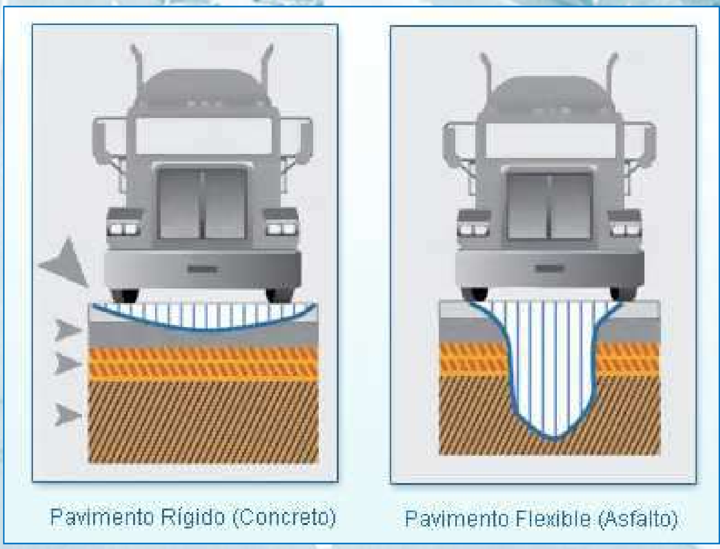
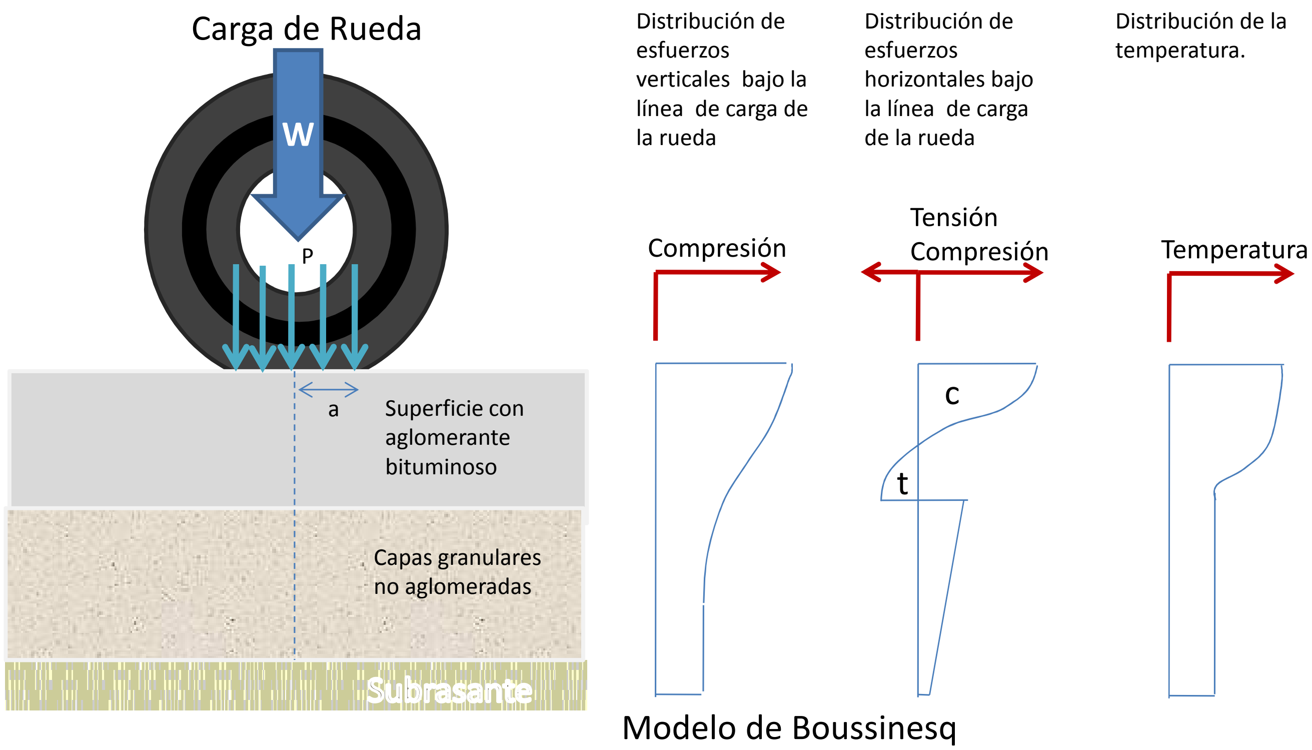
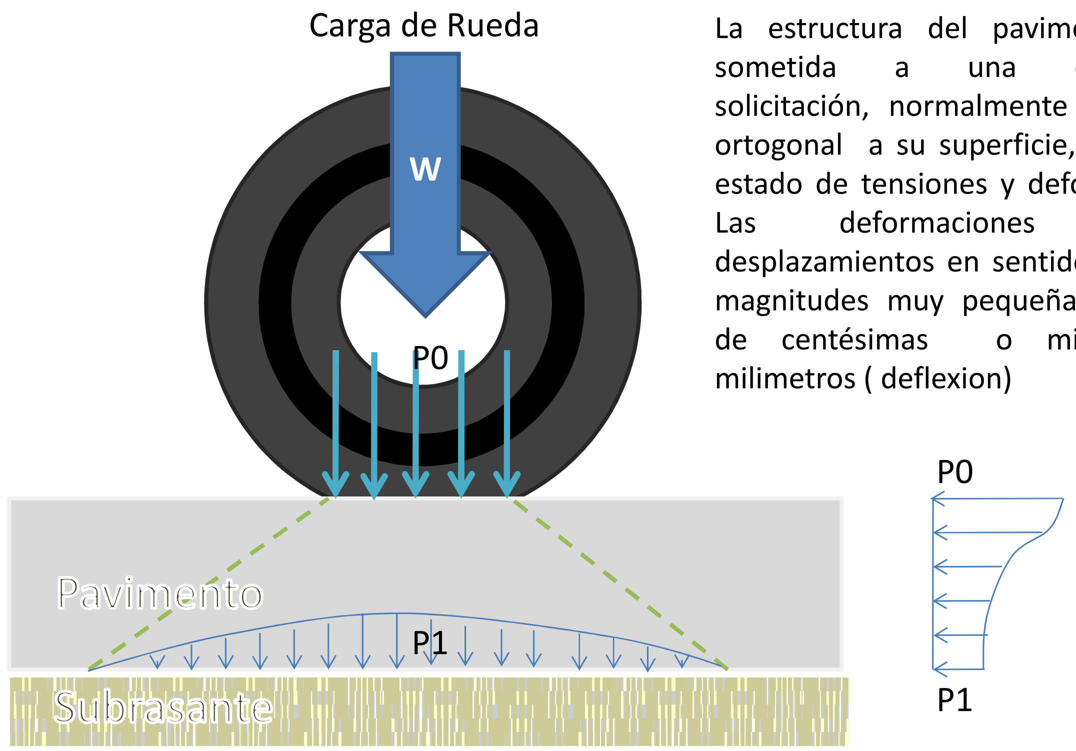
Esfuerzos y deformaciones en pavimentos flexibles



DISTRIBUCIÓN DE PRESIONES DE CARGA DE RUEDA SOBRE LA ESTRUCTURA DEL PAVIMENTO

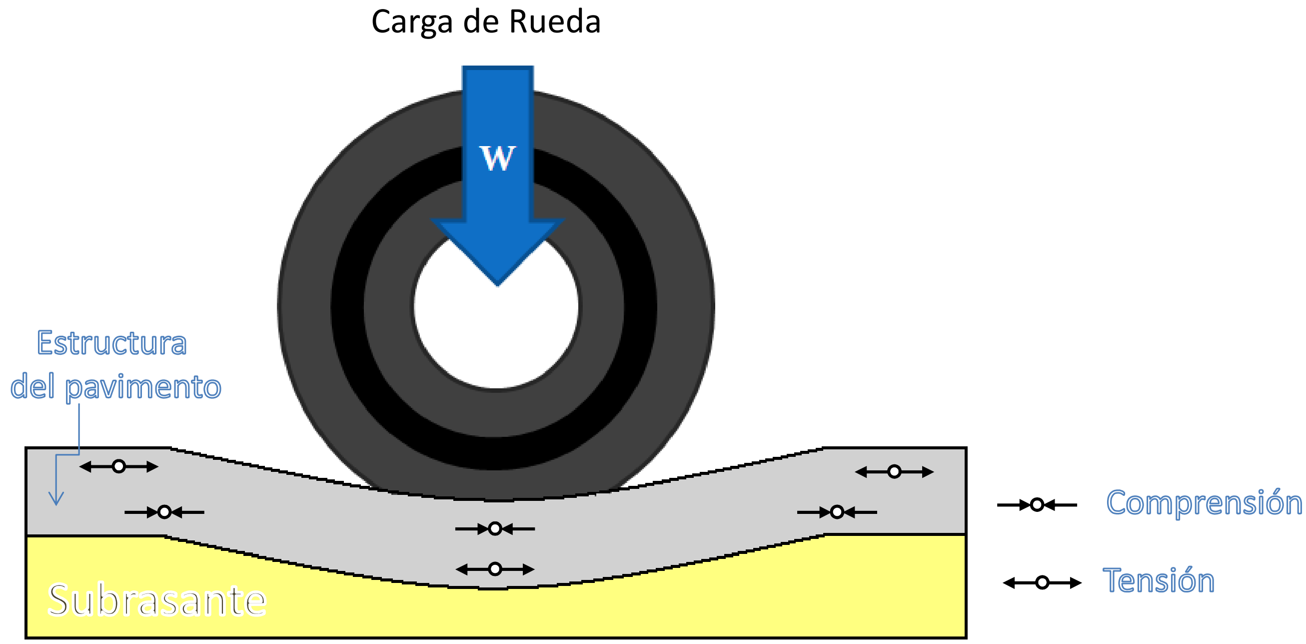


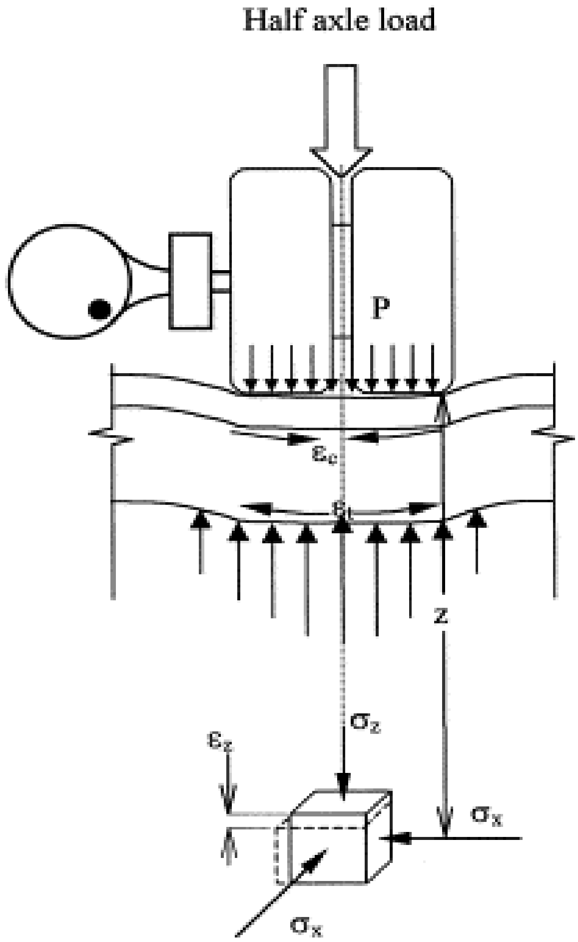
DISTRIBUCION DE PRESIONES DE CARGA DE RUEDA SOBRE LA ESTRUCTURA DEL PAVIMENTO

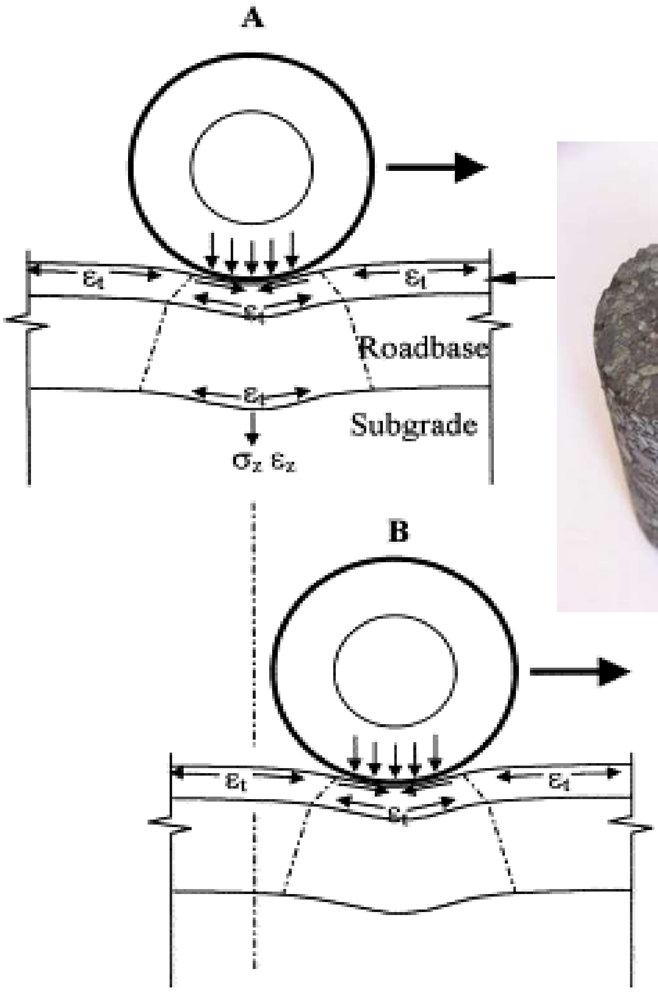


La estructura del pavimento al ser sometida a una determinada solicitación, normalmente una carga ortogonal a su superficie, produce un estado de tensiones y deformaciones . Las deformaciones producen desplazamientos en sentido vertical en magnitudes muy pequeñas del orden de centésimas o milésimas de milimetros (deflexion)

ESQUEMA DE LOS REFUERZOS DE TENSIÓN Y COMPRENSIÓN EN LA ESTRUCTURA DEL PAVIMENTO



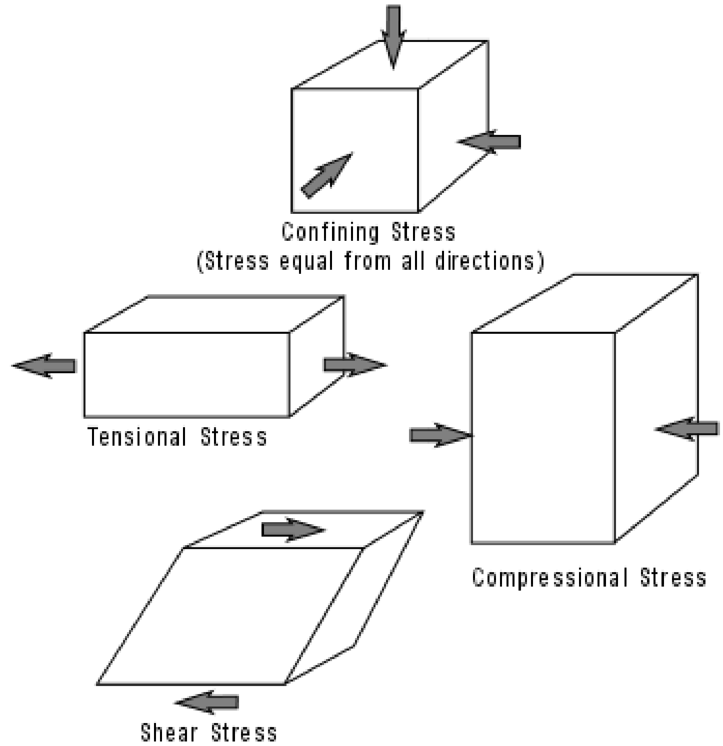




Esfuerzo (stress)

