frac{1}{3}}=\ln x+c$$

$e \qquad e$

$$\left(\frac{x^2-3y^2}{x^2}\right)^{-\frac{1}{3}}=xc$$

Resolución Prueba

① Solución implícita y explícita.

$$(x-y^3+y^2\,\mathrm{Sen}x)\,dx = (3xy^2+2y\cos x)\,dy$$

$$(x-y^3+y^2\,\mathrm{Sen}x) = (3xy^2+2y\cos x)\frac{dy}{dx}$$

$$xy^3+y^2\cos x-\frac{1}{2}x^2=C$$

$$y^3+3xy^2\frac{dy}{dx}+2y\cos x\frac{dy}{dx}-y^2\,\mathrm{Sen}x-\frac{1}{2}(2x)=0$$

$$3xy^2\frac{dy}{dx}+2y\cos x\frac{dy}{dx} = x+y^2\,\mathrm{Sen}x-y^3$$

$$\frac{dy}{dx}\left(3xy^2+2y\cos x\right)=x+y^2\,\mathrm{Sen}x-y^3$$

$$\frac{dy}{dx}=\frac{x+y^2\,\mathrm{Sen}x-y^3}{3xy^2+2y\cos}$$

$$x-y^3+y^2\,\mathrm{Sen}x = (3xy^2+2y\cos x)\left(\frac{x+y^2\,\mathrm{Sen}x-y^3}{3xy^2+2y\cos x}\right)$$

$$x-y^3+y^2\,\mathrm{Sen}x = x+y^2\,\mathrm{Sen}x-y^3$$

② Variables separables

$$\frac{dy}{dx}=\frac{3xy-6y+2x-4}{x^3}$$

$$(3xy-6y)+(2x-4)$$

$$3y(x-2)+2(x-2)$$

$$(x-2)(3y+2)$$

$$\frac{dy}{dx}=\frac{(x-2)(3y+2)}{x^3}$$

$$\int\frac{dy}{3y+2}=\int\frac{x-2}{x^3}\,dx$$

$$u=3y+2$$
$$du=3\,dy$$

$$\frac{du}{3}=dy$$

$$\int\frac{\frac{du}{3}}{u}=\int\frac{x}{x^3}dx-\int\frac{2}{x^3}dx$$

$$\frac{1}{3}\int\frac{du}{u}=\int\frac{1}{x^2}dx-2\int\frac{dx}{x^3}$$

Escaneado con CamScanner

$$\frac{1}{3}\ln u = \int x^{-2}dx - 2\int x^{-3}dx$$

$$\frac{1}{3}\ln(3y+2) = \frac{x^{-1}}{-1} - 2\frac{x^{-2}}{-2} + c$$

$$\frac{1}{3}\ln(3y+2) = -\frac{1}{x} + \frac{1}{x^2} + c$$

③ Ecuación diferencial lineal

$$y' - \frac{3x^2}{3-x^3}y = x^2$$

$$u = 3-x^3$$
$$du = -3x^2dx$$
$$-du = 3x^2dx$$

$$\mu(x) = e^{-\int \frac{3x^2}{3-x^3}dx}$$

$$\mu(x) = e^{-\int -\frac{du}{u}} = e^{\int \frac{du}{u}}$$

$$\mu(x) = e^{\ln u} = u$$

$$\mu(x) = 3-x^3$$

$$y(x) = \frac{1}{3-x^3}\left(\int (3-x^3)x^2dx + c\right)$$

$$y(x) = \frac{1}{3-x^3}\left(\int 3x^2dx - \int x^5dx + c\right)$$

$$y(x) = \frac{1}{3-x^3}\left(\frac{3x^3}{3} - \frac{x^6}{6} + c\right)$$

$$y(x) = \frac{1}{3-x^3}\left(x^3 - \frac{x^6}{6} + c\right)$$

④ Ecuación diferencial exacta.

