

Ejercicios Propuestos

1. Demostrar que son solución de la ecuación diferencial

$$\bullet \quad x = \frac{cy}{\sqrt{1+y^2}} \quad \text{de} \quad x \frac{dy}{dx} - y = y^3$$

$$\bullet \quad 2\sqrt{1+x^3} = 3 \ln(y+1) + c \quad \text{de} \quad \sqrt{1+x^3} \frac{dy}{dx} = x^2 y + x^2$$

$$\bullet \quad y = \frac{a+cx}{1+ax} \quad \text{de} \quad y - x \frac{dy}{dx} = a(1+x^2 y)$$

$$\bullet \quad c + \frac{e^y}{y} = \ln(\ln x) \quad \text{de} \quad (1-y) e^{\frac{y}{x}} \frac{dy}{dx} + \left(\ln x^{-1} \right) \left(\frac{y^2}{x} \right) = 0$$

$$\bullet \quad f(y) = c(1-e^x)^3 \quad \text{de} \quad 3e^x f(y) dx + (1-e^x) \sec^2 y dy = 0$$

2. Hallar la ecuación diferencial correspondiente a

la solución:

$$\bullet \quad (x-a)^2 + (y-b)^2 = r^2$$

$$\bullet \quad y^2 = \frac{x^3}{a-x}$$

$$\bullet \quad y = c_1 x + c_2 e^{-3x}$$

$$\bullet \quad y = A e^{2x} + B x e^{2x}$$

$$\bullet \quad x = A \sin(\omega t + \beta)$$

3. Resolver las ecuaciones diferenciales

$$\bullet \quad (x-y^2 x) dx + (y-x^2 y) dy = 0$$

$$\bullet \quad x y' = \sqrt{1-y^2}$$

$$\bullet \quad x y + y^2 \frac{dy}{dx} = 6x$$

$$\bullet \quad \frac{dy}{dx} = \frac{1+\cos x}{\sin^2 y}$$

$$\bullet \quad y' x \ln x = 0$$

$$\text{Resp. } (x^2-1)(y^2-1) = k$$

$$\text{Resp. } y = \sin(\ln|x| + c)$$

$$\text{Resp. } x^2 y^2 + 12y + 72 \ln|6-y| = c$$

$$\text{Resp. } 2y - \sin^2 y - x - \sin x = c$$

$$\text{Resp. } y = \ln \ln x + c$$

4. Ecuación diferencial con valor inicial.

• $\frac{dy}{dx} = \frac{x}{y} - \frac{x}{1+y}$; $y(0) = 1$ Sol. $3y^2 + 2y^3 = 3x$

Sol. $y = e^{\frac{x}{2}}$

• $\sin x \frac{dy}{dx} = y \ln y$; $y(\frac{\pi}{2}) = e$

Sol. $y = [2(1-x)e^{-1}]^{\frac{1}{2}}$

• $x dx + y e^{-x} dy = 0$; $y(0) = 1$

Sol. $y = -2 \cos^3 2x$

• $y' + 6y + 92x = 0$; $y(0) = 2$

Sol. $y = x$

• $y^2 y' - x^2$; $y(-2) = -2$

5. Ecuaciones diferenciales a variables separables.

• $\frac{dy}{dx} = \frac{1}{y} \ln(x+y)$

Sol. $x - y - \ln | \sin(x+y) + \cos(x+y) | = C$

• $(y-x)^2 dx - (x+x^2y) dy = 0$

Sol. $\ln(\frac{x}{y}) - xy = C$

• $\frac{dy}{dx} = \frac{e^y}{2y - x e^y}$

Sol. $y^2 = x e^y + C$

• $(x^2 y^2 + 1) dx + 2x^2 dy = 0$

Sol. $\frac{1}{1-xy} + \frac{1}{2} \ln x = C$

• $y' = \frac{1}{2} \log^2(x+2y)$

Sol. $4y - 2x + \sin(2x+4y) = C$

6. Ecuaciones diferenciales homogéneas

• $x \frac{dy}{dx} = y + \sqrt{y^2 - x^2}$

Sol. $2cy = c^2 x^2 + 1$

• $x y' = 2(y - \sqrt{xy})$

Sol. $16xy = (y+4x-cx^2)^2$

• $(x \operatorname{cosec} \frac{y}{x} - y) dx + x dy = 0$

Sol. $\ln kx = \cos(\frac{y}{x})$

• $x y' = y + 2x e^{\frac{y}{x}}$

Sol. $\ln(x^2 k) = e^{\frac{y}{x}}$

• $x y' \ln \frac{y}{x} = x + y \ln \frac{y}{x}$

Sol. $\ln x = \frac{y}{x} (\ln(\frac{y}{x}) + 1) + C$

Ecuaciones diferenciales no homogéneas

- $3x + y - 2 + \frac{dy}{dx}(x-1) = 0$ Sol. $(x-1)(3x+2y-1) = k$
- $(x+2y-1)dx - (2x+y-5)dy = 0$ Sol. $(x-y-4)^3 = C(x+y-2)$
- $(1-x^2y)\frac{dy}{dx} + 2xy^2 = 0$ Sol. $x - 2x^2y = C y^2$
- $(x+y^3)dx + 6xy^2dy = 0$ Sol. $y^3 = x^2(C - \ln x)$
- $\frac{dy}{dx} = \frac{1}{2} \left(\frac{x+y-1}{x+2} \right)^2$ Sol. $2 \arctg \left(\frac{y-3}{x+2} \right) = \ln(x+2) + k$

8 Ecuación diferencial reducible homogéneas

- $\frac{dy}{dx} = \frac{2y-x}{2x-y}$ Sol. $|y-x| = C|y+x|^3$
- $3x + y - 2 + y'(x-1) = 0$ Sol. $(x-1)(3x+2y-1) = k$
- $(x+y^3) + 6xy^2y' = 0$ Sol. $y^3 = \frac{C}{x} \frac{x^2-1}{x}$
- $(2x+3y)dx + (y+2)dy = 0$ Sol. $(2x+y-4)^2 = k(y-x-1)$
- $(x+2y-1)dx - (2x+y-5)dy = 0$ Sol. $(x-y-4)^3 = C(x+y-2)$

9 Ecuaciones diferenciales exactas

- $(\frac{y}{x} + 6x)dx + (\ln x - 2)dy = 0$ Sol. $y \ln x + 3x^2 - 2y = k$
- $(2x + \frac{x^3+y^3}{x^2y})dx = (\frac{x^2+y^2}{x^2y^2})dy$ Sol. $x^3y + y^3y^2 = Cxy$
- $(\frac{\sin 2x}{y} + x)dx + (y - \frac{\sin^2 x}{y^2})dy = 0$ Sol. $\frac{\sin^2 x}{y} + \frac{x^2+y^2}{2} = C$
- $4x^3 - e^{xy}(y+xy) = 0$ Sol. $x^4 - e^{xy} = C$
- $(\frac{y}{x} + \ln y)dx + (\frac{x}{y} + \ln x)dy = 0$ Sol. $y \ln x + x \ln y = C$