Fuerza por unidad de área

$$\sigma = \frac{\text{Carga}}{\text{Area}} = \frac{P}{A}$$



- Unidades: N/m2 o Pa, Mpa, psi, ksi
- Tipos: normal, cortante , axial

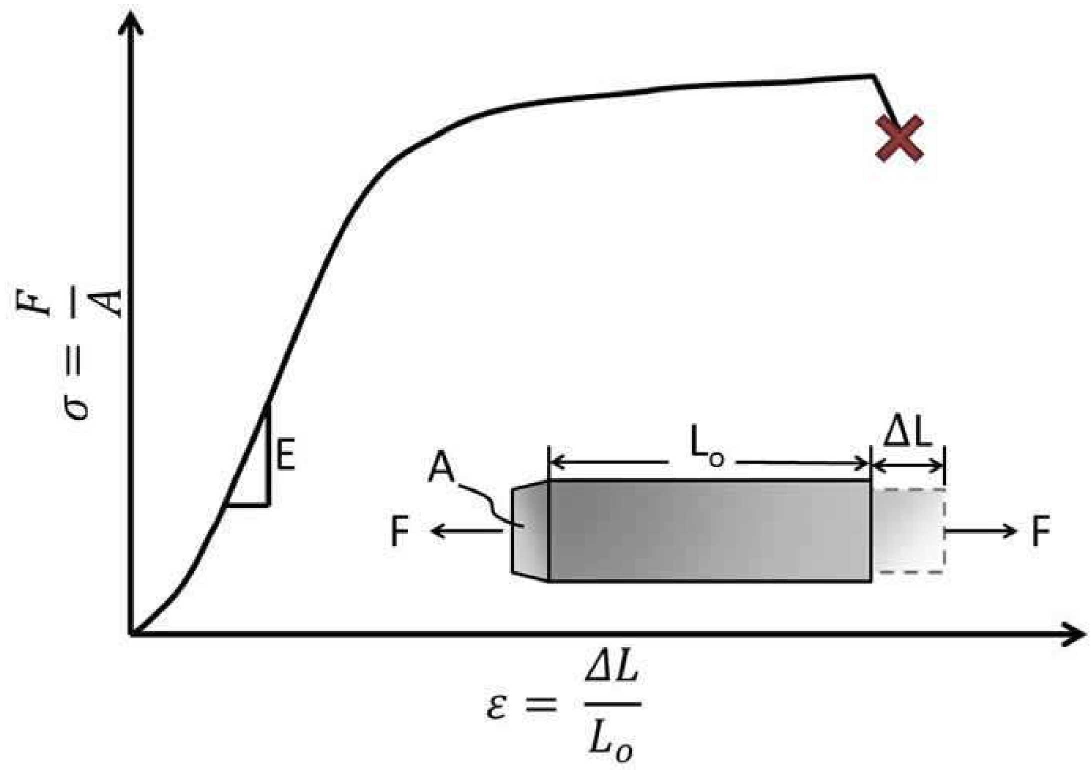
Deformación unitaria (strain)

$$\varepsilon = \frac{\text{Cambio en Longitud}}{\text{Longitud Original}} = \frac{\Delta L}{L}$$

Relación de la deformación causada por la carga y la longitud original del material.

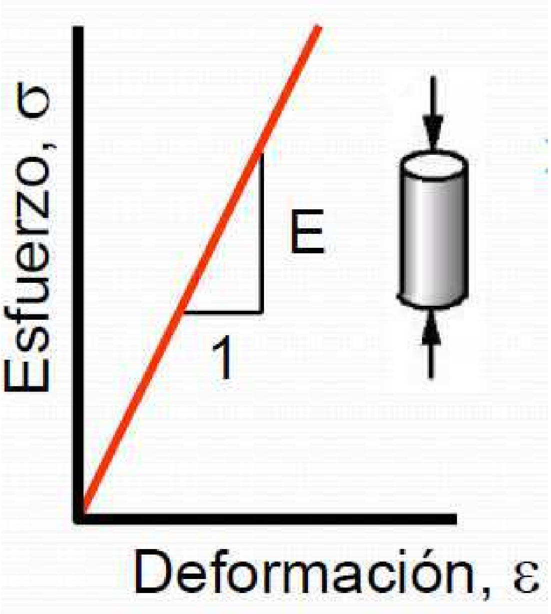
❖ Unidades: Adimensional

Deformación unitaria (strain)



Rigidez (stiffness)

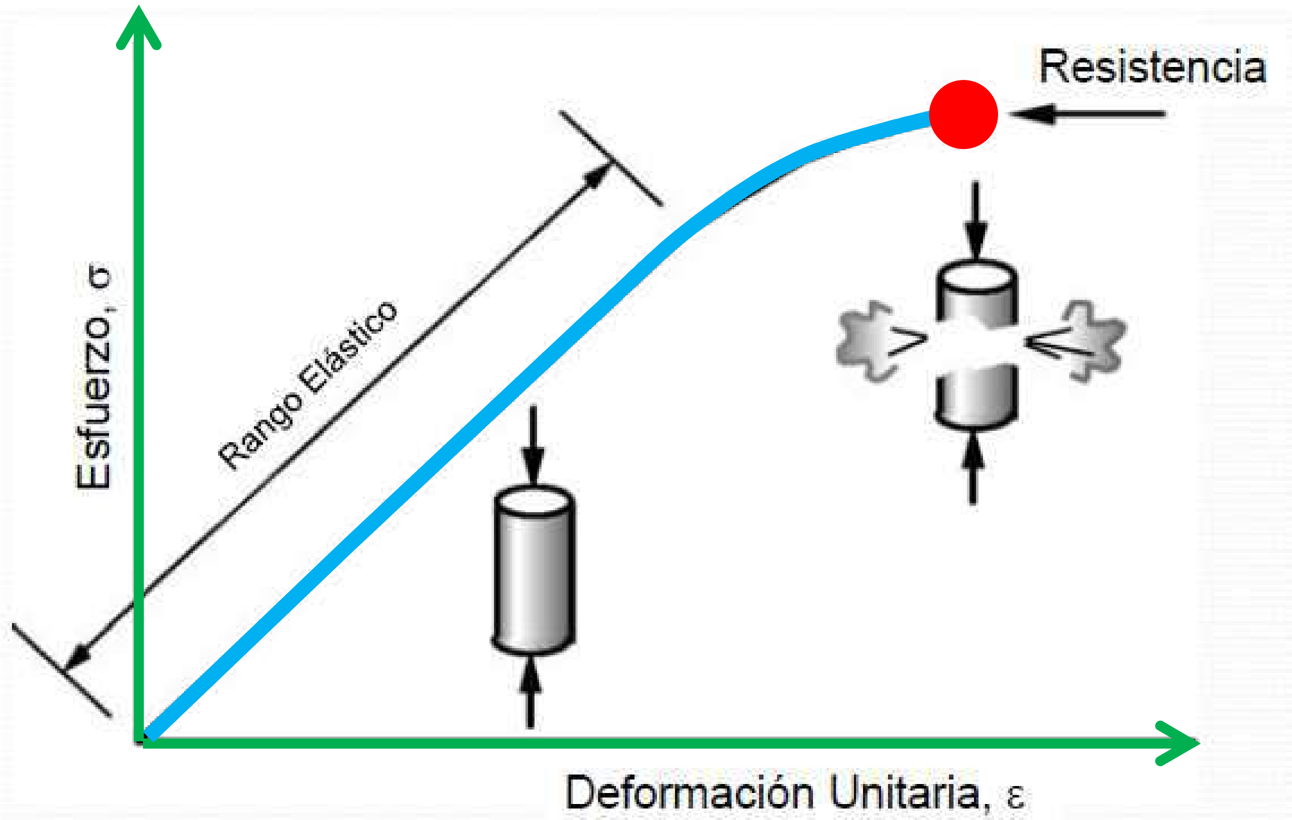
$$\text{Rigidez} = \text{esfuerzo/deformación} = \frac{\sigma}{\varepsilon}$$



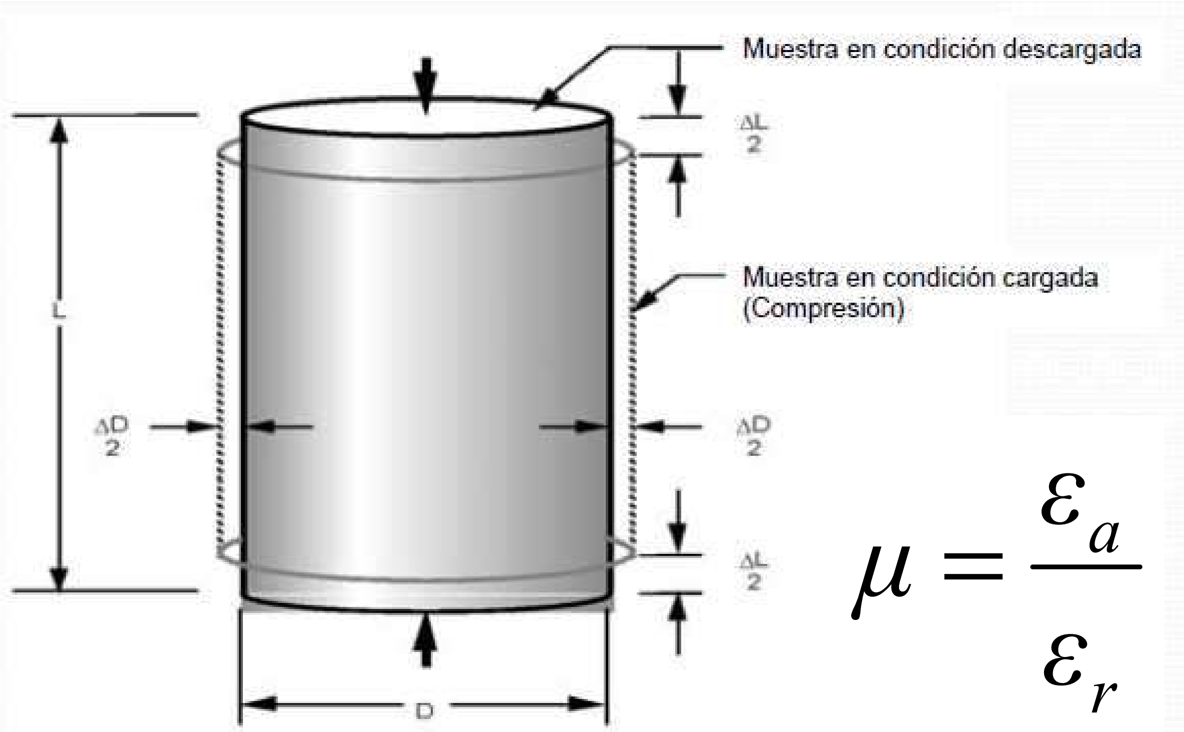
Para materiales elásticos :

- **Modulo de Elasticidad.**
- **Modulo Elástico.**
- **Módulo de Young**

Esfuerzo vs. Deformación de un material en compresión



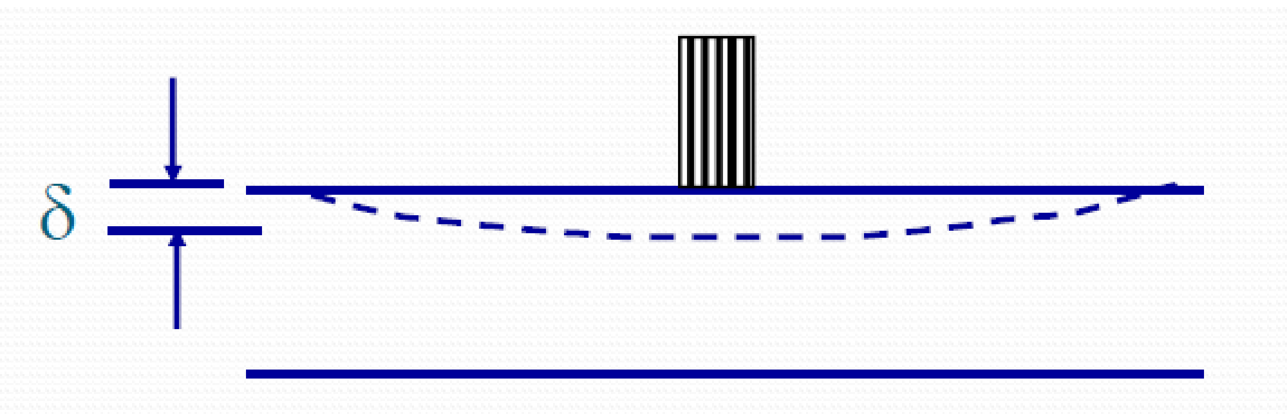
Relación de Poisson



$$\mu = \frac{\varepsilon_a}{\varepsilon_r}$$

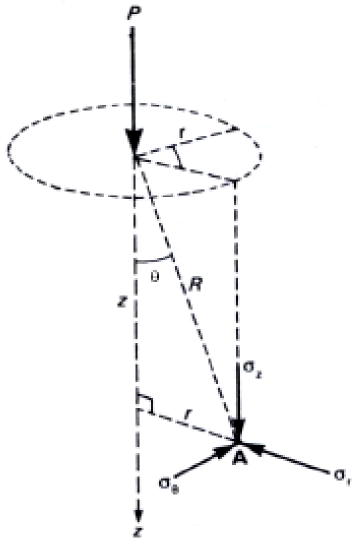
Deflexión (i)

- Cambio en longitud.
- Deformación.
- Unidades: mm, μm (0.001 mm).

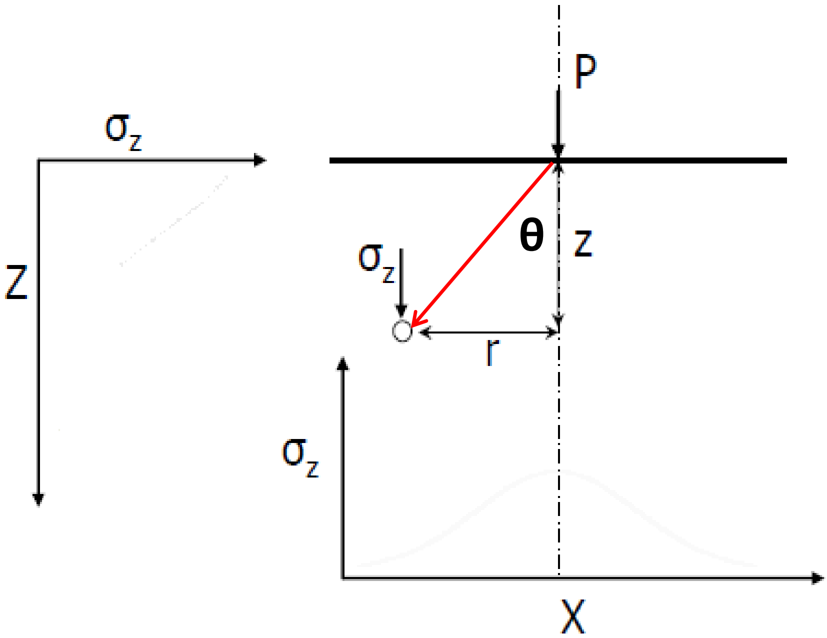


Problema de BOUSSINESQ (1885)

El francés Boussinesq, en 1885, consiguió resolver matemáticamente el problema de calcular las tensiones generadas por una carga puntual actuando normalmente sobre un semiespacio.



