PROPAGACIÓN DE ERRORES

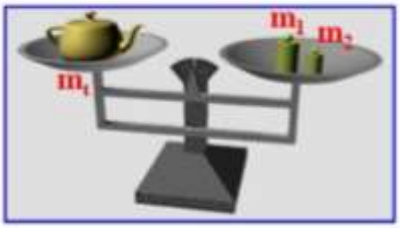
A Partir de datos obtener un resultado, no son exactos por errores asociados

Introducción

$$m_1 = 500 \pm 2\ g$$
$$m_2 = 200 \pm 1\ g$$
$$\Rightarrow m_t = m_1 + m_2 = 700 \pm ?\ g$$

$$m = 0'200 \pm 0'001\ kg$$
$$k = 50'0 \pm 0'1\ N/m$$
$$A = 0'080 \pm 0'001\ m$$
$$x = 0'052 \pm 0'001\ m$$
$$\Rightarrow v = \sqrt{\frac{k}{m}(A^2 - x^2)} = 0'96125 \pm ?\ m/s$$

ERROR DE UNA SUMA

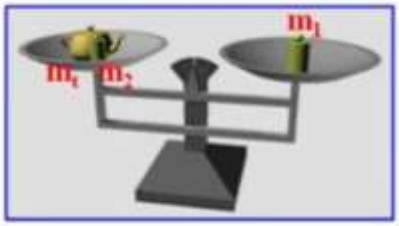


$$m_1 = 500 \pm 2\ g$$
$$m_2 = 200 \pm 1\ g$$
$$m_t = m_1 + m_2$$

$$\left.\begin{array}{l} m_t = (500+2) + (200+1) = 703\ g \\ m_t = (500-2) + (200-1) = 697\ g \end{array}\right\} \Rightarrow m_t = 700 \pm 3\ g$$

$$\varepsilon(m_t) = \varepsilon(m_1) + \varepsilon(m_2)$$

ERROR DE UNA RESTA



$$m_1 = 500 \pm 2\ g$$
$$m_2 = 200 \pm 1\ g$$
$$m_t = m_2 - m_1$$

$$\left.\begin{array}{l} m_t = (500+2) - (200-1) = 303\ g \\ m_t = (500-2) - (200+1) = 297\ g \end{array}\right\} \Rightarrow m_t = 300 \pm 3\ g$$

$$\varepsilon(m_t) = \varepsilon(m_1) + \varepsilon(m_2)$$

ERROR DE UN PRODUCTO

$a = 0'70 \pm 0'01\ m$

$b = 1'00 \pm 0'01\ m$

$S = ab$

$$\left.\begin{array}{l} S = (0'70+0'01)(1'00+0'01) = 0'7171\ m^2 \\ S = (0'70-0'01)(1'00-0'01) = 0'6831\ m^2 \end{array}\right\} \Rightarrow S = 0'700 \pm 0'017\ m^2$$

$$\varepsilon(S) \neq \varepsilon(a)\varepsilon(b)$$

ERROR DE UN MONOMIO

$$M = kA^aB^bC^c$$

$$\ln M = \ln k + a \ln A + b \ln B + c \ln C$$

$$\frac{dM}{M} = a\frac{dA}{A} + b\frac{dB}{B} + c\frac{dC}{C}$$

$$\frac{\varepsilon(M)}{|M|} = |a|\frac{\varepsilon(A)}{|A|} + |b|\frac{\varepsilon(B)}{|B|} + |c|\frac{\varepsilon(C)}{|C|}$$

$$\varepsilon_r(M) = |a|\varepsilon_r(A) + |b|\varepsilon_r(B) + |c|\varepsilon_r(C)$$

ERROR DE UN PRODUCTO

$a = 0'70 \pm 0'01\ m$

$b = 1'00 \pm 0'01\ m$

$S = ab$

$$S=a^1b^1 \Rightarrow \varepsilon_r(S) = |1|\varepsilon_r(a) + |1|\varepsilon_r(b)$$

$$\boxed{\varepsilon_r(S) = \varepsilon_r(a) + \varepsilon_r(b)}$$

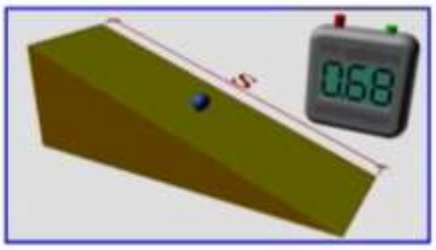
$$\frac{\varepsilon(S)}{0'70 \cdot 1'00} = \frac{0'01}{0'70} + \frac{0'01}{1'00}$$