$$(2xe^y + 5x^2) + (x^2e^y + 1)y' = 0$$

$$(2xe^y + 5x^2)dx + (x^2e^y + 1)dy = 0$$

$$\frac{\partial M}{\partial y} = 2xe^y \qquad \frac{\partial N}{\partial x} = 2xe^y$$

$$F(x,y) = \int (2xe^y + 5x^2)dx$$

$$F(x,y) = \int 2xe^y\,dx + \int 5x^2\,dx$$
$$= e^y \int 2x\,dx + 5\int x^2\,dx$$
$$= 2e^y \int x\,dx + 5\frac{x^3}{3} + g(y)$$

$$F(x,y) = 2e^y\frac{x^2}{2} + \frac{5}{3}x^3 + g(y)$$

$$F(x,y) = x^2e^y + \frac{5}{3}x^3 + g(y)$$

$$\frac{\partial F}{\partial y} = x^2e^y + g'(y)$$

$$\cancel{x^2e^y} + 1 = \cancel{x^2e^y} + g'(y)$$

$$\int g'(y) = \int 1\,dy$$

$$g'(y) = y$$

$$K = x^2e^y + \frac{5}{3}x^3 + y$$

5 Ecuaciones homogéneas

$$-(x^3 - y^3)\,dx + 3xy^2\,dy = 0$$
$$3xy^2\,dy = (x^3 - y^3)\,dx$$
$$3xy^2\frac{dy}{dx} = x^3 - y^3$$

$$\frac{dy}{dx} = \frac{x^3 - y^3}{3xy^2}$$

Si es homogenea

$$y = ux$$
$$\frac{dy}{dx} = x\frac{du}{dx} + u$$

$$x\frac{du}{dx} + u = \frac{x^3 - (ux)^3}{3x(ux)^2}$$

$$x\frac{du}{dx} + u = \frac{x^3 - u^3x^3}{3u^2x^2x}$$

$$x\frac{du}{dx} + u = \frac{\cancel{x^3}(1 - u^3)}{3u^2\cancel{x^3}}$$

$$x\frac{du}{dx} = \frac{1 - u^3}{3u^2} - u$$

$$x\frac{du}{dx} = \frac{1-u^3-3u^3}{3u^2}$$

$$x\frac{du}{dx} = \frac{1-4u^3}{3u^2}$$

$$\int \frac{3u^2}{1-4u^3}\,du = \int \frac{dx}{x}$$

$$w = 1-4u^3$$

$$dw = -12u^2$$

$$-\frac{dw}{4} = 3u^2\,du$$

$$\int \frac{-\frac{dw}{4}}{w} = \ln x + c$$

$$-\frac{1}{4}\int \frac{dw}{w} = \ln x + c$$

$$-\frac{1}{4}\ln w = \ln x + c$$

$$-\frac{1}{4}\ln(1-4u^3) = \ln x + c$$

$$-\frac{1}{4}\ln\left(1-4\left(\frac{y}{x}\right)^3\right) = \ln x + c$$

$$-\frac{1}{4}\ln\left[1-\frac{4y^3}{x^3}\right] = \ln x + c$$

Resolución Prueba.

① Solución implícita y explícita

$$\left(1-\frac{3}{y}+x\right)\frac{dy}{dx}+y=\frac{3}{x}-1$$

$$xy-3\ln x+x+y-3\ln y=C$$

$$y+x\frac{dy}{dx}-\frac{3}{x}+1+\frac{dy}{dx}-\frac{3}{y}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}+x\frac{dy}{dx}-\frac{3}{y}\frac{dy}{dx}=\frac{3}{x}-1-y$$

$$\frac{dy}{dx}\left(1+x-\frac{3}{y}\right)=\frac{3}{x}-1-y$$

$$\frac{dy}{dx}=\frac{\frac{3}{x}-1-y}{1+x-\frac{3}{y}}$$

$$\left(1\cancel{-\frac{3}{y}+x}\right)\left(\frac{\frac{3}{x}-1-y}{1\cancel{-\frac{3}{y}+x}}\right)+y=\frac{3}{x}-1$$