Problema de BOUSSINESQ (1885)



$$\Delta\sigma_z = \frac{3P}{2\pi z^2}\cos^5\theta$$

Donde:

$$\cos\theta = \frac{z}{\sqrt{r^2 + z^2}}$$

$$r = \sqrt{x^2 + y^2}$$

$$\Delta\sigma_z = \frac{3P}{2\pi z^2\left(1+\left(\frac{r}{z}\right)^2\right)^{5/2}}$$

Problema de BOUSSINESQ (1885)

- La más usada en la práctica es ** y puede ser escrita en términos de un factor de influencia Ip:

$$I_p = \frac{3}{2\pi}\left\{\frac{1}{1+(r/z)^2}\right\}^{5/2} \quad ====> \quad \sigma_z = \frac{P}{z^2} I_p$$

- Valores de Ip en términos de r y z están tabulados.

Exactamente debajo del punto de carga Q, Ip= 0.478

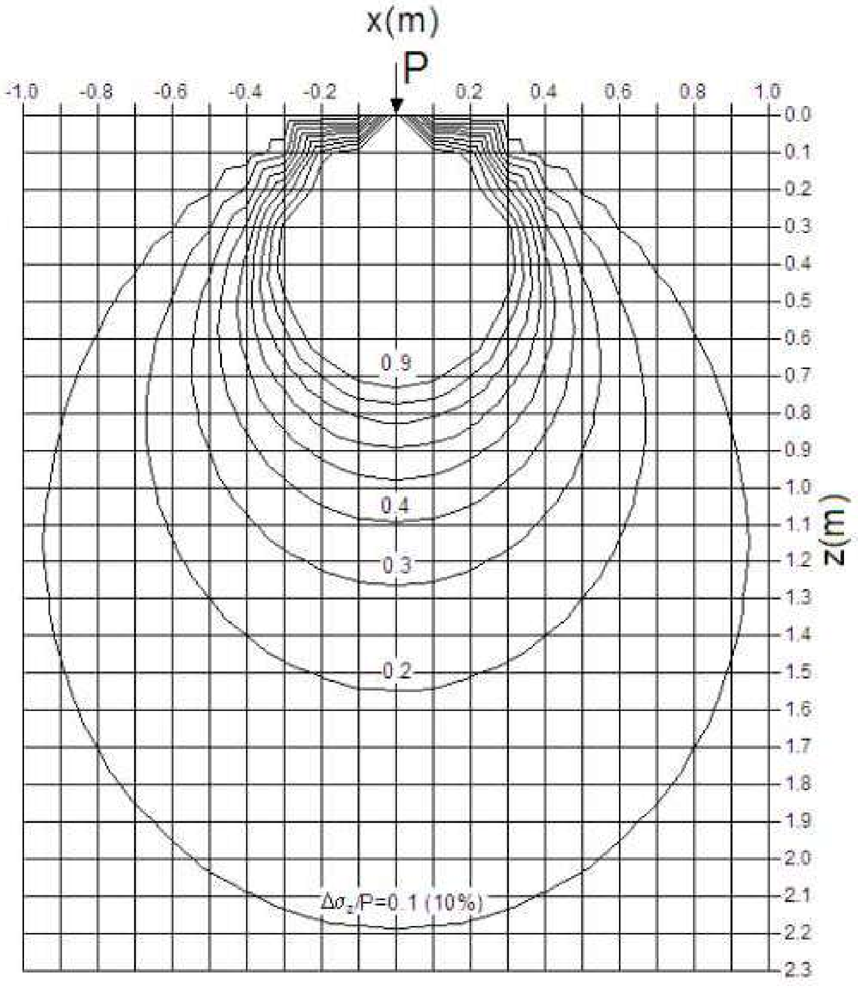


Figura 5.2 – Distribución de esfuerzos en el terreno debido a una carga puntual.

BULBO DE PRESIONES DE BOUSSINESQ

$$\sigma_z = \frac{3}{2\pi}\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}}\frac{P}{z^2}$$

Donde:

- σ_z = Vertical stress
- r = Radial distance from load
- z = Depth
- P = Point load

Problema de BOUSSINESQ (1885)

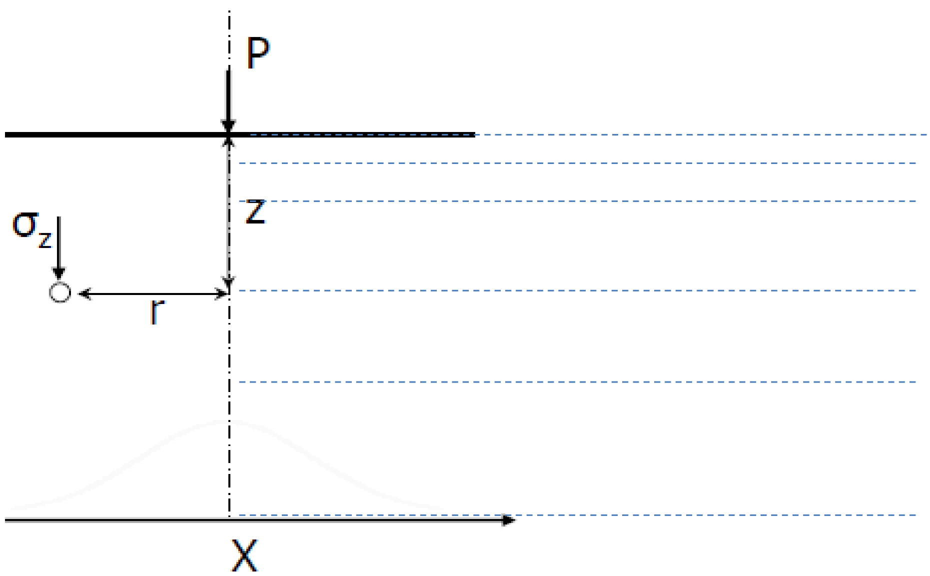
$$\sigma_z = \frac{P}{z^2} I_p$$

Variation of Iz for Various Values of r/z

r/z	Iz	r/z	Iz	r/z	Iz
0.000	0.4775	0.360	0.3521	1.800	0.0129
0.020	0.4770	0.380	0.3408	2.000	0.0085
0.040	0.4756	0.400	0.3295	2.200	0.0058
0.060	0.4732	0.450	0.3011	2.400	0.0040
0.080	0.4699	0.500	0.2733	2.600	0.0028
0.100	0.4657	0.550	0.2466	2.800	0.0021
0.120	0.4607	0.600	0.2214	3.000	0.0015
0.140	0.4549	0.650	0.1978	3.200	0.0011
0.160	0.4482	0.700	0.1762	3.400	0.00085
0.180	0.4409	0.750	0.1565	3.600	0.00066
0.200	0.4329	0.800	0.1386	3.800	0.00051
0.220	0.4243	0.850	0.1226	4.000	0.00040
0.240	0.4151	0.900	0.1083	4.200	0.00032
0.260	0.4054	0.950	0.0956	4.400	0.00026
0.280	0.3954	1.000	0.0844	4.600	0.00021
0.300	0.3849	1.200	0.0513	4.800	0.00017
0.320	0.3742	1.400	0.0317	5.000	0.00014
0.340	0.3632	1.600	0.0200		

Ejercicio 01

Calcular el aumento del esfuerzo vertical a profundidades z = 0.20m, 0.40m, 0.60m, 0.80m, 1.00m, 1.50m, 2.00m, 2.50, 3.00m, Asumir r = 1m. Considerar una carga puntual P=10kN.



Posterior a BOUSSINESQ

- Como puede verse, el modulo elástico no tiene influencia en ninguno de los esfuerzos, que por lo tanto son independientes de los parámetros elásticos.

- Las ecuaciones de Boussinesq fueron originalmente desarrolladas para una carga puntual estática.

- Posteriormente, las ecuaciones de Boussinesq fueron extendidas por otros investigadores para su aplicación con cargas uniformemente distribuidas (Newmark, 1947; Foster y Ahlvin, 1954; Sanborn y Yoder, 1967).

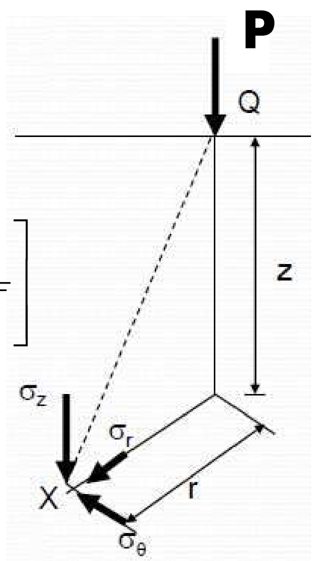
Taylor en 1963 adaptó la ecuación de Boussinesq para que tenga la siguiente forma:

$$\sigma_z = -\frac{P}{2\pi}\frac{3z^3}{\left(r^2+z^2\right)^{5/2}}$$

$$\sigma_r = -\frac{P}{2\pi}\left[\frac{3r^2 z}{\left(r^2+z^2\right)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$$

$$\sigma_\theta = \frac{P}{2\pi}(1-2\mu)\left[\frac{z}{\left(r^2+z^2\right)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$$

$$\tau_{zr} = \frac{P}{2\pi}\frac{3rz^2}{\left(r^2+z^2\right)^{5/2}}$$



Solución ELÁSTICA PARA UNA CAPA

De la Ley de Hooke

$$\varepsilon_z = \frac{\sigma_z - \nu(\sigma_r + \sigma_\theta)}{E}$$

$$\varepsilon_r = \frac{\sigma_r - \nu(\sigma_z + \sigma_\theta)}{E}$$

$$\varepsilon_\theta = \frac{\sigma_\theta - \nu(\sigma_r + \sigma_z)}{E}$$

$$\gamma_{zr} = \frac{2\tau_{zr}(1+\mu)}{E} = \frac{\tau_{zr}}{G}$$

Donde

G = Modulo de Corte

Estas cuatro ecuaciones se pueden reescribir de manera matricial

EC. HOOKE

Solución ELÁSTICA PARA UNA CAPA

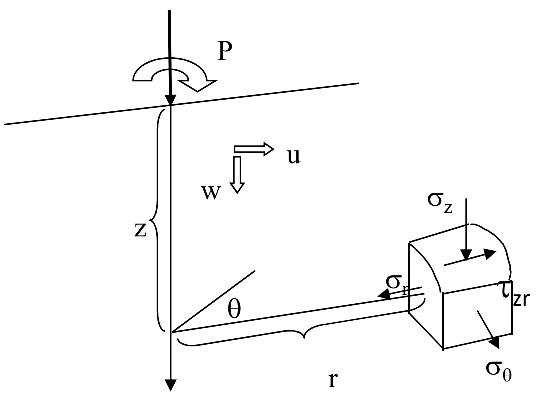
- **Deflexiones Verticales (w) y Horizontales (u)**

$$w = \frac{P}{2\pi E}\left[(1+\mu)z^2\left(r^2+z^2\right)^{-3/2} + 2\left(1-\mu^2\right)\left(r^2+z^2\right)^{-1/2}\right]$$

$$u = P\frac{(1+\mu)(1-2\mu)}{2\pi r E}\left[z\ \left(r^2+z^2\right)^{-1/2} - 1 + \frac{1}{1-2\mu}r^2 z\left(r^2+z^2\right)^{-3/2}\right]$$

For z=0

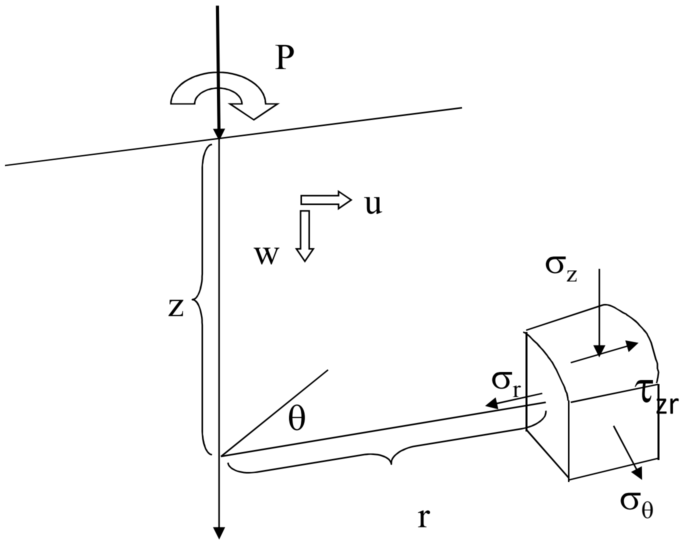
$$w = \frac{P\left(1-\mu^2\right)}{\pi E r}$$



EC. TAYLOR

Ejercicio 02

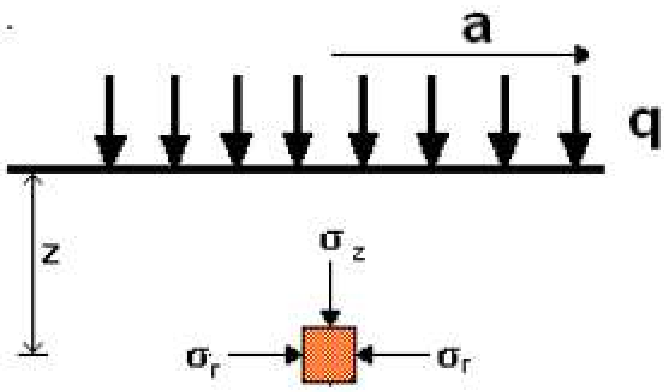
Calcular los esfuerzos y deformaciones resultantes de una carga puntual de 60 kN aplicada a un espacio elástico semi-infinito. El punto de interés a una profundidad de 20 cm y a un distancia radial de 40 cm. Si E = 140 MPa y μ = 0.4



SISTEMA DE UNA CAPA Placa circular

- **Cuando una carga se aplica sobre un área circular, los valores críticos de esfuerzo, deformación y deflexión ocurren en el eje de simetría bajo el centro del área circular.**

- **La carga aplicada a un pavimento por un neumático es similar a un placa flexible con radio a y presión de contacto uniforme q.**



EC. TIMOSHENKO Y GOODIER (1987)

Carga Circular aplicando esfuerzo vertical uniforme para r = 0

$$\sigma_z = p\left[-1 + \frac{z^3}{\left(a^2 + z^2\right)^{3/2}}\right]$$

$$\sigma_r = \sigma_\theta = \frac{p}{2}\left[-(1 + 2\mu) + \frac{2(1+\mu)z}{\sqrt{a^2 + z^2}} - \frac{z^3}{\left(a^2 + z^2\right)^{3/2}}\right]$$

$$\tau_{zr} = 0$$

$$w = \frac{2\left(1 - \mu^2\right)}{E} pa$$

EC. TIMOSHENKO Y GOODIER (1987)

The vertical deflection w can be determined from

$$w = \frac{(1 + v)qa}{E}\left\{\frac{a}{(a^2 + z^2)^{0.5}} + \frac{1 - 2v}{a}\left[(a^2 + z^2)^{0.5} - z)\right]\right\}$$