$$\varepsilon(S) = 0'017 \text{ m}^2$$

$$S = 0'700 \pm 0'017 \text{ m}^2$$

$0'71 \cdot 1'01 = 0'7171$
$0'69 \cdot 0'99 = 0'6831$

ERROR DE UN COCIENTE



$$s = 1'40 \pm 0'01 \text{ m}$$

$$t = 0'95 \pm 0'01 \text{ s}$$

$$v_m = \frac{s}{t}$$

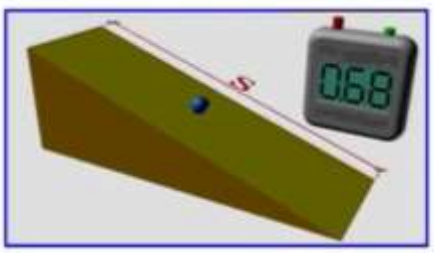
$$\ln v_m = \ln s - \ln t \Rightarrow \frac{dv_m}{v_m} = \frac{ds}{s} - \frac{dt}{t} \Rightarrow \frac{\varepsilon(v_m)}{|v_m|} = \frac{\varepsilon(s)}{|s|} + \frac{\varepsilon(t)}{|t|}$$

$$\boxed{\varepsilon_r(v_m) = \varepsilon_r(s) + \varepsilon_r(t)}$$

$$v_m = s^1 t^{-1} \Rightarrow \varepsilon_r(v_m) = |1|\varepsilon_r(s) + |-1|\varepsilon_r(t)$$

$$\boxed{\varepsilon_r(v_m) = \varepsilon_r(s) + \varepsilon_r(t)}$$

ERROR DE UN COCIENTE



$$s = 1'40 \pm 0'01 \text{ m}$$

$$t = 0'95 \pm 0'01 \text{ s}$$

$$v_m = \frac{s}{t}$$

$$\frac{\varepsilon(v_m)}{1'40/0'95}=\frac{0'01}{1'40}+\frac{0'01}{0'95}$$

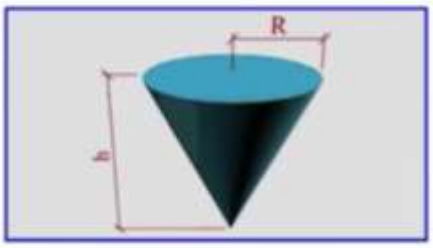
$\varepsilon(v_m)$ =0'026 m/s

$v_m = 1'474 \pm 0'026$ m/s

$v_m = 1'47 \pm 0'03$ m/s

1'41 / 0'94 = 1'500
1'39 / 0'96 = 1'448

ERROR DE UN MONOMIO



$R = 2'50 \pm 0'01$ m

$h = 5'00 \pm 0'01$ m

$\pi = 3'1416 \pm 0'0001$

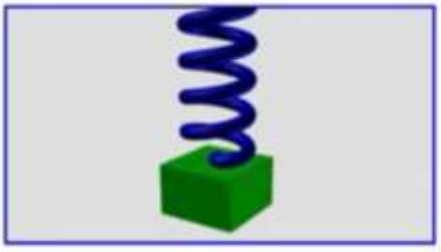
$V=\frac{1}{3}\pi R^2 h$

$$\ln V=\ln\frac{1}{3}+\ln\pi+2\ln R+\ln h \Rightarrow \frac{dV}{V}=\frac{d\pi}{\pi}+2\frac{dR}{R}+\frac{dh}{h}\Rightarrow$$

$$\Rightarrow\frac{\varepsilon(V)}{|V|}=\frac{\varepsilon(\pi)}{|\pi|}+2\frac{\varepsilon(R)}{|R|}+\frac{\varepsilon(h)}{|h|}$$

$$\varepsilon_r(V)=\varepsilon_r(\pi)+2\varepsilon_r(R)+\varepsilon_r(h)$$

ERROR DE UNA COMBINACIÓN



$$v=\sqrt{\frac{k}{m}(A^2-x^2)}=$$
$$=k^{1/2}m^{-1/2}(A^2-x^2)^{1/2}$$

$$\varepsilon_r(A^2)=2\,\varepsilon_r(A)$$

$$\varepsilon_r(x^2)=2\,\varepsilon_r(x)$$

$$\varepsilon(A^2-x^2)=\varepsilon(A^2)+\varepsilon(x^2)$$

$$\varepsilon_r(v)=\frac{1}{2}\varepsilon_r(k)+\frac{1}{2}\varepsilon_r(m)+\frac{1}{2}\varepsilon_r(A^2-x^2)$$