$$\frac{3}{x}-1\cancel{-y}\cancel{+y}=\frac{3}{x}-1$$

$$\frac{3}{x}-1=\frac{3}{x}-1$$

② Variables separables

$$\frac{dy}{dx}=\frac{2xy-4y+x-2}{x^2}$$

$$2xy-4y+x-2$$

$$(2xy-4y)+(x-2)$$

$$2y(x-2)+(x-2)$$

$$(x-2)(2y+1)$$

$$\frac{dy}{dx}=\frac{(x-2)(2y+1)}{x^2}$$

$$\frac{dy}{2y+1}=\frac{x-2}{x^2}dx$$

$$u=2y+1$$

$$du=2dy$$

$$\frac{du}{2}=dy$$

$$\int \frac{\frac{du}{2}}{u} = \int \frac{x}{x^4}dx - 2\int \frac{1}{x^4}dx$$

$$\frac{1}{2}\int \frac{du}{u} = \int \frac{1}{x^3}dx - 2\int x^{-4}dx$$

$$\frac{1}{2}\ln u = \frac{x^{-2}}{-2} - 2\frac{x^{-3}}{-3} + c$$

$$\frac{1}{2}\ln(2y+1) = -\frac{1}{2x^2} + \frac{2}{x^3} + c$$

③ Ecuación diferencial lineal

$$y' - \frac{4x^3}{4-x^4}y = x$$

$$u(x) = e^{-\int + \frac{4x^3}{4-x^4}dx}$$

$u = 4-x^4$

$du = -4x^3dx$

$-du = 4x^3dx$

$$u(x) = e^{-\int -\frac{du}{u}}$$

$$u(x) = e^{\int \frac{du}{u}}$$

$$u(x) = e^{\ln u} = u$$

$$u(x) = 4-x^4$$

$$y(x) = \frac{1}{4-x^4}\left(\int (4-x^4)x\,dx + c\right)$$

$$y(x) = \frac{1}{4-x^4}\left(\int 4x\,dx - \int x^5dx + c\right)$$

$$y(x) = \frac{1}{4-x^4}\left(4\frac{x^2}{2} - \frac{x^6}{6} + c\right)$$

$$y(x) = \frac{1}{4-x^4}\left(2x^2 - \frac{x^6}{6} + c\right)$$

④ Ecuación diferencial exacta

$$(3x^2 \text{Sen}y + x + y) + (x^3 \text{Cos}y - y^2 + x)y' = 0$$

$$(3x^2 \text{Sen}y + x + y)\,dx + (x^3 \text{Cos}y - y^2 + x)\,dy = 0$$

$$\frac{\partial M}{\partial y} = 3x^2 \text{Cos}y + 1 \qquad \frac{\partial N}{\partial x} = 3x^2 \text{Cos}y + 1$$

$$F(x,y) = \int (3x^2 \text{Sen}y + x + y)\,dx$$

$$\int 3x^2 \text{Sen}y\,dx + \int x\,dx + \int y\,dx$$

$$F(x,y) = 3\,\text{Sen}y \int x^2\,dx + \frac{x^2}{2} + y\int dx$$

$$F(x,y) = 3\,\text{Sen}y\,\frac{x^3}{3} + \frac{x^2}{2} + xy + g(y)$$

$$\frac{\partial F}{\partial y} = x^3 \text{Cos}y + x + g'(y)$$

$$x^3 \cancel{\text{Cos}y} - y^2 + \cancel{x} = x^3 \cancel{\text{Cos}y} + \cancel{x} + g'(y)$$

$$\int g'(y) = -\int y^2\,dy$$

$$g(y) = -\frac{y^3}{3}$$

$$k = x^3 \text{Sen}y + \frac{x^2}{2} + xy - \frac{y^3}{3}$$

⑤ Ecuaciones homogéneas.

$$-(x^4 - y^4)\,dx + 4xy^3\,dy = 0$$

Si es homogenea

$$4xy^3\,dy = (x^4 - y^4)\,dx$$

$$y = ux$$

$$\frac{dy}{dx} = \frac{x^4 - y^4}{4xy^3}$$

$$\frac{dy}{dx} = x\frac{du}{dx} + u$$

$$x\cdot\frac{du}{dx} + u = \frac{x^4 - (ux)^4}{4x(ux)^3}$$

$$x\frac{du}{dx} + u = \frac{x^4 - u^4x^4}{4x\,u^3x^3}$$

$$x\frac{du}{dx}+u=\frac{\cancel{x^4}(1-u^4)}{4u^3\cancel{x^4}}$$

$$x\frac{du}{dx}=\frac{1-u^4}{4u^3}-u$$

$$x\frac{du}{dx}=\frac{1-u^4-4u^4}{4u^3}$$

$$x\frac{du}{dx}=\frac{1-5u^4}{4u^3}$$

$$\int\frac{4u^3}{1-5u^4}\,du=\int\frac{dx}{x}$$

$$w=1-5u^4$$

$$dw=-20u^3\,du$$

$$\frac{dw}{-5}=4u^3\,du$$

$$\int\frac{-\frac{dw}{5}}{w}=\ln x+c$$

$$-\frac{1}{5}\int\frac{dw}{w}=\ln x+c$$

$$-\frac{1}{5}\ln w=\ln x+c$$

$$-\frac{1}{5}\ln(1-5u^4)=\ln x+c$$

$$-\frac{1}{5}\ln\left(1-5\left(\frac{y}{x}\right)^4\right]=\ln x+c$$

$$-\frac{1}{5}\ln\left[1-5\frac{y^4}{x^4}\right]=\ln x+c$$

Escaneado con CamScanner