When $v = 0.5$, Eq. 2.6 can be simplified to

$$w = \frac{3qa^2}{2E(a^2 + z^2)^{0.5}}$$

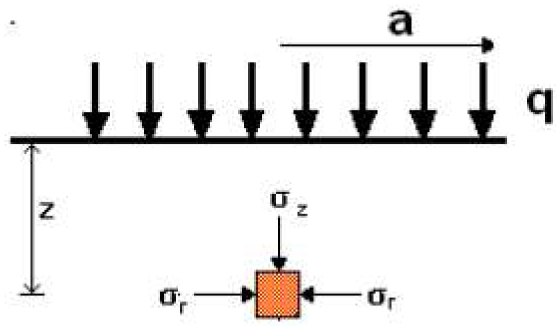
On the surface of the half-space, $z = 0$; from Eq. 2.6,

$$w_0 = \frac{2(1 - v^2)qa}{E}$$

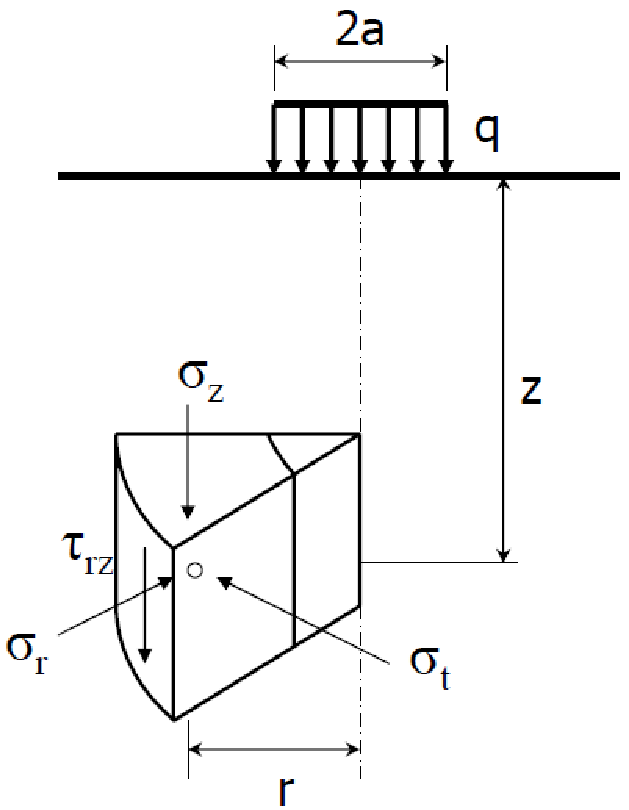
Ejercicio 02

- Calcular los esfuerzos de una llanta inflada a 600 kPa, que sobrelleva una carga de 30 kN descansando en un espacio elástico semi-infinito. La ubicación deseada es a una profundidad de 0.1 m y debajo del centro de la carga (r=0), También calcule la deflexión superficial (cuando z=0) debajo de la llanta

  Datos: E = 140 MPa $\mu = 0.40$

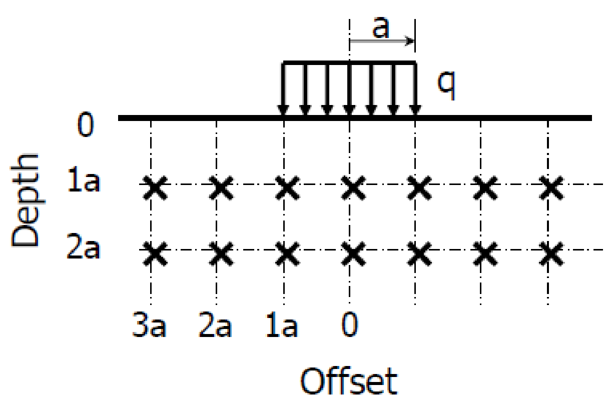


One-layer Solutions (Foster & Ahlvin)



$\sigma_z, \sigma_t, \sigma_r, \tau_{rz}$ & w (v=0.5)

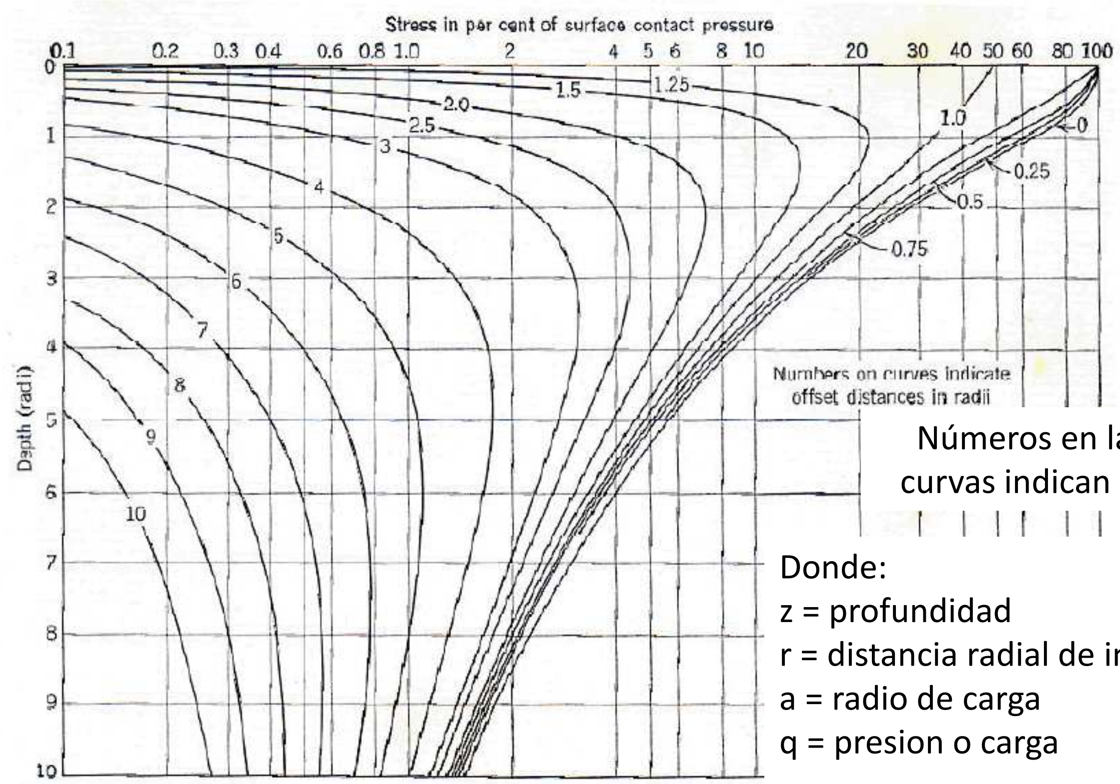
- Axisymmetric loading:
  - σ_z = Vertical stress
  - σ_r = Radial stress
  - σ_t = Tangential stress
  - τ_{rz} = Shear stress
  - w = Deflection
- Pre-solved @ radial distances



Vertical Stress σ_z due to Circular Loading (Foster and Ahlvin, 1954)

$$\frac{\sigma_z}{q} x100\ (\%)$$

z/a



Números en las curvas indican r/a

Donde:
z = profundidad
r = distancia radial de interes
a = radio de carga
q = presion o carga

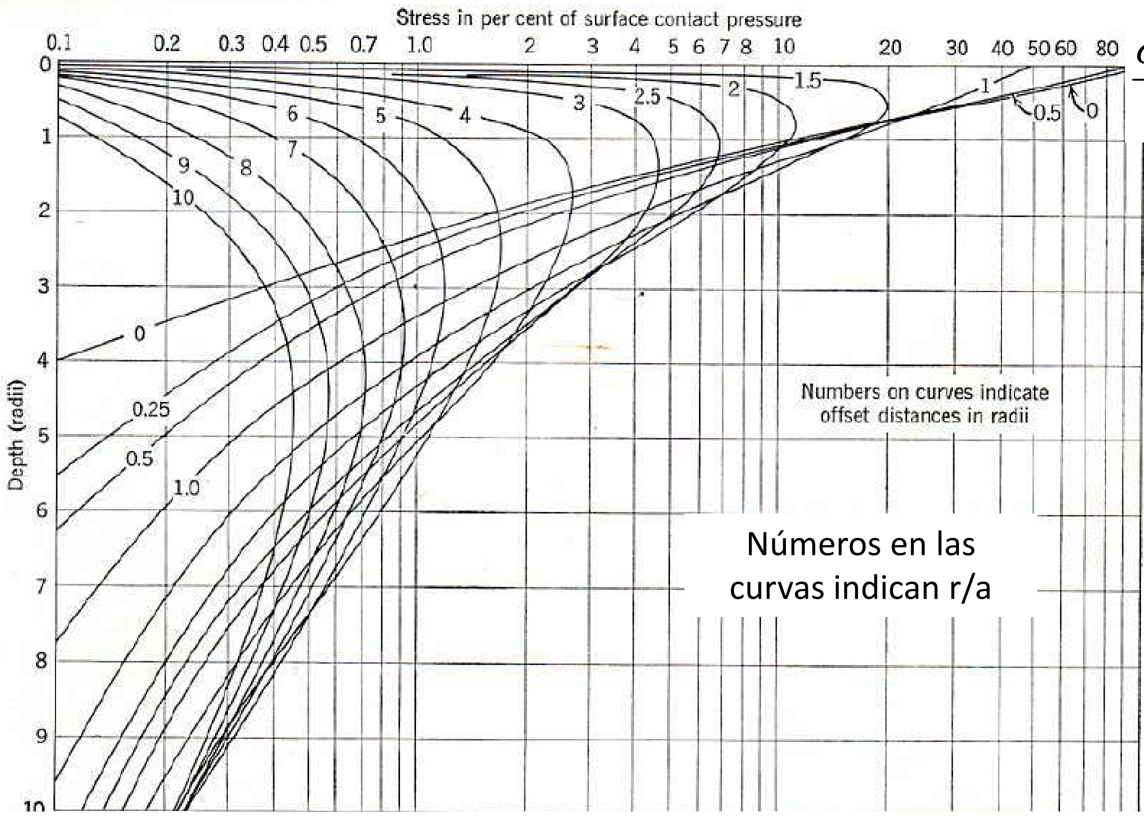
Esfuerzos verticales debido a cargas circulares (Foster y Ahlvin, 1954)

z

Radial Stress σ_r due to Circular Loading (Foster and Ahlvin, 1954)

$$\frac{\sigma_r}{q} x100\ (\%)$$

z/a

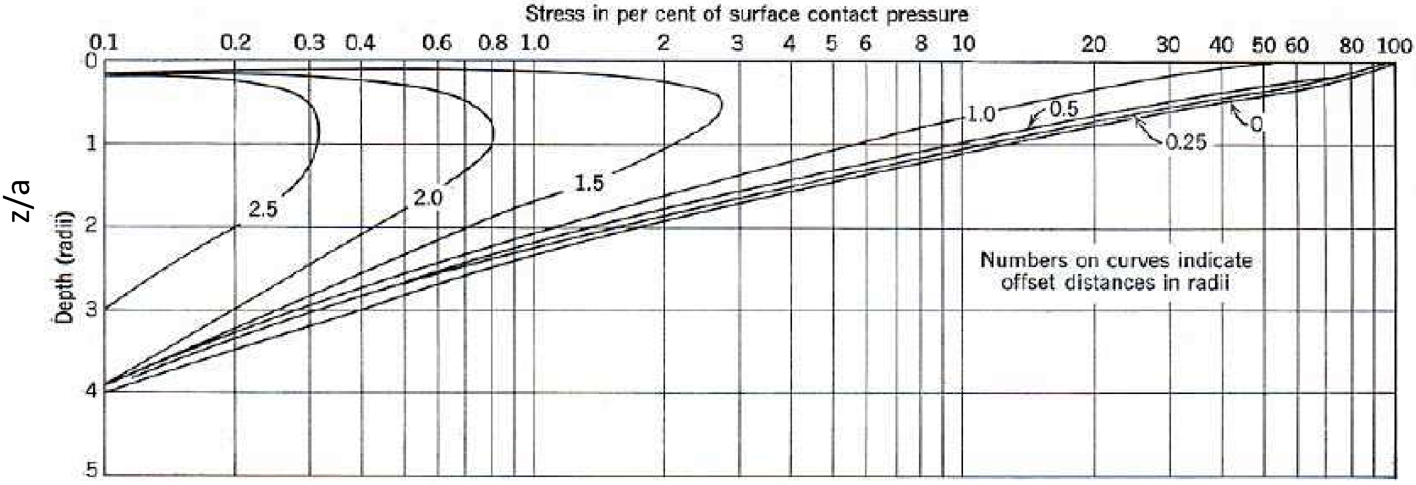


Números en las curvas indican r/a

Esfuerzos radiales debido a cargas circulares (Foster y Ahlvin, 1954)

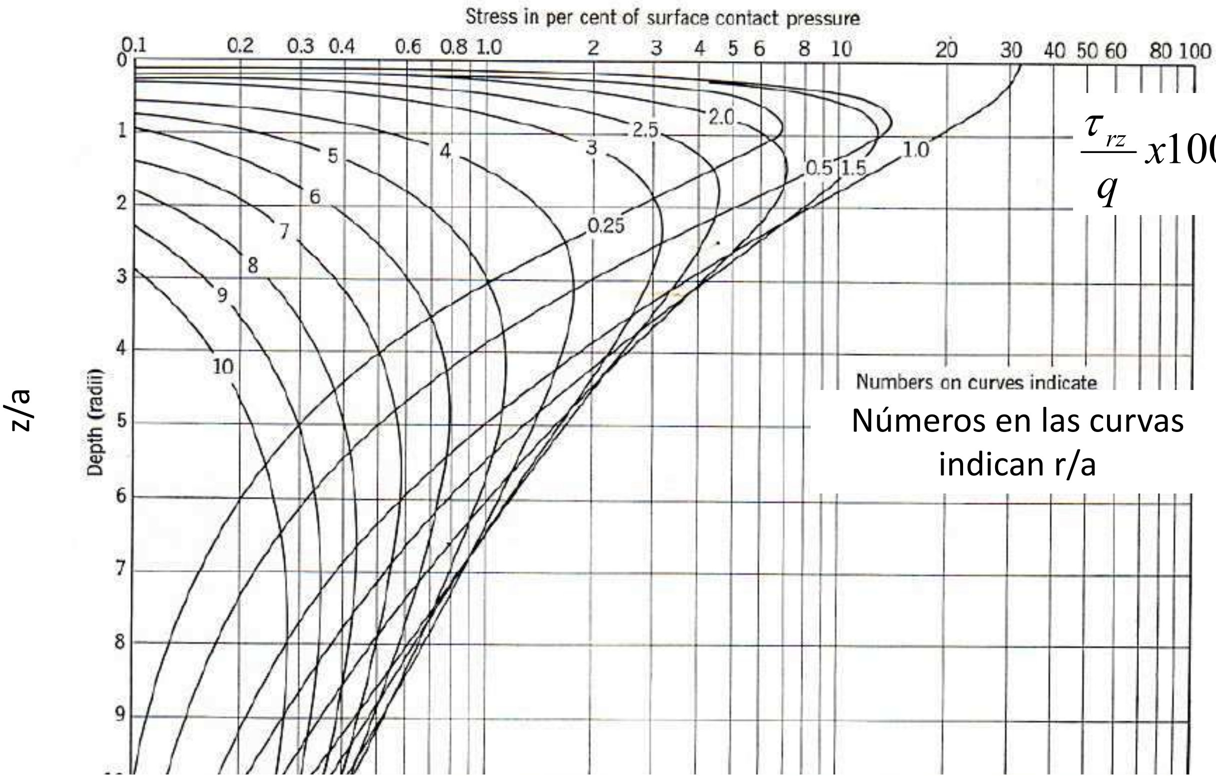
Tangential Stress σt due to Circular Loading (Foster and Ahlvin, 1954)

$$\frac{\sigma_\theta}{q} x100\ (\%)$$



Esfuerzos tangenciales debido a cargas circulares (Foster y Ahlvin, 1954)

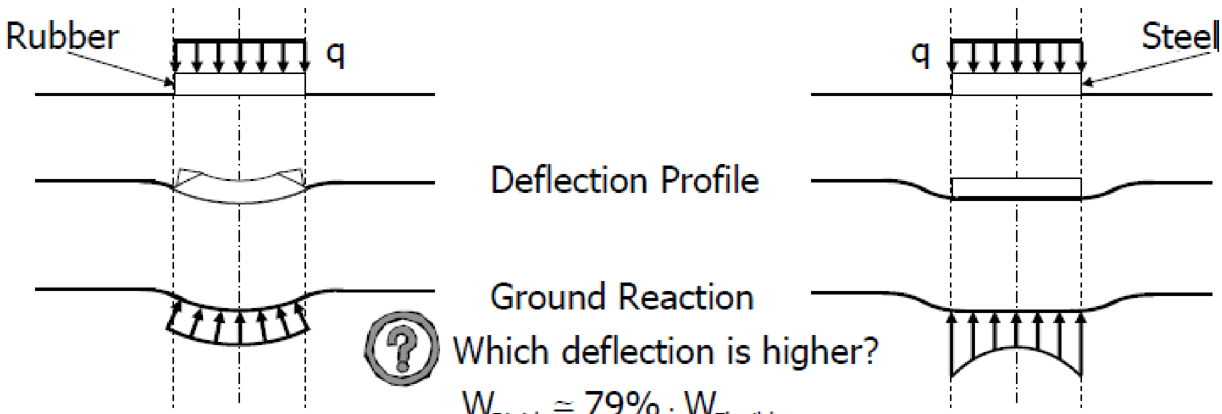
Shear Stress T_{zr} due to Circular Loading (Foster and Ahlvin, 1954)



$$\frac{\tau_{rz}}{q} x100\ (\%)$$

Esfuerzos de Corte debido a cargas circulares (Foster y Ahlvin, 1954)

Deflection



Which deflection is higher?