ERROR PARA CUALQUIER EXPRESIÓN

DE UNA VARIABLE

$$f(x+\Delta x) - f(x) \approx f'(x)\Delta x$$

$$\varepsilon(f(x)) = |f'(x)|\varepsilon(x)$$

DE VARIAS VARIABLES

$$f(x+\Delta x, y+\Delta y, \ldots) - f(x,y,\ldots) \approx \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \ldots$$

$$\varepsilon(f(x,y,\ldots)) = \left|\frac{\partial f}{\partial x}\right|\varepsilon(x) + \left|\frac{\partial f}{\partial y}\right|\varepsilon(y) + \ldots$$

APLICACIÓN A UNA SUMA O RESTA

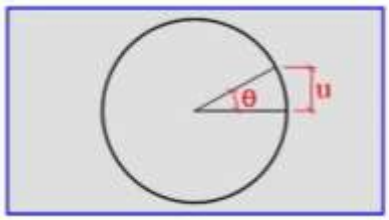
$$\varepsilon(a \pm b) = \left|\frac{\partial(a \pm b)}{\partial a}\right|\varepsilon(a) + \left|\frac{\partial(a \pm b)}{\partial b}\right|\varepsilon(b) =$$
$$= |1|\varepsilon(a) + |\pm 1|\varepsilon(b) = \varepsilon(a) + \varepsilon(b)$$

APLICACIÓN A UN MONOMIO

$$\varepsilon(kA^aB^b) = \left|\frac{\partial(kA^aB^b)}{\partial A}\right|\varepsilon(A) + \left|\frac{\partial(kA^aB^b)}{\partial B}\right|\varepsilon(B) =$$
$$= |akA^{a-1}B^b|\varepsilon(A) + |bkA^aB^{b-1}|\varepsilon(B)$$

$$\frac{\varepsilon(kA^aB^b)}{|kA^aB^b|} = |a|\frac{\varepsilon(A)}{|A|} + |b|\frac{\varepsilon(B)}{|B|} \Rightarrow \varepsilon_r(kA^aB^b) = |a|\varepsilon_r(A) + |b|\varepsilon_r(B)$$

ERROR PARA CUALQUIER EXPRESIÓN



$\theta = 30 \pm 1°$

$u = \text{sen}\,\theta$

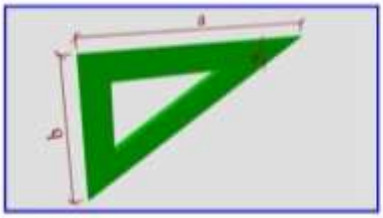
$$\varepsilon(u) = |u'(\theta)|\varepsilon(\theta)$$

$$\varepsilon(u) = |\cos\theta|\varepsilon(\theta)$$

$$\varepsilon(u) = \left|\cos\frac{\pi}{6}\right| \cdot \frac{\pi}{180} = 0'015$$

$u = 0'500 \pm 0'015$

sen 31° = 0'515
sen 29° = 0'485



$\theta = 30 \pm 1°$

$a = 30'0 \pm 0'1$ cm

$b = a\,\mathrm{tg}\,\theta$

$$\varepsilon(b) = \left|\frac{\partial b}{\partial a}\right|\varepsilon(a) + \left|\frac{\partial b}{\partial \theta}\right|\varepsilon(\theta)$$

$$\varepsilon(b) = \mathrm{tg}\,\theta\,\varepsilon(a) + \frac{a}{\cos^2\theta}\varepsilon(\theta)$$

$$\varepsilon(b) = \mathrm{tg}\frac{\pi}{6}\cdot 0'1 + \frac{30'0}{\cos^2\frac{\pi}{6}}\cdot\frac{\pi}{180} = 0'76\,\mathrm{cm}$$

$b = 17'32 \pm 0'76$ cm

30'1 tg 31° = 18'09
29'9 tg 29° = 16'57

$b = 17'3 \pm 0'8$ cm

RESUMEN

Error de suma o resta	$\varepsilon(a \pm b) = \varepsilon(a) + \varepsilon(b)$
Error de monomio	$\varepsilon_r(kA^aB^b) = \lvert a\rvert\varepsilon_r(A) + \lvert b\rvert\varepsilon_r(B)$
Error de expresión de una variable	$\varepsilon(f(x)) = \lvert f'(x)\rvert\varepsilon(x)$
Error de expresión de varias variables	$\varepsilon(f(x,y)) = \left\lvert\frac{\partial f}{\partial x}\right\rvert\varepsilon(x) + \left\lvert\frac{\partial f}{\partial y}\right\rvert\varepsilon(y)$