$W_{Rigid} \cong 79\% \cdot W_{Flexible}$

Flexible Plate:

$$W_0 = \frac{2(1-\nu^2)qa}{E}$$

$$W_0 = \frac{1.5qa}{E}$$

Rigid Plate:

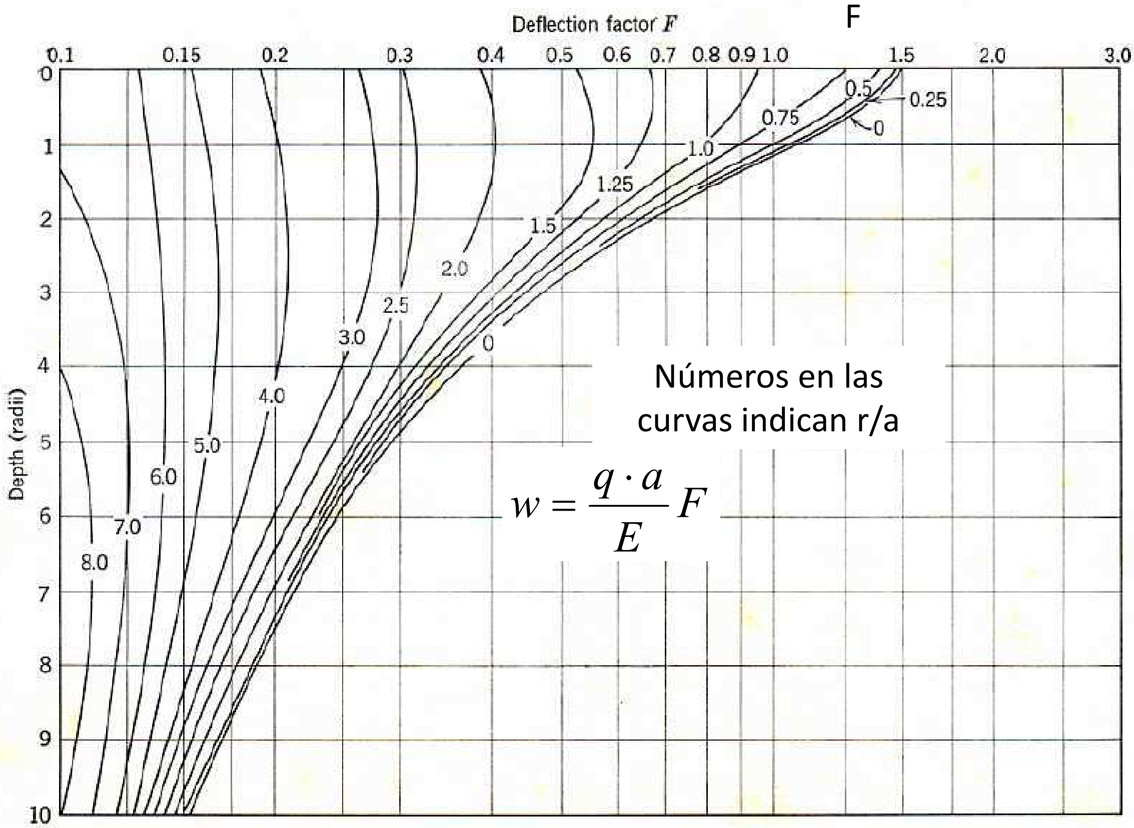
$$W_0 = \frac{\pi(1-\nu^2)qa}{2E}$$

$$W_0 = \frac{1.18qa}{E}$$

Vertical Deflection W due to Circular Loading (Foster and Ahlvin, 1954)

F

z/a

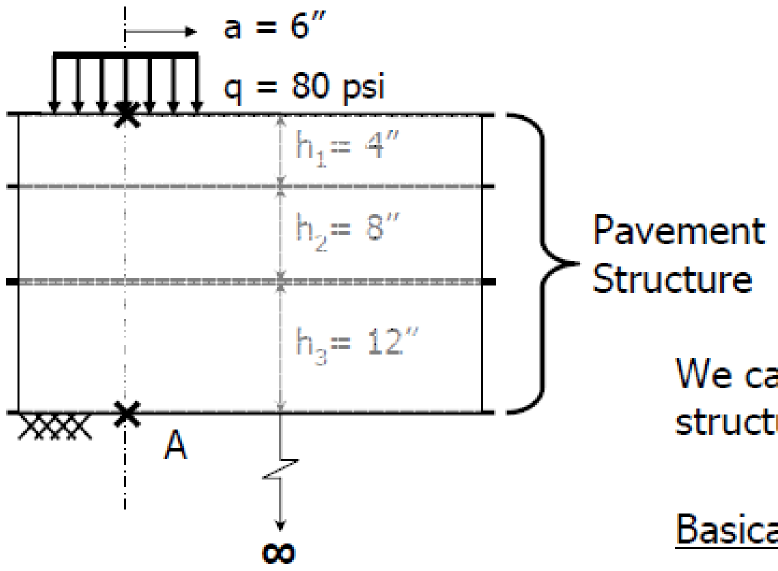


Números en las curvas indican r/a

$$w = \frac{q \cdot a}{E} F$$

Ejemplo 01

Deflection (cont.)



We can assume the pavement structure to be incompressible

Basically: $\delta_{\text{surface}} \equiv \delta_A$

For this case (assuming one-layer):

$$\delta_A = \frac{q \times a}{E} \times F$$

Get F from Fig 2.6

Ejemplo 01

Deflection (cont.)

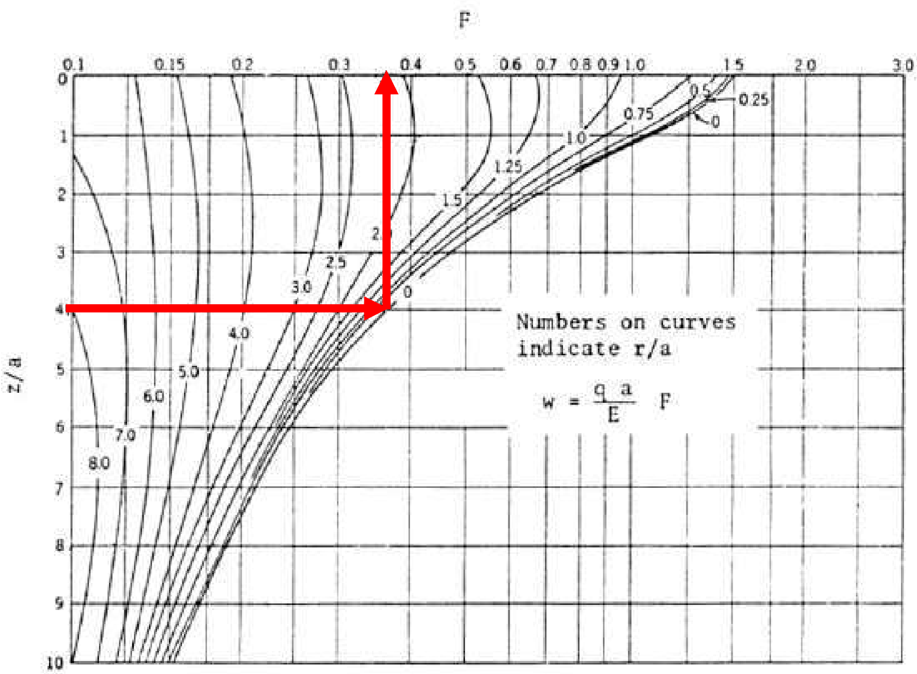


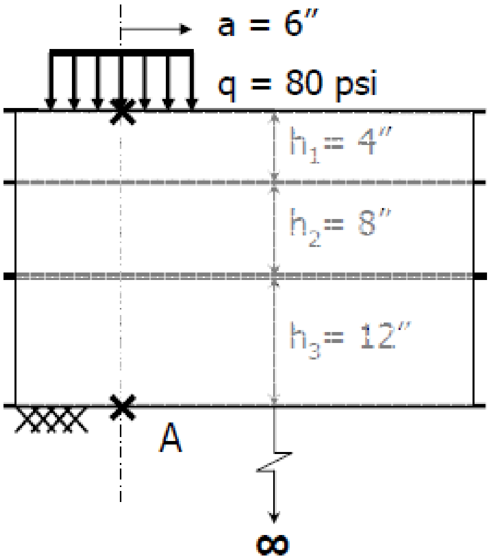
Figure 2.6 Vertical deflections due circular loading. (After Foster and Ahlvin (1954).)

Given:
z/a=24/6=4
r/a=0

Find:
F=0.37

Ejemplo 01

Deflection (cont.)



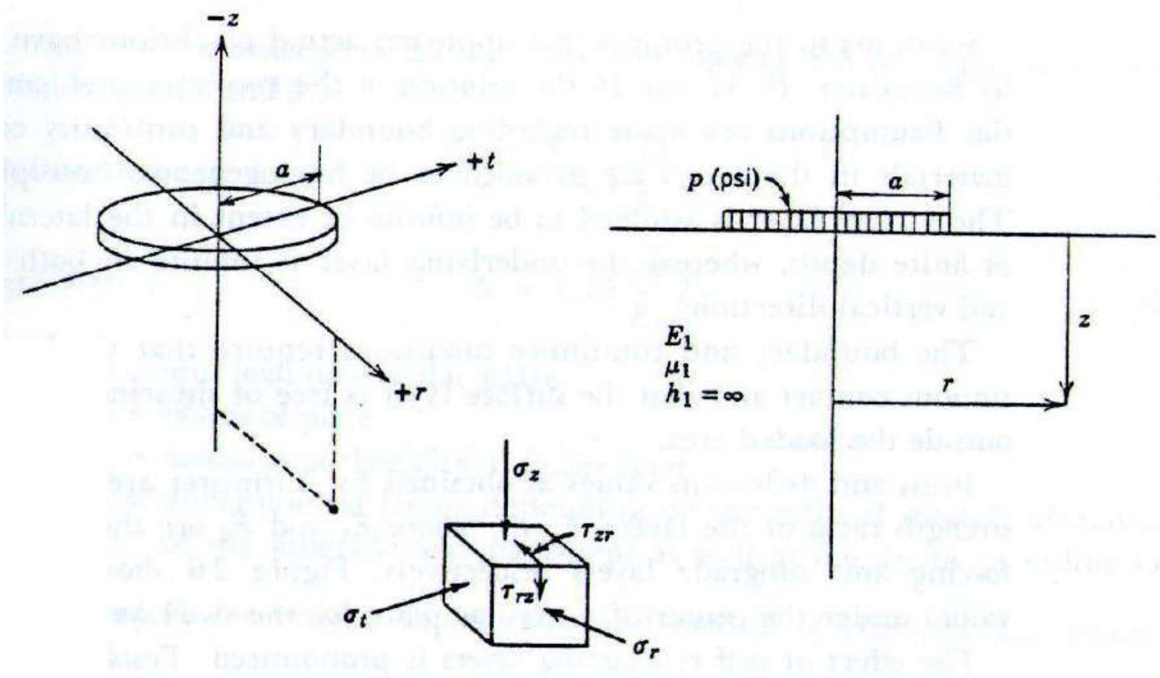
- Examine two cases:

Clay	Dense Sand
E=2,500	E=25,000
$w = \frac{80 \times 6}{2500} 0.37 = 0.071$	$w = \frac{80 \times 6}{25000} 0.37 = 0.0071$
w=71 mils (High)	w=7.1 mils (Low)

Subgrade quality is very important in pavement design

AHLVIN Y ULERY (1962)

Datos para usar en los ábacos / tablas mostrados



One-layer stress diagram.

AHLVIN Y ULERY (1962)

TABLE 2.1. Summary of One-Layer Elastic Equations[a] (after Ahlvin and Ulery)

Parameter	General Case	Special Case ($\mu = 0.5$)
Vertical stress	$\sigma_z = p[A + B]$	(same)
Radial horizontal stress	$\sigma_r = p[2\mu A + C + (1 - 2\mu)F]$	$\sigma_r = p[A + C]$
Tangential horizontal stress	$\sigma_t = p[2\mu A - D + (1 - 2\mu)E]$	$\sigma_t = p[A - D]$
Vertical radial shear stress	$\tau_{rz} = \tau_{zr} = pG$	(same)
Vertical strain	$\epsilon_z = \dfrac{p(1 + \mu)}{E_1}[(1 - 2\mu)A + B]$	$\epsilon_z \quad \dfrac{1.5p}{E_1} B$
Radial horizontal strain	$\epsilon_r = \dfrac{p(1 + \mu)}{E_1}[(1 - 2\mu)F + C]$	$\epsilon_r = \dfrac{1.5p}{E_1} C$
Tangential horizontal strain	$\epsilon_t = \dfrac{p(1 + \mu)}{E_1}[(1 - 2\mu)E - D)]$	$\epsilon_t = -\dfrac{1.5p}{E_1} D$
Vertical deflection	$\Delta_z = \dfrac{p(1 + \mu)a}{E_1}\left[\dfrac{z}{a}A + (1 - \mu)H\right]$	$\Delta_z = \dfrac{1.5pa}{E_1}\left(\dfrac{z}{a}A + \dfrac{H}{2}\right)$

AHLVIN Y ULERY (1962)

Función A

Espaciamiento r/a

profundidad z/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	1	1	1	1	1	0.5	0	0	0	0	0	0	0	0	0	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787	0.00856	0.00211	0.00084	0.00042					
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251	0.01680	0.00419	0.00167	0.00083	0.00048	0.00020			
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199	0.02440	0.00622	0.00250						
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593	0.03118								
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499	0.03701	0.01013	0.00407	0.00209	0.00118	0.00053	0.00025	0.00014	0.00009
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010									
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228	0.04558								
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236									
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094									
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849	0.05185	0.01742	0.00761	0.00393	0.00226	0.00097	0.00050	0.00029	0.00018
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192	0.05260	0.01935	0.00871	0.00459	0.00269	0.00115			
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048	0.05116	0.02142	0.01013	0.00548	0.00325	0.00141	0.00073	0.00043	0.00027
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275	0.04496	0.02221	0.01160	0.00659	0.00399	0.00180	0.00094	0.00056	0.00036
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880	0.03787	0.02143	0.01221	0.00732	0.00463	0.00214	0.00115	0.00068	0.00043
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839	0.03150	0.01980	0.01220	0.00770	0.00505	0.00242	0.00132	0.00079	0.00051
4	0.02986	0.02976	0.02907	0.02832	0.02802	0.02749	0.02651	0.02490	0.02193	0.01592	0.01109	0.00768	0.00536	0.00282	0.00160	0.00099	0.00065
5	0.01945	0.01938				0.01835			0.01573	0.01249	0.00949	0.00708	0.00527	0.00298	0.00179	0.00113	0.00075
6	0.01361					0.01307			0.01168	0.00983	0.00795	0.00628	0.00492	0.00299	0.00188	0.00124	0.00084
7	0.01005					0.00976			0.00894	0.00784	0.00661	0.00548	0.00445	0.00291	0.00193	0.00130	0.00091
8	0.00772					0.00755			0.00703	0.00635	0.00554	0.00472	0.00398	0.00276	0.00189	0.00134	0.00094
9	0.00612					0.00600			0.00566	0.00520	0.00466	0.00409	0.00353	0.00256	0.00184	0.00133	0.00096
10								0.00477	0.00465	0.00438	0.00397	0.00352	0.00326	0.00241			

AHLVIN Y ULERY (1962)

Función B

Espaciamiento r/a

profundidad z/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	-0.07899	-0.02672	-0.00845	-0.00210	-0.00084	-0.00042					
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	-0.07759	-0.04448	-0.01593	-0.00412	-0.00166	-0.00083	-0.00024	-0.00010			
0.3	0.28362	0.26787	0.28018	0.29483	0.27257	0.10757	-0.04316	-0.04999	-0.02166	-0.00599	-0.00245						
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	-0.00766	-0.04535	-0.02522								
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	-0.03455	-0.02651	-0.00991	-0.00388	-0.00199	-0.00116	-0.00049	-0.00025	-0.00014	-0.00009
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	-0.02101									
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	-0.00702	-0.02329								
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614									
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795									
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814	-0.01005	-0.01115	-0.00608	-0.00344	-0.00210	-0.00092	-0.00048	-0.00028	-0.00018
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378	0.00023	-0.00995	-0.00632	-0.00378	-0.00236	-0.00107			
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745	0.01385	-0.00669	-0.00600	-0.00401	-0.00265	-0.00126	-0.00068	-0.00040	-0.00026
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371	0.02836	0.00028	-0.00410	-0.00371	-0.00278	-0.00148	-0.00084	-0.00050	-0.00033
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022	0.03429	0.00661	-0.00130	-0.00271	-0.00250	-0.00156	-0.00094	-0.00059	-0.00039
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354	0.03511	0.01112	0.00157	-0.00134	-0.00192	-0.00151	-0.00099	-0.00065	-0.00046
4	0.05707	0.05666	0.05562	0.05383	0.05145	0.04773	0.04532	0.03995	0.03066	0.01515	0.00595	0.00155	-0.00029	-0.00109	-0.00094	-0.00068	-0.00050
5	0.03772	0.37600				0.03384			0.02474	0.01522	0.00810	0.00371	0.00132	-0.00043	-0.00070	-0.00068	-0.00049
6	0.02666					0.02468			0.01968	0.01380	0.00867	0.00496	0.00254	0.00028	-0.00037	-0.00047	-0.00045
7	0.01980					0.01868			0.01577	0.01204	0.00842	0.00547	0.00332	0.00093	-0.00002	-0.00029	0.00037
8	0.01526					0.01459			0.01279	0.01034	0.00779	0.00554	0.00372	0.00141	0.00035	-0.00008	-0.00025
9	0.01212					0.01170			0.01054	0.00888	0.00705	0.00533	0.00386	0.00178	0.00066	0.00012	-0.00012
10								0.00924	0.00879	0.00764	0.00631	0.00501	0.00382	0.00199			

AHLVIN Y ULERY (1962)

Función C

Espaciamiento r/a

profundidad z/a	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	-0.04926	-0.05142	-0.05903	-0.07708	-0.12108	0.02247	0.12007	0.01475	0.01536	0.00403	0.00164	0.00082					
0.2	-0.09429	-0.09755	-0.10872	-0.12977	-0.14552	0.02419	0.14896	0.07892	0.02951	0.00796	0.00325	0.00164	0.00094	0.00039			
0.3	-0.13181	-0.13484	-0.14415	-0.15023	-0.12990	0.01988	0.13394	0.09816	0.04148	0.01169	0.00483						
0.4	-0.16008	-0.16188	-0.16519	-0.15985	-0.11168	0.01292	0.10014	0.10122	0.05067								
0.5	-0.17889	-0.17835	-0.17497	-0.15625	-0.09833	0.00483	0.08730	0.10125	0.05690	0.01824	0.00778	0.00399	0.00231	0.00098	0.00050	0.00029	0.00018
0.6	-0.18915	-0.18663	-0.17336	-0.14934	-0.08967	-0.00304	0.06731	0.09313									
0.7	-0.19244	-0.18831	-0.17393	-0.14147	-0.08409	-0.01061	0.05028	0.08253	0.06129								
0.8	-0.19046	-0.18481	-0.16784	-0.13393	-0.08066	-0.01744	0.03582	0.07114									
0.9	-0.18481	-0.17841	-0.16024	-0.12664	-0.07828	-0.02337	0.02359	0.05993									
1	-0.17678	-0.17050	-0.15188	-0.11995	-0.07634	-0.02843	0.01331	0.04939	0.05429	0.02726	0.01333	0.00726	0.00433	0.00188	0.00098	0.00057	0.00036
1.2	-0.15742	-0.15117	-0.13467	-0.10763	-0.07289	-0.03575	-0.00245	0.03107	0.04522	0.02791	0.01467	0.00824	0.00501	0.00221			
1.5	-0.12801	-0.12277	-0.11101	-0.09145	-0.06711	-0.04124	-0.01702	0.01088	0.03154	0.02652	0.01570	0.00933	0.00585	0.00266	0.00141	0.00083	0.00039
2	-0.08944	-0.08491	-0.07976	-0.06925	-0.05560	-0.04144	-0.02687	-0.00782	0.01267	0.02070	0.01527	0.01013	0.00321	0.00327	0.00179	0.00107	0.00069
2.5	-0.06403	-0.06063	-0.05839	-0.05259	-0.04522	-0.03605	-0.02800	-0.01536	0.00103	0.01384	0.01314	0.00987	0.00707	0.00569	0.00209	0.00128	0.00083
3	-0.04744	-0.04560	-0.04339	-0.04089	-0.03642	-0.03130	-0.02587	-0.01748	-0.00528	0.00792	0.01030	0.00888	0.00689	0.00392	0.00232	0.00145	0.00096
4	-0.02854	-0.02737	-0.02562	-0.02585	-0.02421	-0.02112	-0.01964	-0.01586	-0.00956	0.00038	0.00492	0.00602	0.00561	0.00389	0.00254	0.00168	0.00115
5	-0.01886	-0.01810				-0.01568			-0.00939	-0.00293	-0.00128	0.00329	0.00391	0.00341	0.00250	0.00177	0.00127
6	-0.01333					-0.01118			-0.00819	-0.00405	-0.00079	0.00129	0.00234	0.00272	0.00227	0.00173	0.00130
7	-0.00990					-0.00902			-0.00678	-0.00417	-0.00180	-0.00004	0.00113	0.00200	0.00193	0.00161	0.00128
8	-0.00763					-0.00699			-0.00552	-0.00393	-0.00225	-0.00077	0.00029	0.00134	0.00157	0.00143	0.00120
9	-0.00607					-0.00423			-0.00452	-0.00353	-0.00235	-0.00118	-0.00027	0.00082	0.00124	0.00122	0.00110
10								-0.00381	-0.00373	-0.00314	-0.00233	-0.00137	-0.00063	0.00040			

AHLVIN Y ULERY (1962)

Function D

Depth (z) in Radii	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	.04926	.04998	.05235	.05716	.06687	.07635	.04108	.01803	.00691	.00193	.00080	.00041					
0.2	.09429	.09552	.09900	.10546	.11431	.10932	.07139	.03444	.01359	.00384	.00159	.00081	.00047	.00020			
0.3	.13181	.13305	.14051	.14062	.14267	.12745	.09078	.04817	.01982	.00927	.00238						
0.4	.16008	.16070	.16229	.16288	.15756	.13696	.10248	.05887	.02545								
0.5	.17889	.17917	.17826	.17481	.16403	.14074	.10894	.06670	.03039	.00921	.00390	.00200	.00116	.00049	.00025	.00015	.00009
0.6	.18915	.18867	.18573	.17887	.16489	.14137	.11186	.07212									
0.7	.19244	.19132	.18679	.17782	.16229	.13926	.11237	.07551	.03801								
0.8	.19046	.18927	.18348	.17306	.15714	.13548	.11115	.07728									
0.9	.18481	.18349	.17709	.16635	.15063	.13067	.10866	.07788									
1	.17678	.17503	.16886	.15824	.14344	.12513	.10540	.07753	.04456	.01611	.00725	.00382	.00224	.00096	.00050	.00029	.00018
1.2	.15742	.15618	.15014	.14073	.12823	.11340	.09757	.07484	.04575	.01796	.00835	.00446	.00264	.00114			
1.5	.12801	.12754	.12237	.11549	.10657	.09608	.08491	.06833	.04539	.01983	.00970	.00532	.00320	.00140	.00073	.00043	.00027
2	.08944	.09080	.08668	.08273	.07814	.07187	.06566	.05589	.04103	.02098	.01117	.00643	.00398	.00179	.00095	.00056	.00036
2.5	.06403	.06565	.06284	.06068	.05777	.05525	.05069	.04486	.03532	.02045	.01183	.00717	.00457	.00213	.00115	.00068	.00044
3	.04744	.04834	.04760	.01548	.04391	.04195	.03963	.03606	.02983	.01904	.01187	.00755	.00497	.00242	.00133	.00080	.00052
4	.02854	.02928	.02996	.02798	.02724	.02661	.02568	.02408	.02110	.01552	.01087	.00757	.00533	.00280	.00160	.00100	.00065
5	.01886	.01950				.01816			.01535	.01230	.00939	.00700	.00523	.00299	.00180	.00114	.00077
6	.01333					.01351			.01149	.00976	.00788	.00625	.00488	.00301	.00190	.00124	.00086
7	.00990					.00966			.00899	.00787	.00662	.00542	.00445	.00292	.00192	.00130	.00092
8	.00763					.00759			.00727 -	.00641	.00554	.00477	.00402	.00275	.00192	.00131	.00096
9	.00607					.00746			.00601	.00533	.00470	.00415	.00358	.00260	.00187	.00133	.00099
10								.00542	.00506	.00450	.00398	.00364	.00319	.00239			

AHLVIN Y ULERY (1962)

TABLE 2.2. (continued)

Function E

Depth (z) in Radii	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	.5	.5	.5	.5	.5	.5	.34722	.22222	.12500	.05556	.03125	.02000	.01389	.00781	.00500	.00347	.00255
0.1	.45025	.44949	.44698	.44173	.43008	.39198	.30445	.20399	.11806	.05362	.03045	.01959					
0.2	.40194	.40043	.39591	.38660	.36798	.32802	.26598	.18633	.11121	.05170	.02965	.01919	.01342	.00762			
0.3	.35633	.35428	.33809	.33674	.31578	.28003	.23311	.16967	.10450	.04979	.02886						
0.4	.31431	.31214	.30541	.29298	.27243	.24200	.20526	.15428	.09801								
0.5	.27639	.27407	.26732	.25511	.23639	.21119	.18168	.14028	.09180	.04608	.02727	.01800	.01272	.00734	.00475	.00332	.00246
0.6	.24275	.24247	.23411	.22289	.20634	.18520	.16155	.12759									
0.7	.21327	.21112	.20535	.19525	.18093	.16356	.14421	.11620	.08027								
0.8	.18765	.18550	.18049	.17190	.15977	.14523	.12928	.10602									
0.9	.16552	.16337	.15921	.15179	.14168	.12954	.11634	.09686									
1	.14645	.14483	.14610	.13472	.12618	.11611	.10510	.08865	.06552	.03736	.02352	.01602	.01157	.00683	.00450	.00318	.00237
1.2	.11589	.11435	.11201	.10741	.10140	.09431	.08657	.07476	.05728	.03425	.02208	.01527	.01113	.00664			
1.5	.08398	.08356	.08159	.07885	.07517	.07088	.06611	.05871	.04703	.03003	.02008	.01419	.01049	.00636	.00425	.00304	.00228
2	.05279	.05105	.05146	.05034	.04850	.04675	.04442	.04078	.03454	.02410	.01706	.01248	.00943	.00590	.00401	.00290	.00219
2.5	.03576	.03426	.03489	.03435	.03360	.03211	.03150	.02953	.02599	.01945	.01447	.01096	.00850	.00546	.00378	.00276	.00210
3	.02566	.02519	.02470	.02491	.02444	.02389	.02330	.02216	.02007	.01585	.01230	.00962	.00763	.00505	.00355	.00263	.00201
4	.01493	.01452	.01495	.01526	.01446	.01418	.01395	.01356	.01281	.01084	.00900	.00742	.00612	.00431	.00313	.00237	.00185
5	.00971	.00927				.00929			.00873	.00774	.00673	.00579	.00495	.00364	.00275	.00213	.00168
6	.00680					.00632			.00629	.00574	.00517	.00457	.00404	.00309	.00241	.00192	.00154
7	.00503					.00493			.00466	.00438	.00404	.00370	.00330	.00264	.00213	.00172	.00140
8	.00386					.00377			.00354	.00344	.00325	.00297	.00273	.00228	.00185	.00155	.00127
9	.00306					.00227			.00275	.00273	.00264	.00246	.00229	.00194	.00163	.00139	.00116
10								.00210	.00220	.00225	.00221	.00203	.00200	.00171			

AHLVIN Y ULERY (1962)

Function F

Depth (z) in Radii	Offset (r) in Radii																
	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	.5	.5	.5	.5	.5	0	−.34722	−.22222	−.12500	−.05556	−.03125	−.02000	−.01389	−.00781	−.00500	−.00347	−.00255
0.1	.45025	.44794	.43981	.41954	.35789	.03817	−.20800	−.17612	−.10950	−.05151	−.02961	−.01917					
0.2	.40194	.39781	.38294	.34823	.26215	.05466	−.11165	−.13381	−.09441	−.04750	−.02798	−.01835	−.01295	−.00742			
0.3	.35633	.35094	.34508	.29016	.20503	.06372	−.05346	−.09768	−.08010	−.04356	−.02636						
0.4	.31431	.30801	.28681	.24469	.17086	.06848	−.01818	−.06835	−.06684								
0.5	.27639	.26997	.24890	.20937	.14752	.07037	.00388	−.04529	−.05479	−.03595	−.02320	−.01590	−.01154	−.00681	−.00450	−.00318	−.00237
0.6	.24275	.23444	.21667	.18138	.13042	.07068	.01797	−.02749									
0.7	.21327	.20762	.18956	.15903	.11740	.06963	.02704	−.01392	−.03469								
0.8	.18765	.18287	.16679	.14053	.10604	.06774	.03277	−.00365									
0.9	.16552	.16158	.14747	.12528	.09664	.06533	.03619	.00408									
1	.14645	.14280	.12395	.11225	.08850	.06256	.03819	.00984	−.01367	−.01994	−.01591	−.01209	−.00931	−.00587	−.00400	−.00289	−.00219
1.2	.11589	.11360	.10460	.09449	.07486	.05670	.03913	.01716	−.00452	−.01491	−.01337	−.01068	−.00844	−.00550			
1.5	.08398	.08196	.07719	.06918	.05919	.04804	.03686	.02177	.00413	−.00879	−.00995	−.00870	−.00723	−.00495	−.00353	−.00261	−.00201
2	.05279	.05348	.04994	.04614	.04162	.03593	.03029	.02197	.01043	−.00189	−.00546	−.00589	−.00544	−.00410	−.00307	−.00233	−.00183
2.5	.03576	.03673	.03459	.03263	.03014	.02762	.02406	.01927	.01188	.00198	−.00226	−.00364	−.00386	−.00332	−.00263	−.00208	−.00166
3	.02566	.02586	.02255	.02395	.02263	.02097	.01911	.01623	.01144	.00396	−.00010	−.00192	−.00258	−.00263	−.00223	−.00183	−.00150
4	.01493	.01536	.01412	.01259	.01386	.01331	.01256	.01134	.00912	.00508	.00209	.00026	−.00076	−.00148	−.00153	−.00137	−.00120
5	.00971	.01011				.00905			.00700	.00475	.00277	.00129	.00031	−.00066	−.00096	−.00099	−.00093
6	.00680					.00675			.00538	.00409	.00278	.00170	.00088	−.00010	−.00053	−.00066	−.00070
7	.00503					.00483			.00428	.00346	.00258	.00178	.00114	.00027	−.00020	−.00041	−.00049
8	.00386					.00380			.00350	.00291	.00229	.00174	.00125	.00048	.00003	−.00020	−.00033
9	.00306					.00374			.00291	.00247	.00203	.00163	.00124	.00062	.00020	−.00005	−.00019
10								.00267	.00246	.00213	.00176	.00149	.00126	.00070			

AHLVIN Y ULERY (1962)

TABLE 2.2 (continued)

Function G

Depth (z) in Radii	Offset (r) in Radii																
	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	.31831	0	0	0	0	0	0	0	0	0	0	0
0.1	0	.00315	.00802	.01951	.06682	.31405	.05555	.00865	.00159	.00023	.00007	.00003					
0.2	0	.01163	.02877	.06441	.16214	.30474	.13592	.03060	.00614	.00091	.00026	.00010	.00005	.00002			
0.3	0	.02301	.05475	.11072	.21465	.29228	.18216	.05747	.01302	.00201	.00059						
0.4	0	.03460	.07883	.14477	.23442	.27779	.20195	.08233	.02138								
0.5	0	.04429	.09618	.16426	.23652	.26216	.20731	.10185	.03033	.00528	.00158	.00063	.00030	.00009	.00004	.00002	.00001
0.6	0	.04966	.10729	.17192	.22949	.24574	.20496	.11541									
0.7	0	.05484	.11256	.17126	.21772	.22924	.19840	.12373	.04718								
0.8	0	.05590	.11225	.16534	.20381	.21295	.18953	.12855									
0.9	0	.05496	.10856	.15028	.18901	.19712	.17945	.28881									
1	0	.05266	.10274	.14566	.17419	.18198	.16884	.12745	.06434	.01646	.00555	.00233	.00113	.00036	.00015	.00007	.00004
1.2	0	.04585	.08831	.12323	.14615	.15408	.14755	.12038	.06967	.02077	.00743	.00320	.00159	.00051			
1.5	0	.03483	.06688	.09293	.11071	.11904	.11830	.10477	.07075	.02599	.01021	.00460	.00233	.00078	.00033	.00016	.00009
2	0	.02102	.04069	.05721	.06948	.07738	.08067	.07804	.06275	.03062	.01409	.00692	.00369	.00129	.00055	.00027	.00015
2.5	0	.01293	.02534	.03611	.04484	.05119	.05509	.05668	.05117	.04099	.01650	.00886	.00499	.00185	.00082	.00041	.00023
3	0	.00840	.01638	.02376	.02991	.03485	.03843	.04121	.04039	.02886	.01745	.01022	.00610	.00241	.00110	.00057	.00032
4	0	.00382	.00772	.01149	.01480	.01764	.02004	.02271	.02475	.02215	.01639	.01118	.00745	.00340	.00167	.00090	.00052
5	0	.00214				.00992		.01343	.01551	.01601	.01364	.01105	.00782	.00404	.00216	.00122	.00073
6	0					.00602		.00845	.01014	.01148	.01082	.00917	.00733	.00432	.00243	.00150	.00092
7	0					.00396			.00687	.00830	.00842	.00770	.00656	.00432	.00272	.00171	.00110
8	0					.00270			.00481	.00612	.00656	.00631	.00568	.00413	.00278	.00185	.00124
9	0					.00177			.00347	.00459	.00513	.00515	.00485	.00381	.00274	.00192	.00133
10	0							.00199	.00258	.00351	.00407	.00420	.00411	.00346			

AHLVIN Y ULERY (1962)

Function H

Depth (z) in Radii	Offset (r) in Radii: 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	2.0	1.97987	1.91751	1.80575	1.62553	1.27319	.93676	.71185	.51671	.33815	.25200	.20045	.16626	.12576	.09918	.08346	.07023
0.1	1.80998	1.79018	1.72886	1.61961	1.44711	1.18107	.92670	.70888	.51627	.33794	.25184	.20081					
0.2	1.63961	1.62068	1.56242	1.46001	1.30614	1.09996	.90098	.70074	.51382	.33726	.25162	.20072	.16688	.12512			
0.3	1.48806	1.47014	1.40979	1.32442	1.19210	1.02740	.86726	.68823	.50966	.33638	.25124						
0.4	1.35407	1.33802	1.28963	1.20822	1.09555	.96202	.83042	.67238	.50412								
0.5	1.23607	1.22176	1.17894	1.10830	1.01312	.90298	.79308	.65429	.49728	.33293	.24996	.19982	.16668	.12493	.09996	.08295	.07123
0.6	1.13238	1.11998	1.08350	1.02154	.94120	.84917	.75653	.63469									
0.7	1.04131	1.03037	.99794	.91049	.87742	.80030	.72143	.61442	.48061								
0.8	.96125	.95175	.92386	.87928	.82136	.75571	.68809	.59398									
0.9	.89072	.88251	.85856	.82616	.77950	.71495	.65677	.57361									
1	.82843	.85005	.80165	.76809	.72587	.67769	.62701	.55364	.45122	.31877	.24386	.19673	.16516	.12394	.09952	.08292	.07104
1.2	.72410	.71882	.70370	.67937	.64814	.61187	.57329	.51552	.43013	.31162	.24070	.19520	.16369	.12350			
1.5	.60555	.60233	.57246	.57633	.55559	.53138	.50496	.46379	.39872	.29945	.23495	.19053	.16199	.12281	.09876	.08270	.07064
2	.47214	.47022	.44512	.45656	.44502	.43202	.41702	.39242	.35054	.27740	.22418	.18618	.15846	.12124	.09792	.08196	.07026
2.5	.38518	.38403	.38098	.37608	.36940	.36155	.35243	.33698	.30913	.25550	.21208	.17898	.15395	.11928	.09700	.08115	.06980
3	.32457	.32403	.32184	.31887	.31464	.30969	.30381	.29364	.27453	.23487	.19977	.17154	.14919	.11694	.09558	.08061	.06897
4	.24620	.24588	.24820	.25128	.24168	.23932	.23668	.23164	.22188	.19908	.17640	.15596	.13864	.11172	.09300	.07864	.06848
5	.19805	.19785				.19455			.18450	.17080	.15575	.14130	.12785	.10585	.08915	.07675	.06695
6	.16554					.16326			.15750	.14868	.13842	.12792	.11778	.09990	.08562	.07452	.06522
7	.14217					.14077			.13699	.13097	.12404	.11620	.10843	.09387	.08197	.07210	.06377
8	.12448					.12352			.12112	.11680	.11176	.10600	.09976	.08848	.07800	.06928	.06200
9	.11079					.10989			.10854	.10548	.10161	.09702	.09234	.08298	.07407	.06678	.05976
10								.09900	.09820	.09510	.09290	.08980	.08300	.07710			

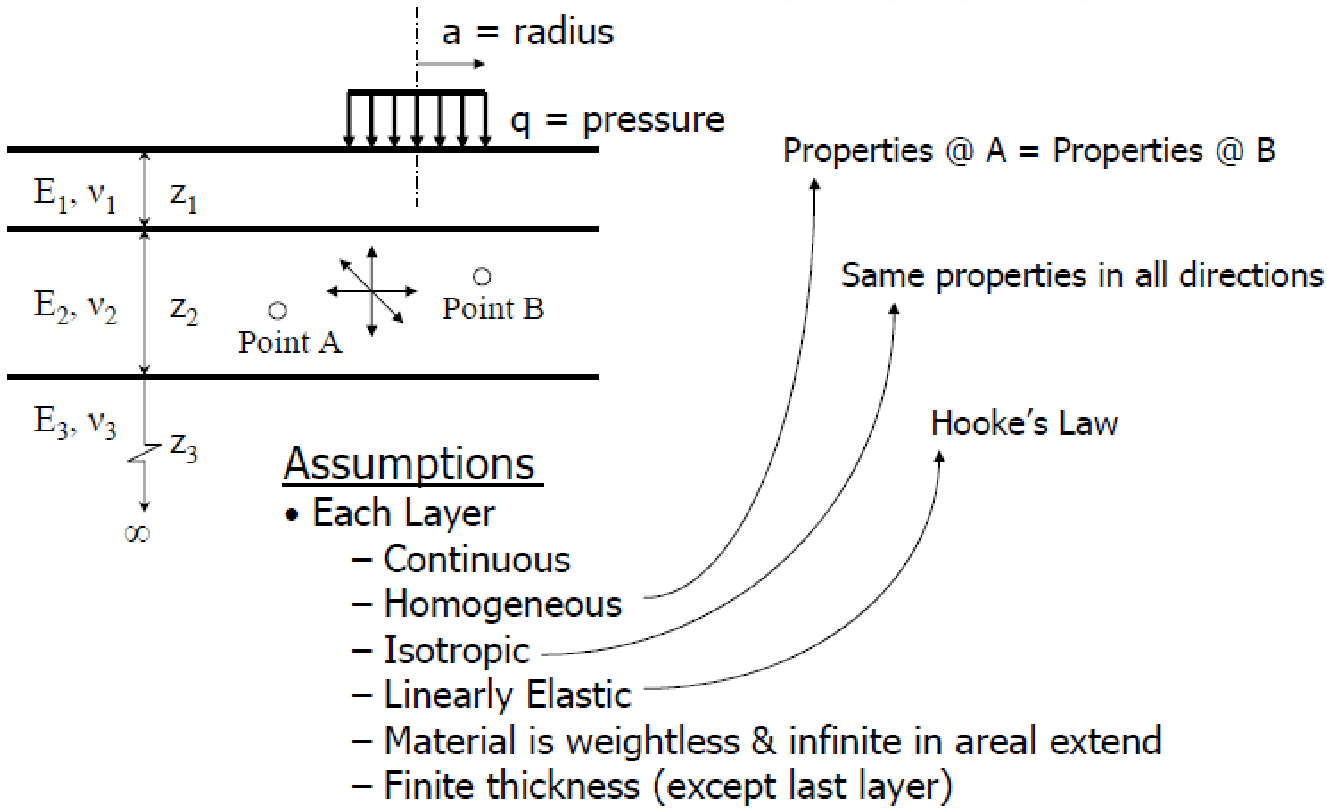
Ejercicio 02

Determinar los esfuerzos σz, σr y σθ usando la teoría de una capa debajo del centro de una llanta que tiene una carga de 50,000 libras-fuerza y una presión de 100 psi para las siguientes proporciones profundidad / radio de llanta (radii)

0	0.2	0.5	
1.0	2.0	4.0	8.0

Asumir que el pavimento tiene un coeficiente de Poisson de 0.5 y módulo de elasticidad E = 3000 psi.

Multilayer Elastic Theory



DOS CAPAS (Carga circular)

- Cálculo de esfuerzos, deformaciones y desplazamientos en función de z/a y r/a (Burmister,1943).

TRES CAPAS (Carga circular)

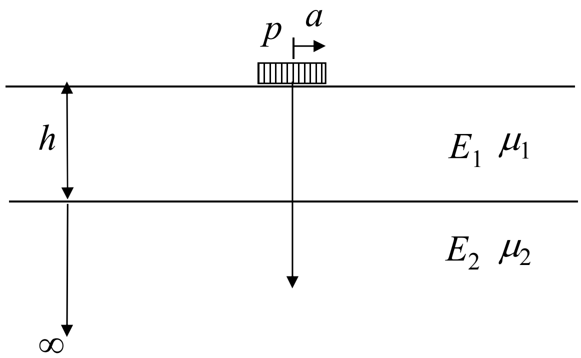
- Expresiones analíticas para cálculo de esfuerzos y desplazamientos (Burmister,1945)
- Tablas para determinar esfuerzos normales y radiales en la intersección del eje de carga con las interfaces (AcumyFox,1951)
- Soluciones gráficas para el cálculo de los esfuerzos verticales (Peattie,1962)

N CAPAS (Carga circular)

- Huang,1967

Solución ELÁSTICA PARA DOS CAPAS

- Burmister propuso en 1943, 1945 y 1958 soluciones al problema de un sistema de dos capas como el que se muestra en la figura adjunta.



- Burmister asumió capas homogéneas, isotrópicas y elásticas. Asimismo, la capa superficial (capa asfáltica) es infinita lateralmente pero con profundidad “h”, mientras que la segunda capa (subrasante) es infinita a los costados y en profundidad

- Las condiciones de limite y continuidad requieren que las capas estén en continuo contacto y que la capa superficial no tenga esfuerzos de corte ni normales mas allá del punto de carga. Además $\mu_1 = \mu_2 = 0.5$

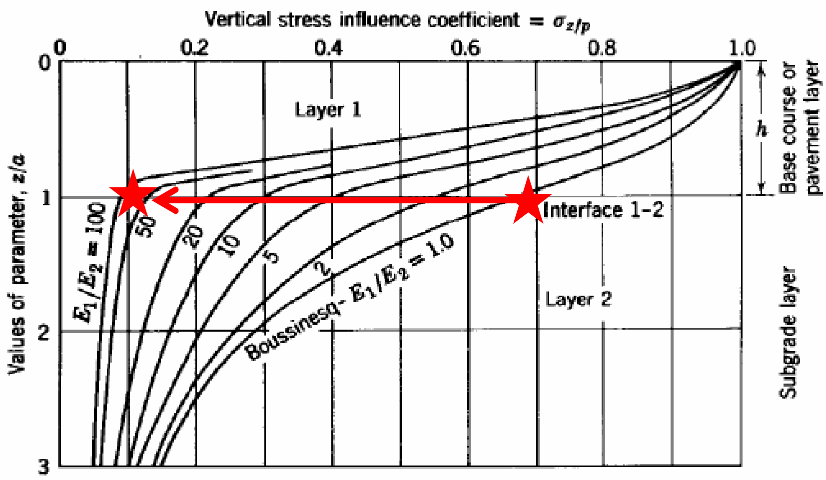
Solución ELÁSTICA PARA DOS CAPAS

- Cuando se asume que el pavimento es homogéneo, $E_1 = E_2$. Esto se ve en la grafica anterior en la línea inferior.

- Al incorporar al sistema una capa con una resistencia 100 veces a la existente, el esfuerzo vertical se ve reducido en la interfase entre la capa 1 y la capa 2 de un 70% a un 10%

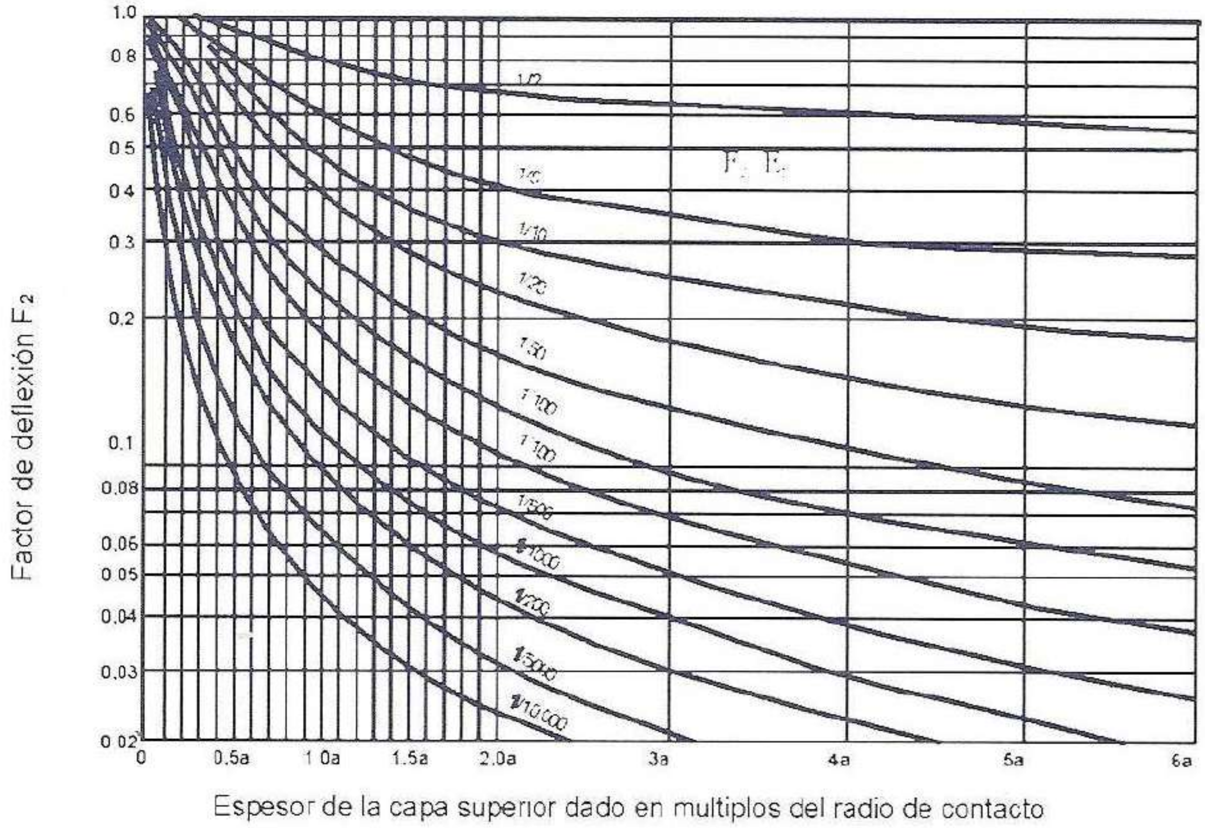
- La profundidad también juega un papel importante.

Muy profundo, el efecto del refuerzo se pierde, así que no tiene sentido el hacer capas muy gruesas porque ya no tiene el mismo efecto



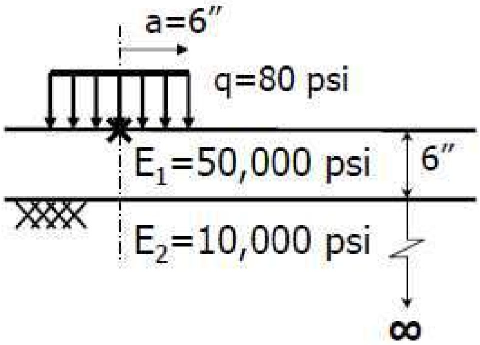
Burmister (1943)

Figura 5.4 – Factor de deflexión F_2 para la teoría Bicapa
(**Fuente:** Instituto Colombiano de Producciones de Cemento – ICPC. Cipriano A. Londoño. *Funcionamiento de los pavimentos*, Medellín, 1989).



Burmister (1943)

Surface Deflections Example



Given:
$h_1/a=6/6=1$
$E_1/E_2=5$
Find:
$F_2=0.6$

$$W_{max} = 1.5\frac{qa}{E_2}F_2 = 1.5\frac{6(80)}{10000}0.6$$

$$W_{max} = 0.0432'' \cong 43 \cdot \text{mils}$$

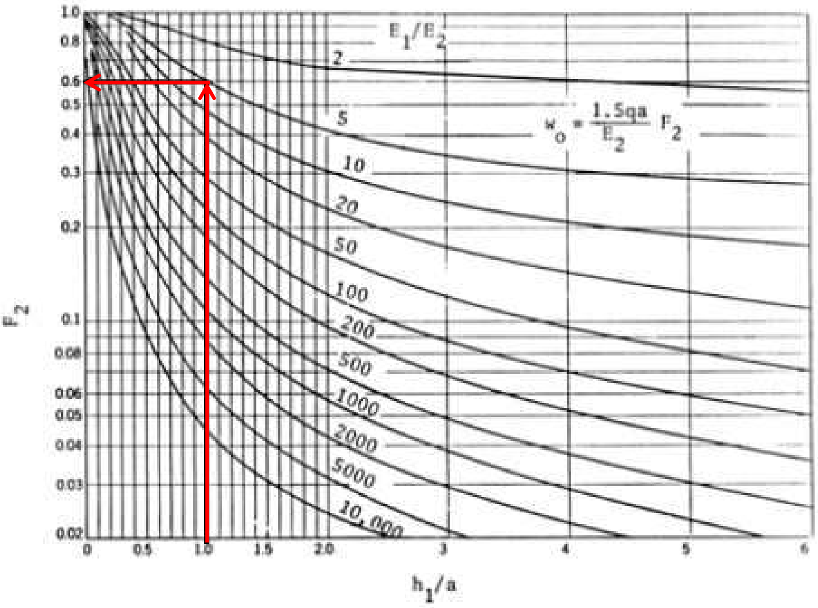


Figure 2.17 Vertical surface deflections for two-layer systems. (After Burmister (1943).)

Burmister (1943)

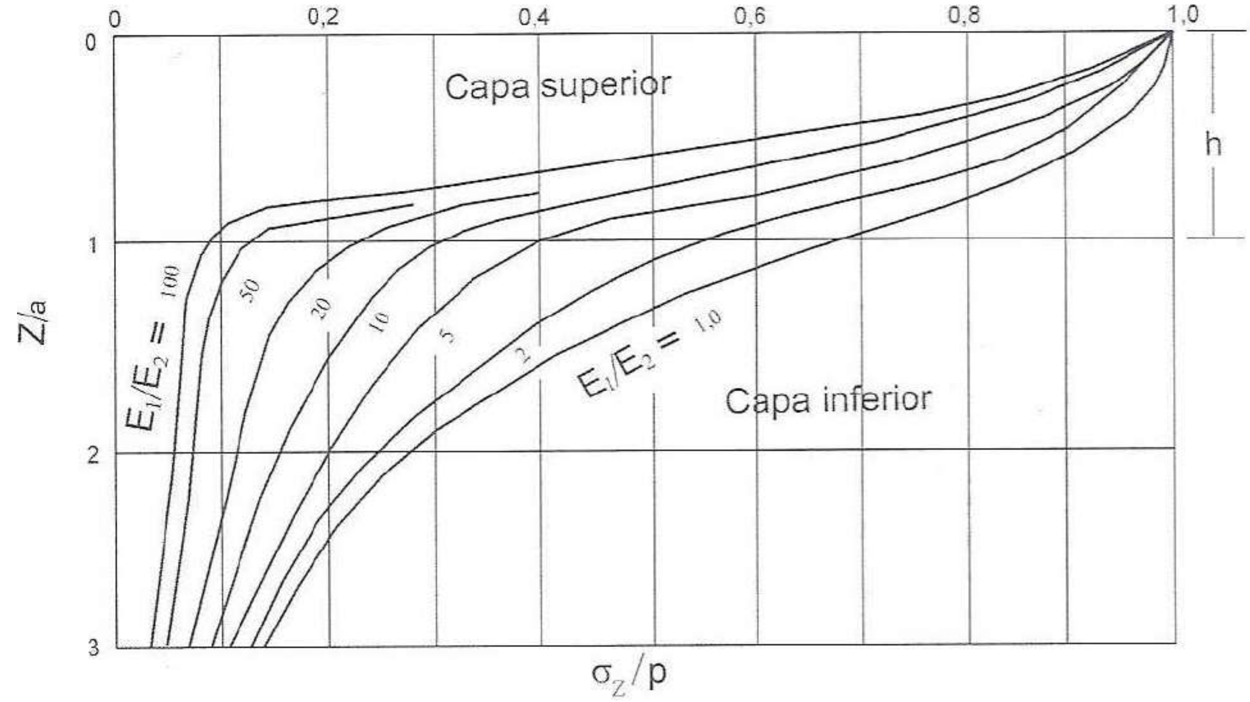


Figura 5.5 – Distribución de esfuerzos en un sistema bicapa según Burmister.
(**Fuente**: Instituto Colombiano de Producciones de Cemento – ICPC. Cipriano A. Londoño. *Funcionamiento de los pavimentos*, Medellín, 1989).

Factor de deflexión – interfaz para Bicapa (Huang)

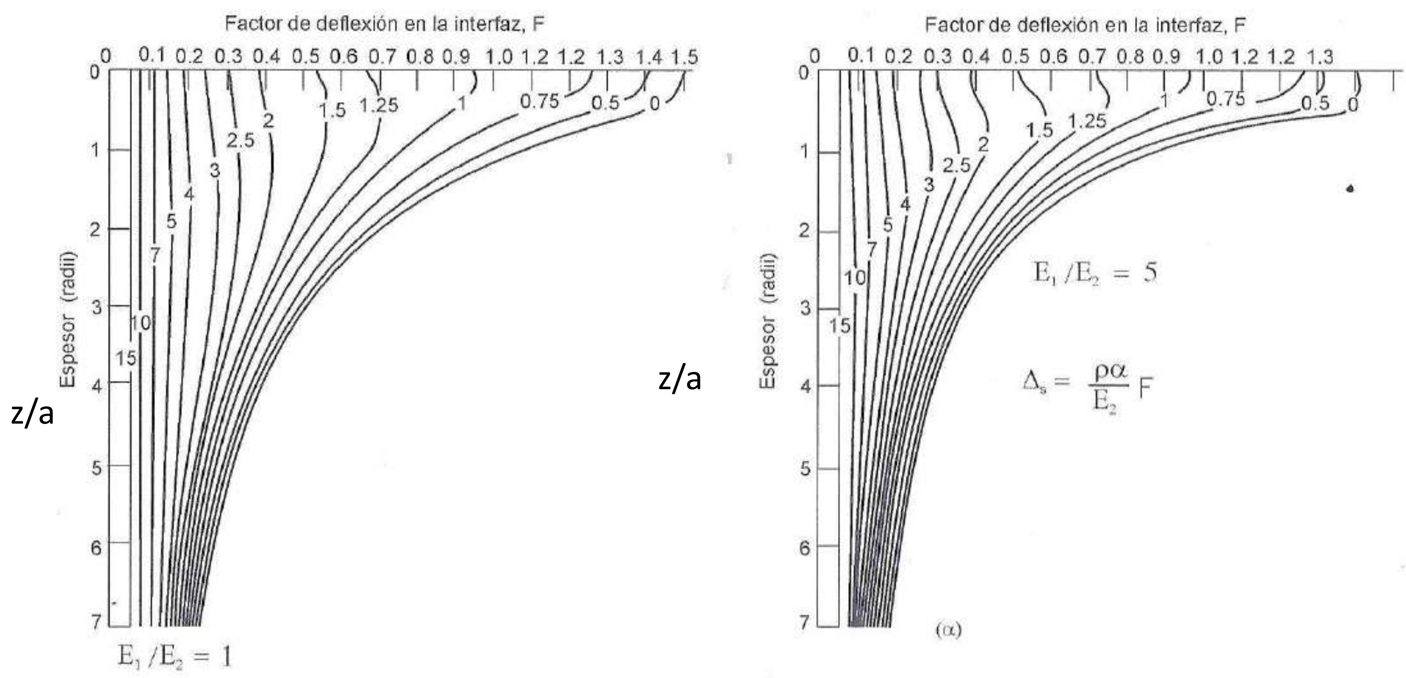
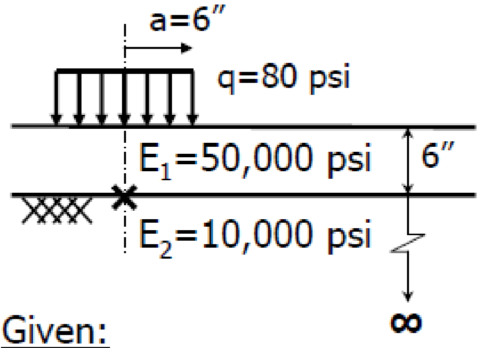


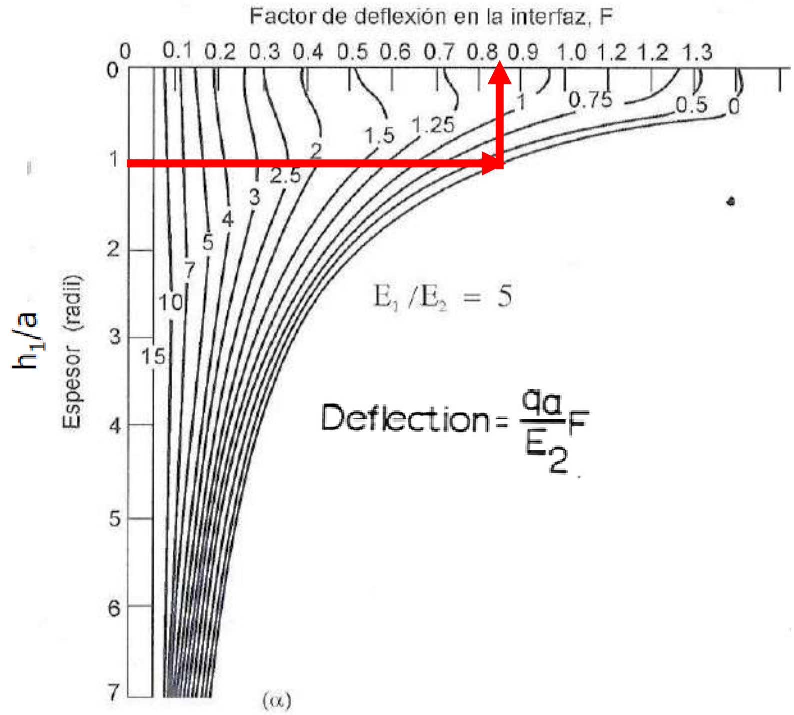
Figura 5.6 – Factor de deflexión de la interfaz para sistemas de dos capas (Huang)
(**Fuente**: Instituto Colombiano de Producciones de Cemento – ICPC. Cipriano A. Londoño. *Funcionamiento de los pavimentos*, Medellín, 1989)

Factor de deflexión – interfaz para Bicapa (Huang)

Interface Deflections Example

- For the same example as above





Given:
$h_1/a=6/6=1$;$r/a=0$
$E_1/E_2=5$

Find:
F=0.83

$$W=\frac{qa}{E_2}F=\frac{6(80)}{10000}0.83$$

$$W=0.0398'' \cong 40\cdot \text{mils}$$

TEORÍA ELÁSTICA vs REALIDAD

Las suposiciones en las cuales se basa la teoría elástica no se cumplen a cabalidad en los materiales y en las estructuras de los pavimentos.

TEORÍA ELÁSTICA	REALIDAD
•Carga estática •Continuidad en los materiales •Homogeneidad •Isotropía •Relación lineal esfuerzo - deformación •Deformaciones elásticas	•Carga dinámica •Discontinuidad en los materiales •No homogeneidad •Anisotropía •Relación compleja esfuerzo-deformación. •Deformaciones elásticas, plásticas, viscosas y viscoelásticas.

USO DE PROGRAMAS DE CALCULO:

- **Programa BISAR 3 (www.camineros.com/software.htm)**
- **Programa ELSYM 5 (www.camineros.com/software.htm)**
- **Programa Kenpave**